

Study of Nuclear Structure and Shell Model

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ABSTRACT

The shell model helps us comprehend the different characteristics of the nucleus and is quite effective in explaining many of these characteristics. Numerous factors determine the potential's nature within the nucleus, making it extremely difficult to characterize. As a result, different physicists have proposed different potentials. One of the most effective and often used models in nuclear physics, the shell model of the nucleus offers important insights into the composition and behavior of atomic nuclei. The goal of this research study is to provide a thorough analysis of the shell model, including its mathematical foundation, historical evolution, and applications to various nuclear processes. We will talk about this in this paper. investigation of the shell model and nuclear structure.

KEYWORDS: Nuclear Structure, Shell Model, Nuclear Physics, Atomic Nuclei, Magic Nuclei, Single-Nucleon Energy, Magic Numbers, Nuclear Density Functional Theory, Magnetic Moments, Stable Nuclei.

INTRODUCTION

The nuclear shell model, which is used in nuclear physics, atomic physics, and nuclear chemistry, models the structure of atomic nuclei in terms of energy levels by applying the Pauli exclusion principle. In 1932, Dmitri Ivanenko and E. Gapon proposed the first shell model. The model was created in 1949 as a result of independent work by a number of physicists, including Eugene Wigner, who shared the 1963 Nobel Prize in Physics with Maria Goeppert Mayer and J. Hans D. Jensen for their earlier groundbreaking work on atomic nuclei. [1-2]

The atomic shell model, which explains how electrons are arranged in an atom, and the nuclear shell model are similar in that a filled shell leads to greater stability. When protons and neutrons are added to a nucleus, there are times when the subsequent nucleon's binding energy is much lower than the preceding one. The shell model originated from the discovery that certain magic quantum numbers of nucleons (2, 8, 20, 28, 50, 82, and 126) are more closely bonded than the subsequent higher number. [3-4]

Protons and neutrons have separate shells from one another. As a result, there can be "doubly magic quantum nuclei," where both nucleon types are at a

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magic number, and "magic nuclei," where one nucleon type or the other is. The quest for the so-called island of stability is influenced by the top magic numbers, which are 126 and, hypothetically, 184 for neutrons but only 114 for protons due to variances in orbital filling. $Z = 40$, which provides the nuclear shell filling for the different elements, is one of the semi-magic numbers that have been discovered; 16 might possibly be a magic number. [5]

Mathematical Framework of the Shell Model: Single-Nucleon Energy Levels

According to the shell model, each nucleon-protons and neutrons-moves autonomously inside an effective potential that is produced by every other nucleon. Like electrons in an atom, nucleons occupy discrete energy levels in this potential, which is frequently represented by a center spherical well. [6]

The energy levels for each nucleon are described by quantum numbers (n), (l), and (j), where:

- (n) is the principal quantum number,
- (l) is the orbital angular momentum quantum number,
- (j) is the total angular momentum quantum number (which combines (l) and the spin of the nucleon).

Usually derived from the nuclear force, the effective potential has a distinct structure with a strong center attraction and a smaller repulsive component at short distances. [7]

Magic Numbers and Shell Structure

Similar to how electrons are arranged in atoms, the "magic numbers" represent entirely filled shells. These numbers represent extremely stable nucleon structures. For instance, the shell model predicts a stable configuration in a nucleus with eight protons because the first shell (for protons) fills completely, creating a closed-shell structure. These closed-shell nuclei have unique nuclear characteristics, including more binding energy and resistance to deformation, and they are especially stable. [8]

Pairing Interaction and Energy Gaps

The pairing interaction is essential to the shell model in addition to the fundamental shell structure. The attractive force between nucleons with parallel spins is what causes this contact. Closed-shell nuclei are made more stable by the pairing interaction, which creates energy gaps between filled and partially filled shells. The Pauli exclusion principle further limits how nucleons can be arranged within each shell, resulting in a degeneracy of energy levels and affecting nuclear characteristics such as spin, parity, and magnetic moment. [9]

Extensions of the Shell Model

Many improvements and expansions of the shell model have been created in order to get around these restrictions:

Interacting Shell Model (ISM): Nucleon-nucleon interactions are taken into account in this model, especially the residual nuclear force that results in nucleon correlations.

Collective Models: These models, such as Bohr and Mottelson's collective model, describe the nucleus as a system of interacting nucleons capable of both individual particle motions and collective motions like rotation and vibration.

