

# Modelling and Simulation of Braking System in Automobile

Sylvester Emeka Abonyi<sup>1</sup>, Emmanuel Emeka Ezendiokwelu<sup>2</sup>, James Akal-karali Obineche<sup>3</sup>

<sup>1,2</sup>Electrical Engineering Department, Nnamdi Azikiwe University, Awka, Anambra State, Nigeria

<sup>3</sup>Technical and Vocational Education Department, Nnamdi Azikiwe University, Awka, Anambra State, Nigeria

## ABSTRACT

In recent years vehicles are fitted with antilock brake systems to enhance braking system in automobiles and prevent wheel lock during braking. To achieve this controller are needed controlled torque that will maintain the optimum wheel slip ratio and coefficient of friction, which is the vehicle speed in relation to wheel rotation. A model of a quarter vehicle was developed and differential equation of motion was formulated and was applied to linear and angular velocity. Control strategies of Proportional Feedback Control, Proportional Integral Feedback Control, Proportional Integral Derivative Feedback Control, were applied to determine the effectiveness of maintaining desired slip ratio and coefficient of friction. Mathlab/Simulink software were used to simulate the response of the different braking strategies. Results shows that by maintaining the slip ratio and coefficient of friction values, the braking time was reduce 3.6 seconds to 2.3 seconds for a distance of 45m.

**KEYWORDS:** ABS, control feedback, slip ratio, coefficient of friction

**How to cite this paper:** Sylvester Emeka Abonyi | Emmanuel Emeka Ezendiokwelu | James Akal-karali Obineche "Modelling and Simulation of Braking System in Automobile"

Published in International Journal of Trend in Scientific Research and Development (ijtsrd), ISSN: 2456-6470, Volume-9 | Issue-3, June 2025, pp.7-14,

URL: [www.ijtsrd.com/papers/ijtsrd79778.pdf](http://www.ijtsrd.com/papers/ijtsrd79778.pdf)



Copyright © 2025 by author (s) and International Journal of Trend in Scientific Research and Development Journal. This is an Open Access article distributed under the terms of the Creative Commons Attribution License (CC BY 4.0) (<http://creativecommons.org/licenses/by/4.0>)



## 1. INTRODUCTION

Since the development of the first motor driven vehicle in 1769 and the occurrence of first driving accident in 1770, engineers were determined to reduce driving accidents and improve the safety of vehicles [1]. By the 1960s, high end automobiles were fitted with rear-only ABS, and with the rapid progress of microcomputers and electronics technology, the trend exploded in the 1980s. Today, all wheel ABS can be found on the majority of our model vehicles and even on select motorcycles [2-6]. It important to note that wheels will slip and lockup during severe braking or when braking on a wet or icy road surface [7-8]. Such breaking results in a long stopping distance and sometimes the vehicle will lose steering stability [9-11].

However, Anti-lock brake systems (ABS) prevent brakes from locking during braking. Under normal braking conditions the driver controls the brakes. However, during severe braking or on slippery roadways, when the driver causes the wheels to approach lockup, the antilock system takes over. ABS modulate the brake line pressure independent of the

pedal force, to bring the wheel speed back to the slip level range that is necessary for optimal braking performance. An antilock system consists of wheel speed sensors, a hydraulic modulator, and an electronic control unit. The ABS have a feedback control system that modulates the brake pressure in response to wheel deceleration and wheel angular velocity to prevent the controlled wheel from locking. The system shuts down when the vehicle speed is below a pre-set threshold.

The main difficulty in the design of ABS control arises from the strong nonlinearity and uncertainty of the problem. It is difficult and, in many cases, impossible to solve this problem by using classical linear, frequency domain methods [12]. ABS systems are designed around system hydraulics, sensors and control electronics. These systems are dependent on each other and the different system components are interchangeable with minor changes in the controller software [13].

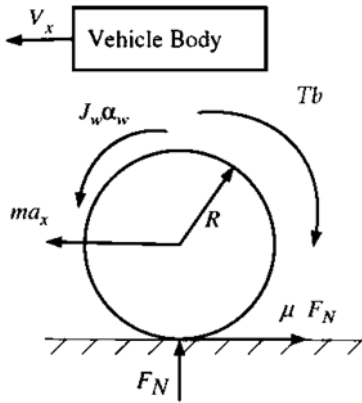
Antilock systems have effectively improved the stopping distances, improve stability, and also

improve steerability during braking. Meanwhile the technology of ABS is also applied in traction control system (TCS) and vehicle dynamic stability control (VDSC) [14].

