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Chaos Suppression and Practical Stabilization of Uncertain Generalized Duffing-Holmes Control Systems with Unknown Actuator Nonlinearity

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ABSTRACT

In this paper, the concept of practical stabilization for nonlinear systems is introduced and the practical stabilization of uncertain generalized Duffing-Holmes control systems with unknown actuator nonlinearity is explored. Based on the time-domain approach with differential inequalities, a single control is presented such that the practical stabilization for a class of uncertain generalized Duffing-Holmes systems with unknown actuator nonlinearity can be achieved. Moreover, both of the guaranteed exponential convergence rate and convergence radius can be correctly calculated Finally, some numerical simulations are given to demonstrate the feasibility and effectiveness of the obtained results.

Keywords: *Practical synchronization, chaotic system, uncertain generalized Duffing-Holmes systems, unknown actuator nonlinearity, chaos suppression*

1. INTRODUCTION

Chaotic dynamic systems have been extensively investigated in past decades; see, for instance, [1-13] and the references therein. Very often, chaos in many dynamic systems is an origin of instability and an origin of the generation of oscillation. Generally speaking, the robust stabilization of uncertain dynamic systems with a single controller is in general not as simple as that with multiple controllers. In the past decades, various methodologies in robust control of chaotic system have been offered, such as control approach, adaptive control approach, variable structure control approach, backstepping control approach, adaptive sliding mode control approach, and others.

In this paper, the concept of practical stabilizability for uncertain dynamic systems is introduced and the practical stabilizability of uncertain generalized Duffing-Holmes control systems with unknown actuator nonlinearity will be studied. Using the time-domain approach with differential inequality, a single control will be designed such that the practical stabilization can be achieved for a class of uncertain generalized Duffing-Holmes systems with unknown actuator nonlinearity. Not only the convergence radius and guaranteed exponential convergence rate can be arbitrarily pre-specified, but also the unknown actuator nonlinearity and mixed uncertainties can be simultaneously overcome by the proposed single control. Several numerical simulations will also be provided to illustrate the use of the main results.

The rest of paper is organized as follows. The problem formulation and main results are presented in Section 2. Numerical simulations are given in Section 3 to illustrate the effectiveness of the developed results. Finally, some conclusions are drawn in Section 4.

2. PROBLEM FORMULATION AND MAIN RESULTS

Nomenclature

$\Re^n$ the n-dimensional real space

$|a|$ the modulus of a complex number a

I the unit matrix

A^T the transport of the matrix A

$\|x\|$ the Euclidean norm of the vector $x \in \Re^n$

$\lambda_{\min}(P)$ the minimum eigenvalue of the matrix P with real eigenvalues

$\sigma(A)$ the spectrum of the matrix A

$P > 0$ the matrix P is a symmetric positive definite matrix

Before presenting the problem formulation, let us introduce a lemma which will be used in the proof of the main theorem.

Lemma 1

If a continuously differentiable real function $s(t)$ satisfies the inequality

$$\dot{s}(t) \le a - 2\alpha\, s(t), \quad \forall\, t \ge 0,$$

with $a > 0$ and $\alpha > 0$, then

$$s(t) \le \left[s(0) - \frac{a}{2\alpha} \right] \cdot e^{-2\alpha t} + \frac{a}{2\alpha}, \quad \forall\, t \ge 0.$$

Proof. It is easy to see that

$$e^{2\alpha t} \cdot \dot{s}(t) + e^{2\alpha t} \cdot 2\alpha\, s(t) = \frac{d}{dt}\left[e^{2\alpha t} \cdot s(t)\right] \le e^{2\alpha t} \cdot a, \quad \forall\, t \ge 0.$$

It results that

$$\int_0^t \frac{d}{dt}\left[e^{2\alpha t} \cdot s(t)\right] dt = e^{2\alpha t} \cdot s(t) - s(0) \le \int_0^t e^{2\alpha t} \cdot a\, dt = \frac{a}{2\alpha}\left(e^{2\alpha t} - 1\right), \quad \forall\, t \ge 0.$$