Nuclear Density Functional Theory (DFT): This contemporary method incorporates the quantum mechanical description of the nuclear many-body system, including collective effects and shell structure, to expand on the shell model. [10]

Review of Literature:

In actuality, the shell model-which was first presented by Mayer [11] and Jensen [12] in 1949-is largely responsible for our knowledge of nuclear structure. Since then, the shell model has undergone substantial development in a number of directions. For example,

Talmi described an early phase as many-body physics in [13], in contrast to Mayer-Jensen's independent particle model. For example, Caurier et al. examined the subsequent advancements up to 2005 in [14] and up to this book. Since many more investigations are to be presented in other articles of the same collection, I would like to sketch emerging conceptions of nuclear structure based on recent shell-model experiments involving the author.

When examining the structure of nuclei, the shell model has proven to be the most basic perspective. Given that it was created in 1949 by Maria Goeppert-Mayer [15] and Hans Jensen and associates [16] at "three-score years and ten," it will not perish. It is predicated on the idea that particles move independently in a spherical mean field with a high spin-orbit interaction. All senior-year undergraduate students should be able to solve the quantum mechanical problem at the level of independent-particle motion in a harmonic-oscillator potential. It offers a broad vocabulary for discussing nuclear structure. The symmetries that result from the harmonic oscillator potential's "gift" to quantum physics mathematics are unparalleled in the quantum realm.

Objectives:

- To study the fundamental concepts of nuclear structure and the arrangement of nucleons inside the atomic nucleus.
- To understand the development and significance of the nuclear shell model in explaining nuclear properties.
- To examine the mathematical framework of the shell model, including quantum numbers, energy levels, and nucleon configurations.
- To provide a comprehensive review of recent advancements in the study of nuclear structure and shell model theory.

Research Methodology:

In this study, we give a comprehensive overview of the core ideas of the shell model, including its theoretical underpinnings, historical evolution, and important predictions. We go over its main uses in nuclear physics and look at the drawbacks that have spurred the creation of substitute models. Lastly, we discuss how contemporary improvements, such as the interacting shell model, nuclear density functional theory, and beyond-mean-field methods, have deepened our grasp of the forces regulating atomic nuclei and increased our comprehension of nuclear structure.

Result and Discussion:
Nuclear Shell Model

In essence, it describes how energy levels are distributed among various atom shells and nucleus atom shells. The energy level where particles with the same energy are found is called a shell. All nuclear particles in this model are paired one to one: protons

are paired with protons, and neutrons are paired with neutrons. When the number of neutrons or protons equals 2, 8, 20, 28, 50, 82, or 126, the paired neutrons and protons in nuclear energy levels are full. The most stable nuclei are displayed by these magic numbers. [17]