## 2. VEHICLE DYNAMICS

The ABS controller must deal with the brake dynamics and the wheel dynamics as a whole plant [15]. Wheel lockup is undesirable since it prolongs the stopping distance and causes the loss of direction control [16,17].

A model capturing the essential features of the vehicle system has to be employed for the controller design. The design considered here belongs to a quarter vehicle model as shown in Fig 2.1. This model has been already used to design the controller for ABS.



**Fig. 1 Vehicle Model**

The longitudinal velocity of the vehicle and the rotational speed of the wheel constitute the degrees of freedom for this model. The governing two equations for the motions of the vehicle model are as follows:

For braking force balance in longitudinal direction (vehicle)

$$m a_x = \mu F_N \rightarrow m \frac{dV_x}{dt} = \mu F_N \quad (1)$$

Summing torque at wheel center

$$J_w \alpha_w - \mu r F_N - T_b \quad (2)$$

For convenience a slip ratio is defined according to:

$$\lambda = \frac{V_x - \omega r}{V_x} \quad (3)$$

Differentiating on both sides with respect to time (t), we get

$$\dot{\lambda} = \frac{\dot{V}_x(1 - \lambda) - \dot{\omega} r}{V_x} \quad (4)$$

Where;  $V_x$  = linear velocity of vehicle  $a_x$  = linear acceleration of vehicle  $\omega$  = rotational speed of wheel  $\alpha_w$  = angular acceleration of wheel  $T_b$  = braking

torque,  $\lambda$  = slip ratio  $\mu$  = friction coefficient  $r$  = radius of tire  $m$  = mass of the model.

The slip ratio is dependent on the input torque  $u$  and the vehicle velocity  $V_x$  during breaking.

$$x_1 = s_x \quad (5)$$

$$x_2 = V_x \quad (7)$$

$$x_3 = \lambda, \quad (8)$$

where  $s_x$  is the stopping distance

The state space equations are

$$\dot{x}_1 = x_2 \quad (9)$$

$$\dot{x}_2 = \frac{-\mu F_N}{m} \quad (10)$$

$$\dot{x}_3 = \frac{-\mu F_N}{x_2} \left( \frac{1 - x_3}{m} + \frac{r^2}{J_w} \right) + \frac{r}{J_w x_2} T_b \quad (11)$$

The slip model in equation (5-8) gives value of coefficient of friction as a function of linear velocity and hence slip ratio.

$$\mu(\lambda, V_x) = [C_1(1 - e^{-C_2 \lambda}) - C_3 \lambda] e^{-C_4 V_x} \quad (12a)$$

$$\lambda = \frac{[C_1(1 - e^{-C_2 \lambda}) - C_3 \lambda] e^{-C_4 V_x}}{\mu V_x} \quad (12b)$$

Where

$C_1$  is the maximum value of friction curve;  $C_2$  the friction curve shapes;

$C_3$  the friction curve difference between the maximum value and the value at  $\lambda = 1$ ; and

$C_4$  is the wetness characteristic value. It lies in the range 0.02–0.04s/m. Where for dry asphalt as the surface condition, above parameters are

From studies it is shown that the effective coefficient of friction between the tire and the road has an optimum value at particular value of wheel slip ratio. Road type determine the value. Hence, frictional coefficient value is optimum for most roads, when the wheel slip ratio is approximately 0.2 and worst when the wheel slip ratio is 1. That is at the point when wheel is locked. Hence, what ABS does is that it tends to control or regulate the wheel slip ratio from 1 to 0.2 to maximize the frictional coefficient ( $\mu$ ) for any given road surface.

## 3. Feedback Control System

In order to regulate the brake system, there is need for a sensor that will monitor the slip ratio and sends signal to the controller which regulates the brake pressure modulator to maintain the correct slip ratio with reference to slip ratio value. Fig 2 shows a diagram of feedback control system.

