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Analysis of Reliability Characteristic of a System

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ABSTRACT

In this Paper, two system models are analyzed. The system have too dissimilar components working independently in parallel. In order to prolong the system operation preventive maintenance (inspection, minor repair) is provided in system at random epochs of time. There are two repair facilities to repair the components. Failure of one component changes the life time parameter of the other component. The failure times of the components are assumed to be exponentially distributed.

Keywords: *Reliability, Exponential distribution, Mean time to system Failure.*

Introduction

Researchers in reliability have shown keen interest in the analysis of two (or more) component parallel systems. Owing to their practical utility in the modern industry and technological set-ups of these systems, we come across with the system in which the failure in one component affects the failure rate of the other component. Taking this concept into consideration, in this paper, system model is analyzed. The system have too dissimilar components working independently in parallel. Several authors have analyzed various system models considering different repair policies. Gupta and Goel (1990) considering a two-dissimilar unit parallel system model assuming that a delay occurs due to administrative action in locating and getting the repairman available to the system. Recently, Gupta et. al. (2000) have analyzed a two-unit standby system with correlated failure, repair, random appearance and disappearance rate of repairman.

System Description and Assumptions

1) The system consists of a single unit having two dissimilar components say A and B are arranged in parallel.
2) Failure of one components affects the failure rate of other component due to increase in working stresses.
3) The system remains operative even if a single component operates.
4) There are two repair facilities to repair the components. When both the components are failed, they work independently on each other.
5) The repair rates are different, when both the repair facilities work on same component and when work on different components.
6) After repair, each component is as good as new.

Notations and States of the System

E	≡	Set of regenerative.
α	≡	constant failure rate of component A when B is also operating.
β	≡	constant failure rate of component B when A is also operating.
α'	≡	failure rate of component A when B has already failed.
β'	≡	failure rate of component B when A has already failed.
γ	≡	repair rate of component A when B is operating.
δ	≡	repair rate of component B when A is operating.
θ	≡	repair rate of component B when A is also under repair.
η	≡	repair rate of component A when B is also under repair.

The system will be one of the following states :

A_N	≡	component A is in normal mode and operative.
B_N	≡	component B is in normal mode
$S_0(A_NB_N)$	≡	Both the components A and B are in normal operative mode.
$S_1(A_RB_N)$	≡	Component A is in under repair and B is normal mode.
$S_2(A_NB_R)$	≡	Component A is in normal operative mode and component B is under repair.
$S_3(A_FB_F)$	≡	Both the components is in failed state.
$S_4(A_{NP}B_F)$	≡	Component A is in operative and under preventive maintenance.
$S_5(A_FB_{NP})$	≡	Component B is operative and under preventive maintenance.
$S_6(A_{NP}B_{NP})$	≡	Both the components are under preventive maintenance.