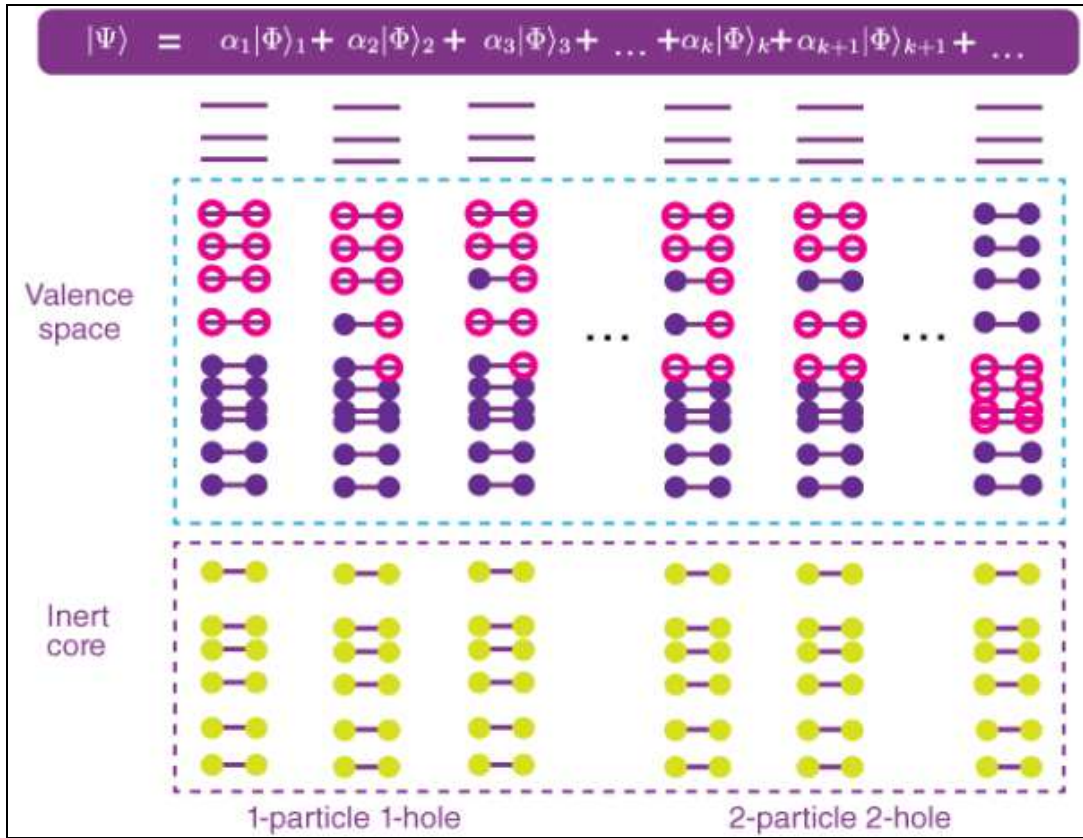


Figure 1. Nuclear Shell Model

Valence electrons are in charge of many elemental chemical properties, while unpaired electrons are in charge of a nucleus's characteristics. We can precisely anticipate nuclear parameters like angular momentum with the aid of the shell model. However, the shell model must be altered or substituted with other models, such as the collective model, liquid-drop model, and compound nucleus model, for nuclei that are in a highly unstable state. [18]

Shell Model of the Atom

It describes how various electrons are arranged according to energy levels around an atom's nucleus.

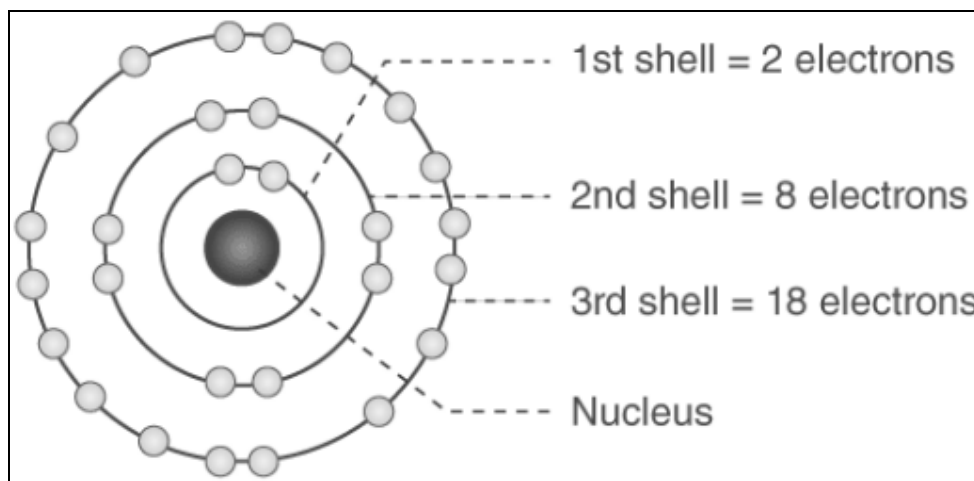


Figure 2: Shell Model of the Atom

Atomic Number = number of electrons in an atom

Atomic structure is explained by the atomic shell concept. In the area surrounding the positively charged nucleus, the negatively charged fundamental particles known as electrons are thought to reside in diffuse shells. The initial shell is the one that is closest to the nucleus. [19]

Nuclear Shell Structure

According to the empirical relations, the well should have a center elevation (Elsasser's wine bottle potential) for heavy nuclei and a central depression for light nuclei (N or $Z \leq 20$). The Coulomb repulsion between protons in heavy nuclei results in a particle density that varies from a minimum at the center to a maximum near the boundary, and consequently a similarly varying interaction energy. This is the qualitative physical explanation for these modifications. For light nuclei, the particle density and consequently the nuclear interaction energy are greatest at the center of the nucleus.