Consequently, one has

$$s(t) \le \left[s(0) - \frac{a}{2\alpha} \right] \cdot e^{-2\alpha t} + \frac{a}{2\alpha}, \quad \forall\, t \ge 0.$$

This completes the proof. □

In this paper, we consider the following uncertain generalized Duffing-Holmes control systems with uncertain actuator nonlinearity described as

$$\dot{x}_1 = x_2, \tag{1a}$$

$$\dot{x}_2 = q_1 x_1 + q_2 x_2 + q_3 x_1^3 + q_4 x_1^5 + q_5 \cos(w_1 t) + \Delta f(x_1, x_2) + \Delta\phi(u), \ \forall\, t \ge 0. \tag{1b}$$

where $x = [x_1 \quad x_2]^T \in \Re^{2\times 1}$ is the state vector, $u \in \Re$ is the input, $\Delta f(x_1, x_2)$ represents the mixed uncertainties (unmodeled dynamics, parameter mismatchings, external excitations, and disturbance), and $\Delta\phi(u)$ represents the uncertain actuator nonlinearity. For the existence of the solutions of (1), we assume that the unknown terms $\Delta f(x_1, x_2)$ and $\Delta\phi(u)$ are all continuous functions. It is well known that the system (1) without any uncertainties (i.e., $\Delta f(x_1, x_2) = \Delta\phi(u) = 0$) displays chaotic behavior for certain values of the parameters [1]. In this paper, the concept of practical stabilization will be introduced. Motivated by time-domain approach with differential inequality, a suitable control strategy will be established. Our goal is to design a single control such that the practical stabilization for a class of uncertain nonlinear systems of (1) can be achieved.

Throughout this paper, the following assumption is made:

(A1) There exist continuous function $f(x_1, x_2) \ge 0$ and positive number r_1 such that, for all arguments,

$$|\Delta f(x_1, x_2)| \le f(x_1, x_2),$$

$$u \cdot \Delta\phi(u) \ge r_1 u^2.$$

Remark 1

Generally speaking, if the uncertain actuator nonlinearity satisfies

$$r_1 \cdot u^2 \le u \cdot \Delta\phi(u) \le r_2 \cdot u^2, \quad \forall u \in \Re,$$

we often ascribe r_2 as the gain border and r_1 as the gain reduction endurance. Thus, we know $r_2 = \infty$ from (A1); see Figure 1.

A precise definition of the practical stabilization is given as follows, which will be used in subsequent main results.

Definition 1

The uncertain system (1) is said to realize the practical stabilization, provided that, for any $\alpha > 0$ and $\varepsilon > 0$, there exist a control $u := (\alpha, \varepsilon)$ such that the state trajectory satisfies

$$\|x(t)\| \le \kappa \cdot e^{-\alpha t} + \varepsilon, \quad \forall t \ge 0,$$

for some $\kappa > 0$. In this case, the positive number ε is called the convergence radius and the positive number α is called the exponential convergence rate. Namely, the practical stabilization means that the states of system (1) can converge to the equilibrium point at $x = 0$, with any pre-specified convergence radius and exponential convergence rate. Obviously, a control system, having small convergence radius and large exponential convergence rate, has better steady-state response and transient response.

Now we present the main result for the practical stabilization of uncertain systems (1) via time-domain approach with differential inequalities.

Theorem 1

The uncertain systems (1) with (A1) realize the practical stabilization under the following control

$$u(t) = -r(t) \cdot (p_3 x_1 + p_2 x_2), \quad (2)$$

$$r(t) := \frac{h^2(t)}{r_1 \cdot h(t) \cdot |p_3 x_1 + p_2 x_2| + \alpha \cdot r_1 \cdot \varepsilon^2 \cdot \lambda_{\min}(P)}, \quad (3)$$

$$\begin{aligned} h(t) := &\left| q_1 x_1 + q_2 x_2 + q_3 x_1^3 + q_4 x_1^5 \right. \\ &+ q_5 \cos(w_1 t) + (\alpha+1)^2 x_1 \\ &\left. + 2(\alpha+1) x_2 \right| + f, \end{aligned} \quad (4)$$