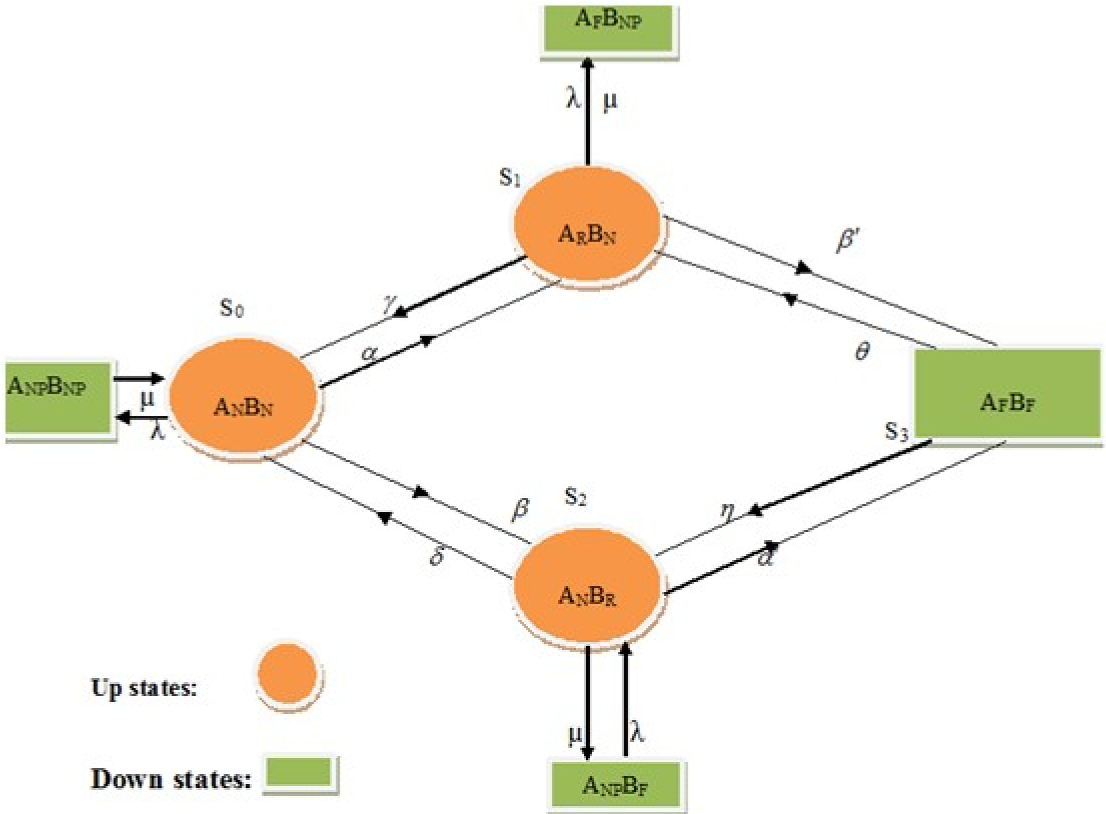

Fig : State Transition Diagram for the System

Transition Probabilities and Mean Sojourn Times.

Steady state transition probabilities p_{ij} are achieve from the conditional transition time cdf

$Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \leq t | X_n = i]$.

Expressed as integrals by simple probabilistic considerations the non-zero elements of $Q_{ij}(t)$ are :

$$Q_{01}(t) = \gamma \int_0^t e^{-(\nu+\beta)u} du = \frac{\left[1 - e^{-(\nu+\beta)t}\right]}{(\nu+\beta)}$$

$$Q_{02}(t) = \beta \int_0^t e^{-(\nu+\beta)u} du = \frac{\left[1 - e^{-(\nu+\beta)t}\right]}{(\nu+\beta)},$$

$$Q_{10}(t) = \nu \int_0^t e^{-(\alpha'+\nu+\lambda)u} du = \frac{\left[1 - e^{-(\alpha'+\nu+\lambda)t}\right]}{(\alpha'+\nu+\lambda)}$$

$$Q_{13}(t) = \alpha' \int_0^t e^{-(\alpha'+\nu+\lambda)u} du = \frac{\left[1 - e^{-(\alpha'+\nu+\lambda)t}\right]}{(\alpha'+\nu+\lambda)}$$

$$Q_{15}(t) = \lambda \int_0^t e^{-(\alpha'+\alpha+\lambda)u} du = \frac{\left[1 - e^{-(\alpha'+\alpha+\lambda)t}\right]}{(\alpha'+\alpha+\lambda)}$$

$$Q_{20}(t) = \delta \int_0^t e^{-(\delta+\beta'+\lambda)u} du = \frac{\left[1 - e^{-(\delta+\beta'+\lambda)t}\right]}{(\delta+\beta'+\lambda)}$$

$$Q_{24}(t)=\lambda\int_0^t e^{-(\delta+\beta'+\lambda)u}du=\frac{\left[1-e^{-(\delta+\beta'+\lambda)t}\right]}{(\delta+\beta'+\lambda)}$$

$$Q_{23}(t)=\beta'\int_0^t e^{-(\delta+\beta'+\lambda)u}du=\frac{\left[1-e^{-(\delta+\beta'+\lambda)t}\right]}{(\delta+\beta'+\lambda)}$$

$$Q_{31}(t)=\theta\int_0^t e^{-(\theta+\eta)u}du=\frac{\left[1-e^{-(\theta+\eta)t}\right]}{(\theta+\eta)}$$

$$Q_{51}(t)=\mu\int_0^t e^{-\mu u}du \quad , \quad Q_{60}(t)=\mu\int_0^t e^{-\mu u}du \quad .$$

By the subsequent relation

$$p_{ij}=Q_{ij}(\infty)=\lim_{t\to\infty}Q_{ij}(t).$$

The non-zero elements of p_{ij} are given below :

$$p_{01}=\gamma/(\gamma+\beta);\ p_{02}=\beta/(\gamma+\beta);\ p_{10}=\gamma/(\alpha'+\gamma+\lambda);$$

$$p_{13}=\alpha'/(\alpha'+\gamma+\lambda);\ p_{15}=\lambda/(\alpha'+\gamma+\lambda);\ p_{20}=\delta/(\delta+\lambda+\beta');$$

$$p_{24}=\lambda/(\lambda+\beta'+\delta);\ p_{23}=\beta'/(\delta+\lambda+\beta');\ p_{31}=\theta/(\theta+\eta);$$

$$p_{32}=\eta/(\theta+\eta);\ p_{42}=p_{51}=p_{60}=1.$$

It can be established that :

$$p_{01}+p_{02}=1;\ p_{10}+p_{13}+p_{15}=1;\ p_{20}+p_{23}+p_{24}=1;\ p_{30}+p_{31}=1;\ p_{42}=p_{51}=p_{60}=1.$$

