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Analysis of 1 out of 2 Main Units and 2 out of 4 Subunits System

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ABSTRACT

Introduction of redundancy is one of the well-known methods by which the reliability of a system can be improved. A standby redundant system is one in which one operating unit is followed by spare units called standbys. On failure of operating unit, a standby unit is in working mode. In the present paper we investigate the probabilistic analysis of a two-main unit and four subunit system. The system remain operative if one main unit and two subunit are in working mode. System is failed when the main unit fails. A failed system is replaced by new one. The failed sub-unit are repairable.

Keywords: *Reliability, Availability, Mean time to system failure, busy period analysis, Markov renewal process*

Introduction

Two/three unit standby systems with two stages, that is, working or failed, have been widely discussed by a large number of researchers including (Mokaddis et. Al. 1997, Taneja, 2005, Goel et. Al. 2010). Introduction of redundancy is one of the well-known methods by which the reliability of a system can be improved. A standby redundant system is one in which one operating unit is followed by spare units called standbys. On failure of operating unit, a standby unit is in working mode. Ritu Mittal (2006) Analyzed stochastic analysis of a compound redundant system consisting three subsystems.

System Description and Assumption

Initially, the system starts operation with two main units and four subunits. If one main unit and two subunits work then system is in operative mode. After failure of both the main units, it is replaced by new – one. Failure time distributions of main unit and sub-units are arbitrary functions of time. A single repair facility is available to repair the sub-unit. The repair rate of the sub-unit is constant. The repair of sub-unit is as good as new. Using regenerative point technique with the Markov-renewal process, the following reliability characteristics have been obtained:

(i) Reliability of the system.
(ii) Steady state availability of the system.

Notation and States of the System

α ≡ Constant failure rate of main unit

β ≡ Constant failure rate of sub-unit.

θ ≡ repair rate of sub-unit.

f (.) = replacement time pdf of main unit.

S_0 (M_2S_4) ≡ 2 main units and 4 sub-units are in normal mode.

S_1(M_1S_4) ≡ 1 main unit and 4 sub-units are in working mode.

S_2(M_2S_3) ≡ 2 main units and 3 sub-units are in working mode.

S_3(M_2S_2) ≡ 2 main units and 4 sub-units are in working mode.

S_4(M_1S_3) ≡ 1 main unit and 3 sub-units are in working mode.

S_5(M_1S_2) ≡ 1 main unit and 2 sub-units are in working mode.

S_6(M_2S_1) ≡ System is in failed mode.

S_7(M_1S_1) ≡ System is in failed mode.

S_8(M_1S_4) ≡ System is in failed mode.