where $P := \begin{bmatrix} p_1 & p_3 \\ p_3 & p_2 \end{bmatrix} > 0$ is the unique solution to the following Lyapunov equation

$$\begin{aligned} &\begin{bmatrix} \alpha & 1 \\ -(\alpha+1)^2 & -\alpha-2 \end{bmatrix}^T P \\ &+ P \begin{bmatrix} \alpha & 1 \\ -(\alpha+1)^2 & -\alpha-2 \end{bmatrix} = -2I, \end{aligned} \quad (5)$$

with $\alpha > 0$. In this case, the guaranteed convergence radius and exponential convergence rate are ε and α, respectively.

Proof. From (1), the state equation can be represented as

$$\begin{aligned} \dot{x} = &\begin{bmatrix} 0 & 1 \\ -(\alpha+1)^2 & -2\alpha-2 \end{bmatrix} x \\ &+ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \left[\Delta\phi + \Delta f + q_1 x_1 + q_2 x_2 + q_3 x_1^3 + q_4 x_1^5 \right. \\ &\left. + q_5 \cos(w_1 t) + (\alpha+1)^2 x_1 + (2\alpha+2) x_2 \right] \\ = &Ax + B \cdot \left[\Delta\phi + \Delta f + q_1 x_1 + q_2 x_2 + q_3 x_1^3 \right. \\ &+ q_4 x_1^5 + q_5 \cos(w_1 t) + (\alpha+1)^2 x_1 \\ &\left. + (2\alpha+2) x_2 \right], \forall t \ge 0, \end{aligned}$$

where $A := \begin{bmatrix} 0 & 1 \\ -(\alpha+1)^2 & -2\alpha-2 \end{bmatrix}$ and $B := \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Clearly, one has $\sigma(A) = \{-\alpha-1\}$. This implies $A + \alpha I$ is Hurwitz and the Lyapunov equation of (5) has the unique posive definite solution *P*. Let

$$V(x(t)) = x^T(t) P x(t). \quad (6)$$

The time derivative of $V(x(t))$ along the trajectories of the system (1) with (2)-(6) is given by

$$\begin{aligned} \dot{V}(x(t)) = &x^T \left[A^T P + PA \right] x + 2x^T PB \\ &\cdot \left[\Delta\phi + \Delta f + q_1 x_1 + q_2 x_2 + q_3 x_1^3 + q_4 x_1^5 \right. \\ &\left. + q_5 \cos(w_1 t) + (\alpha+1)^2 x_1 + (2\alpha+2) x_2 \right] \end{aligned}$$

$$\le x^T[-2\alpha P-2I]\,x+2x^TPB\cdot\Delta\phi+2\left|x^TPB\right|\left[\left|q_1x_1+q_2x_2+q_3x_1^3+q_4x_1^5+q_5\cos(w_1t)+(\alpha+1)^2x_1+(2\alpha+2)x_2\right|+f\right]$$

$$\le -2\alpha x^TPx+2x^TPB\cdot\Delta\phi+2h(t)\cdot\left|x^TPB\right|$$

$$=-2\alpha x^TPx-\left(\frac{2}{r}\right)\cdot(u\cdot\Delta\phi)+2h\cdot\left|x^TPB\right|$$

$$=-2\alpha V-\left(\frac{2}{r}\right)\cdot\left(r_1u^2\right)+2h\cdot\left|x^TPB\right|$$

$$=-2\alpha V-2r_1\cdot r\cdot\left|x^TPB\right|^2+2h\cdot\left|x^TPB\right|$$

$$=-2\alpha V-2r_1\cdot r\cdot\left|p_3x_1+p_2x_2\right|^2+2h\cdot\left|p_3x_1+p_2x_2\right|$$

