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Fuzzy Strongly g*s Closed sets in Fuzzy Topological spaces

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ABSTRACT

In this paper, we have introduced and study the concept of strongly g*s closed set in Fuzzy topological spaces and discuss some of its properties.

Keywords and Phrases: *Fuzzy set, fuzzy topological spaces, fg-closed set, fuzzy strongly g*-closed set.*

1. Introduction

In 1965 Zadeh introduced the concept of fuzzy sets. Subsequently many researcher have been worked in this area and related areas which have applications in different field based on this concept. As generalizations of topological spaces Chang introduced the concept of fuzzy topological spaces in 1968. Veerakumar introduced and studied g*closed sets in general topology. Recently Parimezhagan and subramaniapillai introduced strongly g*-closed sets in topological spaces. In this paper, we have introduced and study the concept of strongly g*s – closed sets in fuzzy topological spaces and obtained the relationship among the various closed sets.

2. Preliminaries

Definition 2.1: [5] A subset of a topological space (X, τ) is called a fuzzy generalized closed set(briefly *f*g-closed) if cl(A) ≤ U whenever A ≤U and U is fuzzy open in X.

Definition 2.2: [9]A subset of a topological space (X, τ) is called a fuzzy generalized semi closed set(briefly *f*gs-closed) if scl(A) ≤ U whenever A ≤U and U is fuzzy open in X.

Definition 2.3: [12]A subset of a topological space (X, τ) is called a fuzzy semi generalized closed set(briefly *f*sg-closed) if cl(A) ≤ U whenever A ≤U and U is fuzzy semi- open in X.

Definition 2.4: [4]A subset of a topological space (X, τ) is called a fuzzy g*-closed set(briefly *fg**-closed) if cl(int(A))≤ U whenever A ≤U and U is fuzzy gopen in X.

Definition 2.5:[1]A subset of a topological space (X, τ) is called a fuzzy wg-closed set(briefly *fwg*-closed) if cl(int(A))≤ U whenever A ≤U and U is fuzzy open in X.

Definition 2.6: [8]A subset of a topological space (X, τ) is called a fuzzy strongly g*-closed set(briefly strongly *fg**-closed) if cl(A) ≤ U whenever A ≤U and U is fuzzy gopen in X.

3. Some Properties of Strongly Generalized closed sets with Respect to an Ideal

In this section we introduce the concept of fuzzy strongly g*s – closed sets and study some of its properties in fuzzy topological spaces.

Definition 3.1: Let (X, τ) be a fuzzy topological spaces. A fuzzy set A of (X, τ) is called fuzzy strongly g*s – closed set if int(scl(A)) ≤ U whenever A ≤U and U is fuzzy gs-open
in X

Theorem 3.2: Every fuzzy closed set is fuzzy strongly g*s – closed set in a fuzzy topological spaces (X, τ)

Proof: Let A be fuzzy closed set in X. Let H be a fgs open in X such that A≤ H. Since A is fuzzy closed, cl(A) = A. Therefore cl(A) ≤ H. Now int(scl(A)) ≤cl(A) ≤ H. Hence A is fuzzy strongly g*s – closed set in X.

The converse of the above theorem is not true in general as seen from the following example.

Example 3.3: Let X={a, b, c}. Fuzzy sets A and B are defined by A(a) = 0.7, A(b) = 0.3, A(c) = 0.5; B(a) = 0.7, B(b) = 0.3, B(c) = 0.5.Let τ={0, A, 1}. Then B is a fuzzy strongly g*s – closed set but it is not a fuzzy closed set in (X, τ)

Theorem 3.4: Union of two fuzzy strongly g*s closed set is fuzzy strongly g*s closed.

Proof: Let A and B be two fuzzy strongly g*s closed sets in X. Let U be a fgs open in X such that A [B _ U . Then A ≤ U and B ≤U. Since A and B are fuzzy strongly g*sclosed set, int(scl(A)) ≤ U and int(scl(B))≤U. Hence int(scl(A ∪ B)) = intscl(A) ∪ int(scl(B)) ≤ U .
Therefore A ∪ B is a fuzzy strongly g*s closed.

Theorem 3.5: Every fuzzy g – closed is fuzzy strongly g*s closed set in X.

Proof: Let A be fuzzyg-closed set in X. Let H be af open in X such that A≤ H. Since A is fuzzy g-closed,cl(A) ≤ H. Now int(scl(A)) ≤cl(A) ≤ H. Hence A is fuzzy strongly g*s – closed set in X.

The converse need not be true, as this may be seen from the following example.

Example 3.6: Let X = fa; bg . Fuzzy sets A and B are defned by A(a) = 0.3 , A(b) = 0.3 ;
B(a) = 0.5 , B(b) = 0.4 . Let τ = {0, B, 1}. Then A is a fuzzy strongly g*s closed set but it is
not a fg closed, since Cl(B) ≰B.