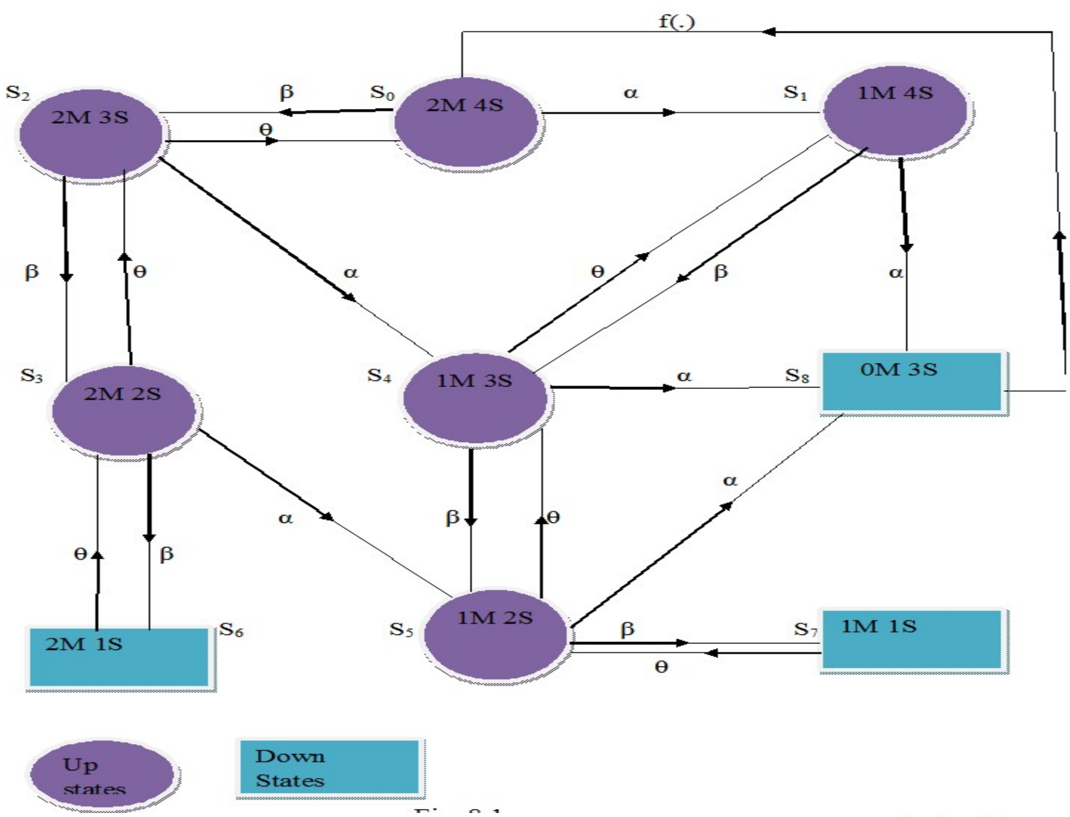

Figure 1: 1 out of 2 main units and 2 out of 4 subunits system

Analysis of Reliability and Mean Time to system Failure

Assuming that the failed states S_7 and S_8 to be the absorbing states and employing the arguments used in the theory of regenerative process, the following relation among $R_i(t)$ be obtained :

$$R_0(t) = Z_0(t) + q_{01}(t) © R_1(t) + q_{02}(t) © R_2(t)$$

$$R_1(t) = Z_1(t) + q_{14}(t) © R_4(t)$$

$R_2(t) = Z_2(t) + q_{20}(t) © R_0(t) + q_{24}(t) © R_4(t) + q_{23}(t) © R_3(t)$

$R_3(t) = Z_3(t) + q_{35}(t) © A_5(t) + q_{32}(t) © A_2(t)$

$R_4(t) = Z_4(t) + q_{41}(t) © A_1(t)$

$R_5(t) = Z_5(t) + q_{54}(t) © A_4(t)$ (1-5)

where,

$$Z_0(t) = e^{-(\alpha+\beta)t}, \qquad Z_1(t) = e^{-(\alpha+\beta)t},$$

$$Z_2(t) = e^{-(\theta+\delta+\beta)}, \qquad Z_3(t) = e^{-(\theta+\delta+\beta)},$$

$$Z_4(t) = e^{-(\theta+\alpha+\beta)}, \qquad Z_5(t) = e^{-(\theta+\alpha+\beta)}.$$

For an illustration, the equation for $R_0(t)$ is the sum of the following mutually exclusive contingencies :

(i) System sojourns in state S_0up to time t. The probability of this contingency is

$$e^{-(\alpha+\beta)t} = z_0(t), \text{(Say)}.$$

(ii) System transits from state S_0 to S_1 during time (u, u+du); u ≤ t and then starting from S_1, it survives for the remaining time duration (t – u). The probability of this event is

$$\int_0^t q_{01}(u)du R_1(t-u) = q_{01}(t) © R_1(t).$$

(iii) System transits from state S_0 to S_2 during time (u, u+du); u ≤ t and then starting from state S_2, it survives for the remaining time duration (t – u). The probability of this event is

$$\int_0^t q_{02}(u)du R_2(t-u) = q_{02}(t) © R_2(t).$$

Taking Laplace Transform of relations (1-5) and writing the solution of resulting set of algebraic equations in the matrix form as follows

$$\begin{bmatrix} R_0^* \\ R_1^* \\ R_2^* \\ R_3^* \\ R_4^* \\ R_5^* \end{bmatrix} = \begin{bmatrix} 1 & -q_{01}^* & -q_{02}^* & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -q_{14}^* & 0 \\ -q_{20}^* & 0 & 1 & -q_{23}^* & -q_{24}^* & 0 \\ 0 & 0 & -q_{32}^* & 1 & 0 & -q_{35}^* \\ 0 & -q_{41}^* & 0 & 0 & 1 & -q_{45}^* \\ 0 & 0 & 0 & 0 & -q_{54}^* & 1 \end{bmatrix}^{-1} \begin{bmatrix} Z_0^* \\ Z_1^* \\ Z_2^* \\ Z_3^* \\ Z_4^* \\ Z_5^* \end{bmatrix}. \qquad (6)$$