$$=-2\alpha V-\frac{2r_1\cdot\left|p_3x_1+p_2x_2\right|^2\cdot h^2}{r_1\cdot h\cdot\left|p_3x_1+p_2x_2\right|+\alpha\cdot r_1\cdot\varepsilon^2\cdot\lambda_{\min}(P)}+2h\cdot\left|p_3x_1+p_2x_2\right|$$

$$=-2\alpha V+\frac{2h\cdot\left|p_3x_1+p_2x_2\right|\cdot\alpha\cdot r_1\cdot\varepsilon^2\cdot\lambda_{\min}(P)}{r_1\cdot h\cdot\left|p_3x_1+p_2x_2\right|+\alpha\cdot r_1\cdot\varepsilon^2\cdot\lambda_{\min}(P)}$$

$$=-2\alpha V+\left(\frac{2}{r_1}\right)\left[\frac{r_1\cdot h\cdot\left|p_3x_1+p_2x_2\right|\cdot\alpha\cdot r_1\cdot\varepsilon^2\cdot\lambda_{\min}(P)}{r_1\cdot h\cdot\left|p_3x_1+p_2x_2\right|+\alpha\cdot r_1\cdot\varepsilon^2\cdot\lambda_{\min}(P)}\right],\ \forall\,t\ge 0.$$

By the inequality

$$x\left(\frac{yz}{y+z}\right)\le xz,\quad \forall\,x>0,\ \ y\ge 0,\ \text{and}\ z>0.$$

It can be deduced that

$$\dot V(x(t))\le -2\alpha V(x(t))+2\alpha\cdot\varepsilon^2\cdot\lambda_{\min}(P),\quad \forall\,t\ge 0. \tag{7}$$

Thus, from Lemma 1, (6), and (7), it can be readily obtained that

$$\lambda_{\min}(P)\|x(t)\|^2\le V(x(t))\le e^{-2\alpha t}\left[V(x(0))-\varepsilon^2\lambda_{\min}(P)\right]+\varepsilon^2\lambda_{\min}(P),\quad \forall\,t\ge 0.$$

Consequently, we conclude that

$$\|x(t)\|\le\sqrt{e^{-2\alpha t}\left[\frac{V(x(0))+\varepsilon^2\lambda_{\min}(P)}{\lambda_{\min}(P)}\right]+\varepsilon^2}\le\sqrt{e^{-2\alpha t}\left[\frac{V(x(0))+\varepsilon^2\lambda_{\min}(P)}{\lambda_{\min}(P)}\right]}+\sqrt{\varepsilon^2}$$
$$=\sqrt{\left[\frac{V(x(0))+\varepsilon^2\lambda_{\min}(P)}{\lambda_{\min}(P)}\right]}\cdot e^{-\alpha t}+\varepsilon,\quad \forall\,t\ge 0.$$

This completes the proof. □

3. NUMERICAL SIMULATIONS

Consider the following uncertain generalized Duffing-Holmes control systems with unknown actuator nonlinearity described as

$$\dot x_1=x_2, \tag{8a}$$

$$\dot x_2=q_1x_1+q_2x_2+q_3x_1^3+q_4x_1^5+q_5\cos(w_1t)+\Delta f(x_1,x_2)+\Delta\phi(u),\ \forall\,t\ge 0, \tag{8b}$$

where

$q_1=1$, $q_2=-0.25$, $q_3=-1$, $q_4=0$, $q_5=0.3$, $w_1=1$,

$\Delta f(x_1,x_2)=\Delta a\cdot x_1^2$, $\quad\Delta\phi(u)=\Delta b\cdot u+\Delta c\cdot u^3$,

$-1\le\Delta a\le 1$, $\quad 1\le\Delta b\le 3$, $\quad\Delta c\ge 0$.

Our objective, in this example, is to design a feedback control such that the uncertain systems (8) realize the practical stabilization with the exponential convergence rate $\alpha=3$ and the convergence radius $\varepsilon=0.1$. The condition (A1) is clearly satisfied if we let $f(x_1,x_2)=x_1^2$, $r_1=1$. From (5) with $\alpha=3$, we have

$$P=\begin{bmatrix}141 & 26.5\\ 26.5 & 5.5\end{bmatrix},\quad \lambda_{\min}(P)=0.5,$$

$p_2=5.5$, $\quad p_3=26.5$.