Theorem 3.7: Every fuzzy strongly g*s closed set is fuzzy gs closed sets in X
Proof: Let A be a fuzzy strongly g*s closed set in X. Let U be open set such that A ≤U .
Since every open set is gs open and A is fuzzy strongly g*s closed. We have int(scl(A)) ≤ U .
Therefore A is fgs closed in X .
The converse of the above theorem is not true in general as seen from the following example.

Example 3.8: Let X = {a, b}. Fuzzy sets A and B are defined by A(a) = 0.3 , A(b) = 0.3 ;
B(a) = 0.5 , B(b) = 0.4 . Let τ = {0, B, 1}. Then A is a fuzzy strongly g*s closed set but it is
not a fgs closed, since sCl(B)≰ B .

Theorem 3.9: Every fuzzy strongly g*s closed set is fuzzy sg closed sets in X
Proof: Let A be a fuzzy strongly g*s closed in X. Let U be semi-open set such that A . Since
every semi-open set is gs open and A is fuzzy strongly g*s closed. We have int(scl(A))≤U .
Therefore A is fsg closed in X .
The converse of the above theorem is not true in general as seen from the following example.

Example 3.10: Let X = {a, b}, τ= {0, A, B, D, 1} and fuzzy sets A , B ,D and H are
defined by A(a) = 0.2 , A(b) = 0.4 ; B(a) = 0.6 , B(b) = 0.7 ;D(a) = 0.4 ,D(b) = 0.6; H(a) = 0.4, H(b) = 0.5 . Then H is a fuzzy strongly g*s-closed set but it is not a sg closed, since
Cl(H)≰D where as H ≤D and D is fg open.

Theorem 3.11: Every fuzzy strongly g*s closed set is fuzzy wg -closed sets in X
Proof: Let A be a fuzzy strongly g*s closed in X. Let U be open set in X such that A≤U .
Since every open set is gs open and A is fuzzy strongly g*s closed. We have int(scl(A)) ≤ U .
Therefore A is fwg -closed in X.
The converse of the above theorem is not true in general as seen from the following example.

Example 3.12: Let X = {a, b}, τ= {0, A, B, D, 1} and fuzzy sets A , B ,D and H are
defined by A(a) = 0.4 , A(b) = 0.2; B(a) = 0.7, B(b) = 0.6 ;D(a) = 0.6 ,D(b) = 0.4 ;H(a) =0.5 ,H(b) = 0.4 . Then H is a fuzzy strongly g*s-closed set but it is not a fwg - closed, since
Cl(int(A)≰D where asH≤D and D is f open in X .

Theorem 3.13: Every fuzzy strongly g*s closed set is fuzzy g*closed sets in X
Proof: Let A be a fuzzy strongly g*s closed in X. Let U be gs open set in X such that A≤U .
Since U is gs open set and A is fuzzy strongly g*s closed. We have int(scl(A))≤ U . ThereforeA is fuzzy g*closed in X .

Example 3.14: Let X = {a, b}τ= {0, A,B 1} and fuzzy sets A, B and Hare defined by

A(a) = 0.3 , A(b) = 0.6 ; B(a) = 0.5 , B(b) = 0.6 ; H(a) = 0.5 , H(b) = 0.7 . Then H is
a strongly g*s closed set but it is not fuzzy g*closed.

Theorem 3.15: If A is a fuzzy strongly g*sclosed set in (X, τ) and A≤ B ≤sCl(Int(A)) , thenB is fuzzy strongly g*s closed in (X,τ).
Proof: Let A be a fuzzy strongly g*s closed in X. Let U be fuzzy open set such that A . Since every fuzzy open set is gs open and A is fuzzy strongly g*s closed. We have B≤int(scl(A))≤ U . Therefore B is fg*sclosed in X .

Theorem 3.16: A subset A of X is fuzzy strongly g*s closed set in (X, τ) if and only if
int(scl(A)) − A contains no nonempty gs closed sets in X
Proof: Suppose that F is a non-empty gs -closed subset of int(scl(A)−A) . Now F≤scl(A)−A. then F ≤int(scl(A))∩ $(1 - A)$. Therefore F ≤int(scl(A)) and $F \leq 1 - A$. Since F^C is fuzzy gs -open and A is fuzzy stronlgy g*s -closed, int(scl(A) −F^C) . That is F ≤int(scl(AC)) . Since $F^C$is gs -open set and A is strongly g*s closed, int(scl(A)) ≤F^C . (i.e) F ≤int(scl(A))int(scl(A))^C .Hence F≤int(scl(A)) ∩int(scl(A))^c= ∅. Thus int(scl(A)) − A contains no non emptygs -closed set.

Conversely, Assume that int(scl(A))−A contains no non empty gs -closed set. Let A≤U , U isgs -open. Suppose that int(scl(A)) is not contained in U. then int((scl(A)) ∩UC is a non-emptygs-closed set and contained in int(scl(A)−A). Which is contradiction. Therefore int(scl(A)≤ U and hence A is fuzzy strongly g*s-closed set.

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