The mean sojourn times □$_I$ in states S_i are :

$$\mu_0=\int_0^\infty e^{-(\alpha+\beta)t}dt=\frac{1}{(\alpha+\beta)},\ \mu_1=\int_0^\infty e^{-(\alpha+\gamma+\lambda)t}dt=\frac{1}{(\alpha+\gamma+\lambda)},$$

$$\mu_2=\int_0^\infty e^{-(\delta+\lambda+\beta')t}dt=\frac{1}{(\delta+\lambda+\beta')},\ \mu_3=\frac{\theta}{(\theta+\eta)}.$$

Taking Laplace-Stieltje's transform and using relation $m_{ij}=\tilde{Q}'_{ij}(s)\Big|_{s=0}$,

$$m_{01}=\alpha/(\alpha+\beta)^2,\ m_{10}=\gamma/(\alpha'+\gamma+\lambda)^2,$$

$$m_{13}=\alpha'/(\alpha'+\gamma+\lambda)^2,\ m_{15}=\lambda/(\alpha'+\gamma+\lambda)^2,\ m_{32}=\eta/(\theta+\eta)^2.$$

From above relations, it is evident that

$$m_{01}+m_{02}=\mu_0,\ m_{10}+m_{13}+m_{15}=\mu_1,\ m_{31}+m_{32}=\mu_3.$$

Reliability and Mean Time to System Failure

To determine $R_i(t)$, we assume that the failed state S_3, S_4 and S_5 of the system as absorbing. By probabilistic arguments as the following recursive relation are as follows:

$$R_0(t)=Z_0(t)+q_{01}(t)\copyright R_1(t)+q_{01}(t)\copyright R_2(t).$$
$$R_1(t)=Z_1(t)+q_{01}(t)\copyright R_0(t)$$
$$R_2(t)=Z_2(t)+q_{20}(t)\copyright R_0(t). \qquad (1\text{-}3)$$

where,

$$Z_0(t)=e^{-(\gamma+\beta')t},\qquad Z_1(t)=e^{-(\gamma+\alpha'+\lambda)t},\ Z_2(t)=e^{-(\delta+\beta'+\lambda)t}.$$

Taking Laplace Transform of relation (1-3), we can write solution of algebraic equations in the matrix form as follows –

$$\begin{bmatrix} R_0^* \\ R_1^* \\ R_2^* \end{bmatrix} = \begin{bmatrix} 1 & -q_{01}^* & -q_{02}^* \\ -q_{10}^* & 1 & 0 \\ -q_{20}^* & 0 & 1 \end{bmatrix} \begin{bmatrix} Z_0^* \\ Z_1^* \\ Z_2^* \end{bmatrix}. \quad (4)$$

For brevity, we have omitted the argument 's' from $q_{ij}^*(s)$, $Z_1^*(s)$ and $R_i^*(s)$. Computing the above matrix equation from $R_0^*(s)$, we get

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \quad (5)$$

where

$N_1(s) = Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^*$ and $D_1(s) = 1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^*$.

Taking the inverse Laplace transform of the (4), we can get the reliability of the system when initially it starts from state S_0. The mean time to system failure is given by –

$$E(T_0) = \int R_0(t)dt = \lim_{s \to 0} R_0^*(s) = \frac{N_1(0)}{D_1(0)}.$$

To determine $N_1(0)$ and $D_1(0)$, we can use the results

$$\lim_{s \to 0} Z_0^*(s) = \int Z_i(t)dt = \mu_i.$$

Therefore,

$$N_1(0) = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2.$$

Conclusion:

This paper describes an improvement over the Said and Sherbeny (2010).They analyzed a two-unit cold standby system with two stage repair and waiting time. In this paper we analyzed a two dis-similar component system. The system operates even if a single component operates. A single repair facility is available with some fixed probability for the repair of failed components. . Several measures of system effectiveness such as MTSF, A, B etc. are obtained by using regenerative point technique which shows that the proposed model is better than Said and Sherbeny.

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