The argument's' has been omitted from $R_i^*(s)$, $q_{ij}^*(s)$ and $Z_1^*(s)$ for brevity. Computing the above matrix equation for $R_0^*(s)$, we get

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \tag{7}$$

Where

$$N_1(s) = \begin{vmatrix} Z_0^* & -q_{01}^* & -q_{02}^* & 0 & 0 & 0 \\ Z_1^* & 1 & 0 & 0 & -q_{14}^* & 0 \\ Z_2^* & 0 & 0 & -q_{23}^* & 0 & 0 \\ Z_3^* & 0 & 0 & 1 & 0 & -q_{35}^* \\ Z_4^* & -q_{41}^* & 0 & 0 & 1 & 0 \\ Z_5^* & 0 & 0 & 0 & -q_{54}^* & 1 \end{vmatrix}$$

$$D_1(s) = \begin{vmatrix} 1 & -q_{01}^* & -q_{02}^* & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -q_{14}^* & 0 \\ 0 & 0 & 0 & -q_{23}^* & 0 & 0 \\ 0 & 0 & -q_{32}^* & 1 & 0 & -q_{35}^* \\ 0 & -q_{41}^* & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -q_{54}^* & 1 \end{vmatrix}$$

Taking the inverse Laplace transforms of the expression (7), we can get the reliability of the system when system initially starts from state S_0.

To get the MTSF, we have a well-known formula

$$E(T_0) = \int R_0(t)dt = \lim_{S \to 0} R_0^*(S)$$

So that, using $q_{ij}^*(0) = p_{ij}$ and $Z_i^*(0) = \mu_i$ we get

$$E(T_0) = \frac{\mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{01}p_{23}\mu_3 + p_{02}p_{14}\mu_4}{1 - p_{01}p_{14}p_{24} - p_{02}p_{35}p_{32}p_{54}}. \tag{8}$$

Availability Analysis

A_i (t) as the probability that the system is operative at epoch t due to both the units respectively. When initially system starts from state $S_i \in E$. Using the similar probabilistic arguments, point wise availability are satisfied the recursive relations as follows:

$$A_0(t) = Z_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t)$$

$$A_1(t) = Z_1(t) + q_{14}(t) © A_4(t) + q_{18}(t) © A_8(t)$$

$$A_6(t) = q_{63}(t)©A_3(t)$$

$$A_7(t) = q_{75}(t)©A_5(t)$$

$$A_8(t) = q_{80}(t)©A_0(t). \tag{1-9}$$

Taking Laplace transform (L.T.) of the set of relations (1-9), the solution for $A_i^*(s)$ can be written in the matrix form as follows-

Taking Laplace transform of relation (1-9), and solving for , $A_0^*(s)$, we get

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)}$$

Where $N_2(s) = \left[q_{01}^*\left(q_{23}^*q_{14}^* + q_{32}^*q_{41}^*\right) + q_{02}^*\left(q_{23}^*q_{14}^* + q_{32}^*q_{41}^*\right)\right]Z_1^*$

$$+q_{80}^*\left(q_{45}^*q_{32}^* + q_{54}^*q_{01}^*\right)Z_3^*. \tag{11}$$

and

$$D_2(s) = \left(1-q_{23}^*q_{24}^*\right)\left(1-q_{01}^*q_{10}^*\right) - q_{01}^*\left(q_{23}^* + q_{14}^*q_{53}^*\right)q_{35}^*$$

$$-q_{02}^*\left(q_{24}^*q_{36}^* + q_{63}^*q_{75}^*\right)q_{32}^* - q_{03}^*q_{36}^*\left(q_{14}^*q_{75}^* + q_{14}^*\right)q_{30}^*$$

$$-q_{02}^*\left(q_{24}^* + q_{63}^*q_{75}^*\right) - q_{24}^*\left(q_{75}^* + q_{58}^*\right)q_{45}^*. \tag{12}$$