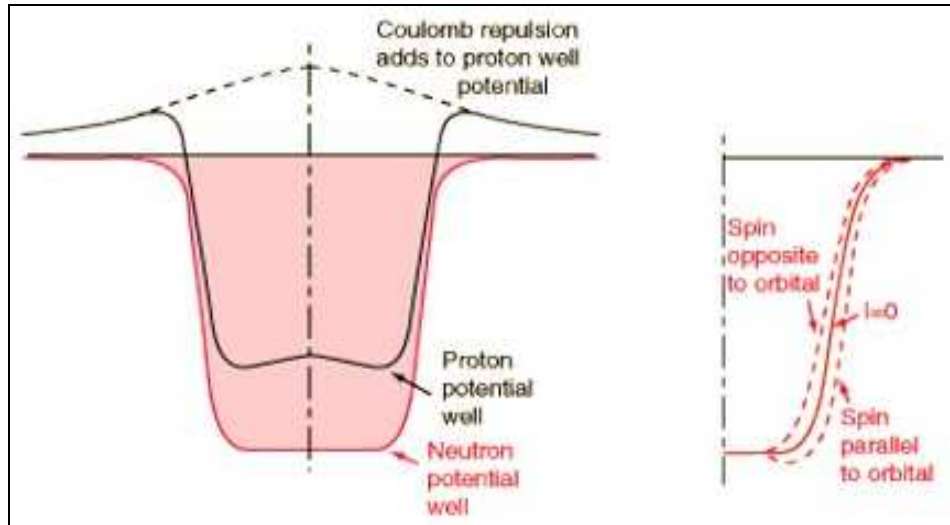


Figure 3: Nuclear Shell Structure

The hypothesis is generally supported by experimentally determined spins, magnetic moments, and quadrupole moments of nuclei with few particles outside of closed shells or absent from closed shells. For odd nuclei with one type of nucleon forming a closed shell \pm one particle, the agreement is especially good (and even better if the even group of nucleons constitutes a closed shell). In these circumstances, the state of the nucleus is often determined by the orbit of the odd particle, as provided by the shell model, and is verified to be consistent with the known spins and moments. In certain instances, the theory's crossing points between energy levels are fixed by reversing the rationale.

Although there is some debate on whether protons and neutrons are distinct substructures of a nucleus, it is generally accepted that nuclei are made up of these two elements. It's possible that a nucleus is simply a collection of the quarks that make up protons and neutrons. Some once believed that a nucleus was made up of alpha particle substructures (helium nuclei). Protons and neutrons are thought to make up a nucleus for the purposes of this analysis. [20]

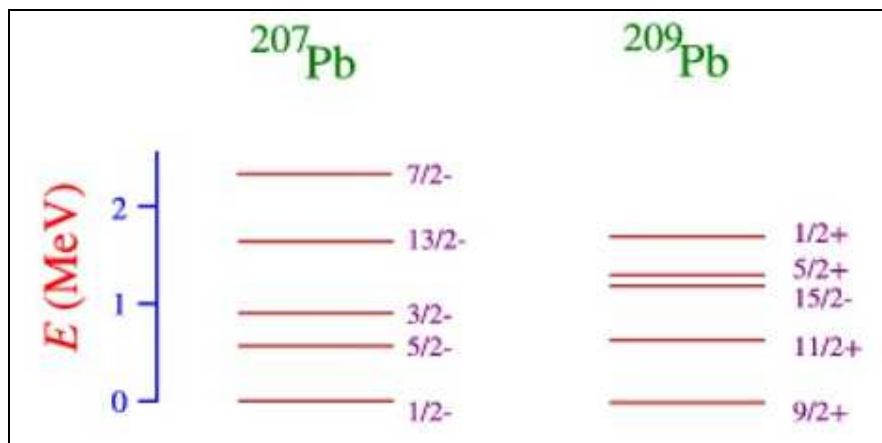


Figure 4: Atomic Physics

Nevertheless, in contrast to atomic physics, it might occasionally be less expensive to promote a nucleon from an inner closed shell into a high energy state as opposed to a nucleon from an outside shell. Furthermore, compared to atomic physics, nuclear physics is far more likely to have an excited state when many nucleons are elevated above their ground state.

Evidence for existence of magic number:

1. Mayer proposed in 1948 that there are particularly many nuclei in nature that have a magic number of nucleons.
2. If we plot the binding energy per nucleon versus the total number of nucleons A, the binding energy curve will exhibit a peak that corresponds to magic numbers, indicating a dramatic rise in binding energy.
3. ${}^4_2\text{He}$ and ${}^{16}_8\text{O}$ nuclei are more stable in comparison of nearby nucleus because such nuclei are doubly magic nuclei. In lighter nuclei binding energy curve shows peaks corresponding to such nuclei.
4. Number of stable isotopes per element

Elements	Number of stable isotopes
K_{16}	3
In_{49}	2
Ti_{22}	2
Ca_{20}	6
Sn_{50}	10
Pb_{82}	4
Sc_{16}	1
Sb_{51}	2
Bi_{83}	1
Ni_{28}	4
Be_4	4
S_{16}	6
Mo_{42}	7

This table suggests that the number of stable isotopes for Z=20, 28, 50, and 82 is significantly more than that of neighboring isotopes. Neutron counts for sulfur and molybdenum are N = 20 and N = 50, respectively.

5. Number of stable isotones

Number of neutrons (N)	Stable isotones
19	0
20	5
21	1
49	1
50	6
51	1
81	1
82	7
83	1

It may be inferred from this table that there are significantly more stable isotones for N=20, 50, and 82 than there are for nearby isotones.