From (4), it can be readily obtained that

$$h(t):=\left|x_1-0.25x_2-x_1^3+0.3\cos t+16x_1+8x_2\right|+x_1^2.$$

From (3) with $\varepsilon = 0.1$, one has $r(t) := \dfrac{h^2(t)}{h(t)\cdot|26.5x_1 + 5.5x_2| + 0.015}$. Finally, the desired control, given by (2), can now be calculated as

$$u(t) = -r(t)\cdot(26.5x_1 + 5.5x_2). \qquad (9)$$

Consequently, by Theorem 1, we conclude that system (8) with the control (9) is practically stable, with the exponential convergence rate $\alpha = 3$ and the guaranteed convergence radius $\varepsilon = 0.1$. The typical state trajectories of uncontrolled systems and controlled systems are depicted in Figure 2 and Figure 3, respectively. Besides, the time response of the control signal is depicted in Figure 4. From the foregoing simulations results, it is seen that the uncertain dynamic systems of (8) achieves the practical stabilization under the control law of (9).

4. CONCLUSION

In this paper, the concept of practical stabilization for nonlinear systems has been introduced and the practical stabilization of uncertain generalized Duffing-Holmes control systems with unknown actuator nonlinearity has been studied. Based on the time-domain approach with differential inequalities, a single control has been presented such that the practical stabilization for a class of uncertain generalized Duffing-Holmes systems with unknown actuator nonlinearity can be achieved. Not only the unknown actuator nonlinearity and mixed uncertainties can be simultaneously overcome by the proposed control, but also the convergence radius and guaranteed exponential convergence rate can be arbitrarily pre-specified. Finally, some numerical simulations have been offered to show the feasibility and effectiveness of the obtained results.

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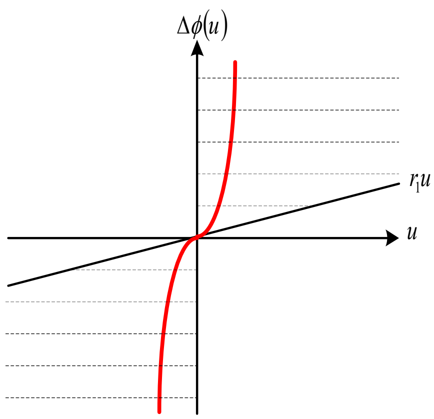

Figure 1: Uncertain actuator nonlinearity.

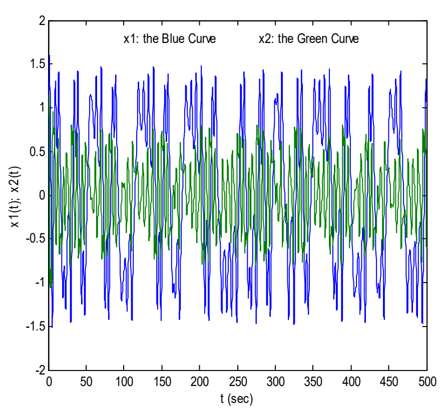

Figure 2: Typical state trajectories of the uncontrolled system of (8).

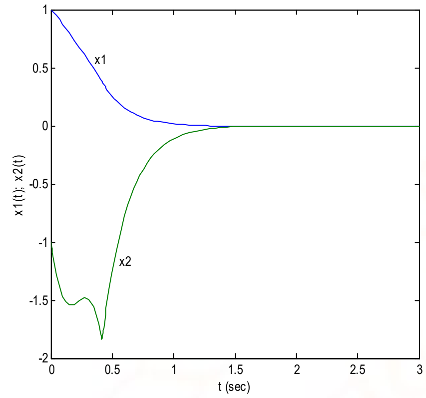

Figure 3: Typical state trajectories of the feedback-controlled system of (8) with (9).