The steady state availability of the system due to both the units is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to\infty} A_0^*(s)$$

$$= \lim_{s\to 0} S\frac{N_1(s)}{D_1(s)}.$$

Now,

$$D_2(0) = \left(1-p_{23}p_{24}\right)\left(1-p_{10}p_{01}\right) - p_{01}\left(p_{23} + p_{14}p_{53}\right)p_{35}$$

$$-p_{02}\left(p_{24}p_{36} + p_{63}p_{75}\right)p_{32} - p_{03}p_{36}\left(p_{14}p_{75} + p_{14}\right)$$

$$-p_{02}\left(p_{24} + p_{63}p_{75}\right) - p_{24}\left(p_{75} + p_{58}\right)p_{32}$$

$$= p_{20}\left(1-p_{01}p_{10}\right) - p_{01}p_{23} - p_{01}p_{14}p_{53} - p_{02}p_{24}p_{36}p_{32}$$

$$-p_{03}p_{75}p_{32} - p_{03}p_{36}p_{14}p_{15} - p_{03}p_{36}p_{14} - p_{02}p_{24}$$

$$-p_{02}p_{03}p_{75}-p_{24}p_{75}p_{45}-p_{24}p_{58}p_{45}$$

$= 0.$

Hence, by L. Hospital's rule

$$A_0 = \lim_{s\to 0}\frac{N_1(s)}{D_1^{'}(s)} = \frac{N_1(0)}{D_1(0)}$$

where,

$$N_1(0) = \left[p_{01}\left(p_{23}p_{24}+p_{32}p_{41}\right)+p_{02}\left(p_{23}p_{24}+p_{32}p_{41}\right)\right]\mu_1$$
$$+\left(p_{45}p_{32}+p_{54}p_{01}\right)\mu_3$$

In order to obtain $D_1^{'}(0)$ we collect the coefficient of m_{ij} in $D_1^{'}(s)\Big|_{s=0}$ as follows -

Coefficient of $m_{01} = p_{23}p_{24}+p_{32}p_{41}$

Coefficient of $m_{02} = p_{23}p_{24}+p_{32}p_{41}$

Coefficient of $m_{23} = p_{01}p_{24}+p_{02}p_{24}$

$$= p_{24}\left(p_{01}+p_{02}\right) = p_{24}$$

Coefficient of $m_{24} = p_{01}p_{23}+p_{02}p_{23}$

$$= p_{23}\left(p_{01}+p_{02}\right) = p_{23}$$

Coefficient of $m_{32} = p_{01}p_{41}+p_{02}p_{41}+p_{54}$

$$=\left(p_{01}+p_{02}\right)p_{41}+p_{54}$$
$$= p_{41}+p_{54}$$

Coefficient of $m_{41} = p_{02}p_{32}+p_{01}p_{32}$

$$=\left(p_{01}+p_{02}\right)p_{32}$$
$$= p_{32}$$

Coefficient of $m_{45} = p_{32}$

Coefficient of $m_{54} = p_{01}.$

Busy Period Analysis

Let $B_i(t)$ be the probabilities that the repairman is busy in the repair of sub-unit at time t when system initially starts from regenerative state S_i using some the probabilistic arguments in respect to the definition of $B_i^{'}(t)$, we have the following recursive relation –

$$B_0^0(t) = q_{01}(t)©B_1(t)+q_{02}(t)©B_2(t)$$
$$B_1^k(t) = q_{14}(t)©B_4(t)+q_{18}(t)©B_8(t)$$
$$B_2(t) = W_2+q_{20}(t)©B_0(t)+q_{24}(t)©B_4(t)+q_{23}(t)©B_3(t)$$
$$B_3(t) = q_{35}(t)©B_5(t)+q_{36}(t)©B_6(t)+q_{32}(t)©B_2(t)$$
$$B_4(t) = W_4+q_{41}(t)©B_1(t)+q_{48}(t)©B_8(t)+q_{45}(t)©B_5(t)$$