6. For a given value of Z, alpha decay energies are a highly smooth function of A; however, at N = 126, they exhibit a severe discontinuity. It displays the N=126 magic character.

7. Decay product of radioactive series

Mass no.	Series	Parents	Half life	Stable end product
4n	Thorium	$^{232}_{90}\text{Th}$	1.39×10^{10}	$^{208}_{82}\text{Pb}$
4n+1	Neptunium	$^{237}_{93}\text{Np}$	2.25×10^6	$^{209}_{83}\text{Bi}$
4n+2	Uranium	$^{238}_{92}\text{U}$	4.51×10^9	$^{206}_{82}\text{Pb}$
4n+3	Actinium	$^{235}_{92}\text{U}$	7.07×10^8	$^{207}_{82}\text{Pb}$

From the table it may be concluded that $Z=82$ and $N=126$ are particularly stable nuclei.

8. Low probabilities of neutrons being captured by nuclei indicate weak binding of the initial nucleon outside a closed shell.

Number of neutrons	Cross-section in milli barn (mb) approx...
19	2
20	0.41
21	12
28	2
49	19
50	0.65
51	6.40
82	10
126	8

These nuclei's cross-sections are significantly smaller than those of nearby nuclei; this suggests that the compound nucleus created by neutron capture has a considerable level spacing. The cross-sections of calcium, tin, and lead are significantly smaller than those of their neighbors. [21]

The doubly magic nuclei when Z and N are both magic numbers like as ^4_2He , $^{16}_8\text{O}$, $^{40}_{20}\text{Ca}$ and $^{238}_{82}\text{Pb}$ are particular stable and such nuclei shows peaks in binding energy curve.

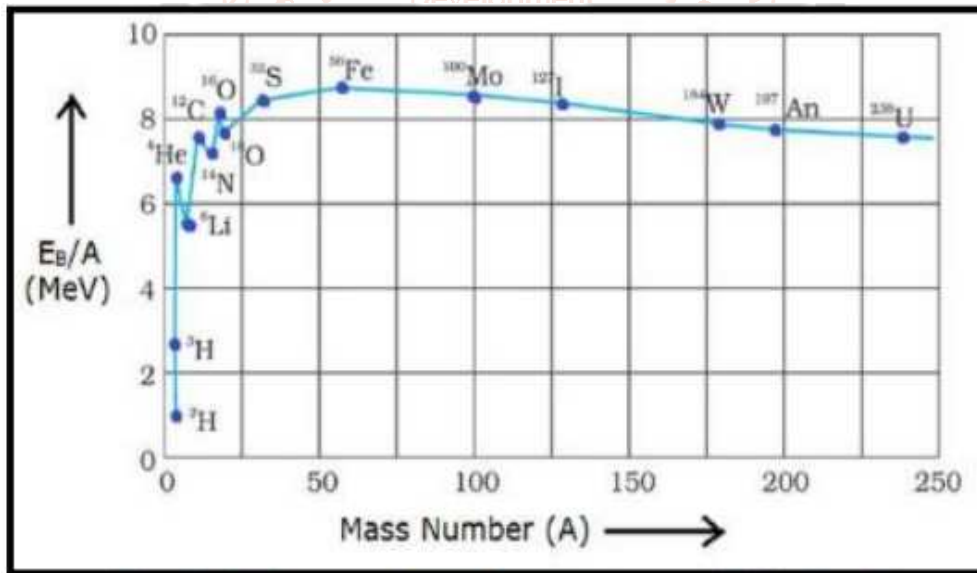


Figure 5: Binding Energy Curve

Conclusion:

Nuclear physics has benefited greatly from the shell model of the nucleus, which successfully explains many nuclear phenomena and offers an intuitive picture of the arrangement of nucleons within the nucleus. It has become a crucial framework in the study of nuclear structure due to its capacity to predict nuclear magic numbers, spin-parity

configurations, and nuclear stability. The internal organization and behavior of nucleons within the atomic nucleus can be better understood by studying nuclear structure and the shell model. Because it explains key nuclear phenomena like magic numbers, nuclear spin, parity, magnetic moments, and binding energy, the nuclear shell model has become one of the most popular models in nuclear physics. The validity

of this model has been significantly confirmed by the idea of closed shells and the existence of stable nuclei with magic numbers. All things considered, the shell model is still a key concept in nuclear physics and is still used extensively in contemporary nuclear research, such as the investigation of unusual nuclei, nuclear processes, and the hunt for new magic numbers and superheavy elements.

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