$$B_5(t) = W_5 + q_{58}(t)©B_8(t) + q_{57}(t)©B_7(t) + q_{54}(t)©B_4(t)$$
$$B_6(t) = W_6 + q_{63}(t)©B_3(t)$$
$$B_7(t) = W_7 + q_{75}(t)©B_5(t)$$
$$B_8(t) = q_{80}(t)©B_0(t) \tag{1-9}$$

where

$$W_2 = e^{-(\alpha+\beta+\theta)t}; \qquad W_4 = e^{-(\alpha+\beta+\theta)t};$$
$$W_5 = e^{-(\alpha+\beta+\theta)}; \qquad W_6 = e^{\theta t}; \quad W_7 = e^{-\theta t}.$$

For an illustration, $B_2^{'}(t)$ is the sum of the following mutually exclusive contingencies –

(i) The repairman remains busy in state S_2 continuously up to time t in the repair of sub-unit. The probability of this event is -

$$e^{-(\alpha+\beta+\theta)t} = W_2(t).$$

(ii) System transists from state S_2 to S_0 during time (u, u+du); u ≤ t and then repairman remains busy in the repair of epoch t starting from state S_0 at epoch u. The probability of this contingency is –

$$\int_0^t q_{20}(u)du B_4^{'}(t-u) = q_{20}(t)©B_0(t)$$

(iii) System transits from state S_2 to S_4 during the time (u, u+du); u ≤ t and then starting from state S_4 at epoch u, the repairman may be observed to be busy in the repair at epoch t. The probability of this contingency is

$$\int_0^t q_{24}(u)du B_4^{'}(t-u) = q_{24}(t)©B_4^{'}(t)$$

(iv) Systemtransits from state S_2 to S_3 during the time (u, u+du); u ≤ t and then starting from state S_3 at epoch u, the repairman may be observed to be busy in the repair at epoch t. The probability of this contingency is

$$\int_0^t q_{23}(u)du B_3^{'}(t-u) = q_{23}(t)©B_3^{'}(t)$$

Taking Laplace transform of relations (1-9) and solving the resulting set of the algebraic equation for $B_0^*(s)$, we get

$$B_0^*(s) = \frac{N_3(s)}{D_2(s)}. \tag{10}$$

$$N_3(s) = \left(1 - q_{24}^* q_{41}^* q_{14}^* q_{57}^*\right) q_{35}^* W_3^* + \left(q_{01}^* + q_{02}^* q_{23}^* q_{58}^*\right)\left(q_{14}^* W_4^* + q_{18}^* q_{63}^*\right)$$
$$+ \left(q_{02}^* + q_{14}^* q_{45}^*\right)\left(q_{25}^* W_5^* + q_{24}^* q_{75}^* W_3^* + q_{80}^* q_{75}^*\right).$$

In the long run, the expected fraction of the time for which the repairman is busy in the repair of the system, is given by

$$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} sB_0^*(s)$$
$$= \frac{N_3(0)}{D_2^{'}(0)}$$

where

$$N_3(0) = \left(1 - p_{24}p_{41}p_{14}p_{57}\right)p_{35}\mu_3 + \left(p_{01} + p_{02}p_{23}p_{58}\right)\left(p_{14}\mu_4 + p_{18}\right)$$
$$+ \left(p_{02} + p_{14}\mu_5\right)\left(p_{25}\mu_5 + p_{24}p_{75}\mu_3 + p_{75}\right).$$

Conclusion

This paper describes an improvement over the Gupta, R, Goel, C .K. and Tomar, (2010) Analysis of a two unit standby system with correlated failure and repair and random appearance and disappearance of repairman. Using regenerative point technique reliability analysis, availability analysis, busy period analysis which shows that the proposed model is better than Gupta, R, Goel, C .K. and Tomar, (2010)because we investigate the probabilistic analysis of a two-main unit and four subunit system. On failure of operating unit, a standby unit is in working mode. The system remains operative if one main unit and two subunit are in working mode.

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