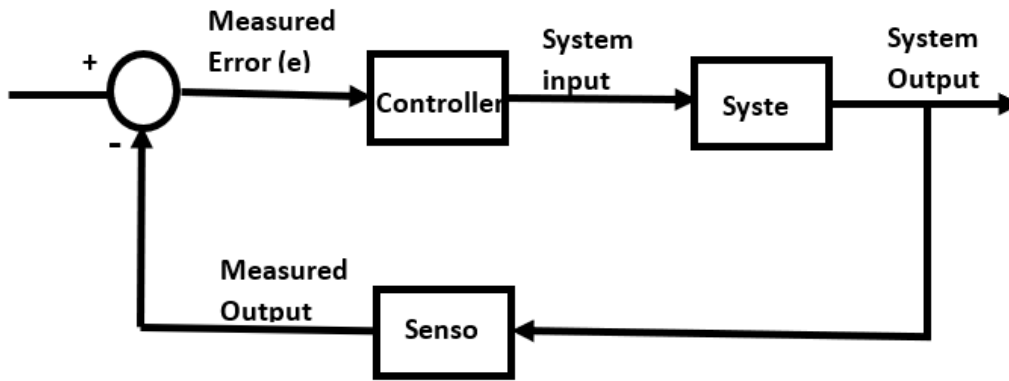


Fig 2 Diagram of Feedback Control System

The feedback control was performed under different conditions. These includes;

### 3.1. Proportional Feedback Control.

Where input to the system which is in proportion to measured error (e) between the output and the set-point.

Here control torque is

$$\mu = k_p e \quad (13)$$

Where  $k_p$  is known as the proportional gain of the controller.

$$e = \lambda_d - \lambda \quad (14)$$

where  $\lambda_d$  is desired output and is actual output measured by sensor Proportional Derivative Feedback Control (PD-type)

This controller feeds both the error with constant gain ( $k_p$ ) and the differentiation of error with constant gain ( $k_d$ ) to the system in order to maintain the output of system at the set point.

$$\mu = k_p e + k_d \frac{de}{dt} \quad (15)$$

Where  $k_d$  is differential gain of the controller

### 3.2. Proportional Integral Feedback Control.

Here input to the system is the error with constant gain ( $k_p$ ) plus the integral of error with constant gain ( $k_i$ ) to control the system output.

$$\mu = k_p e + k_i \int e dt \quad (16)$$

Where  $K_i$  is integral gain of the controller.

### 3.3 Proportional Integral Derivative Feedback Control.

Here the input is the addition of error with constant gain ( $k_p$ ), integral of error with constant gain ( $k_i$ ), and differential of error with constant gain ( $k_d$ ).

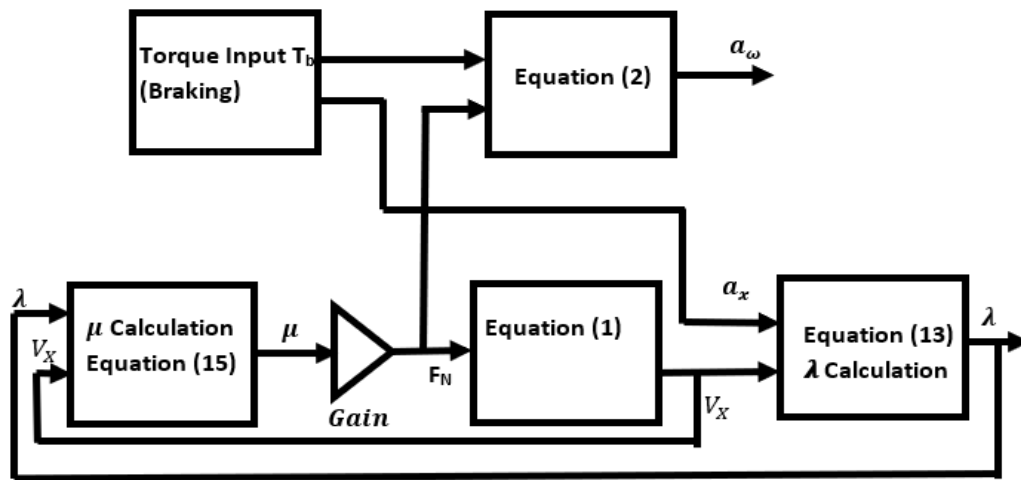
$$\mu = k_p e + k_i \int e dt + k_d \frac{de}{dt} \quad (17)$$

Simulink software was used to determine the ABS braking system with control system having the following parameters;  $k_p$ ,  $k_i$  and  $k_d$ .

## 4. SIMULUATION MODELS

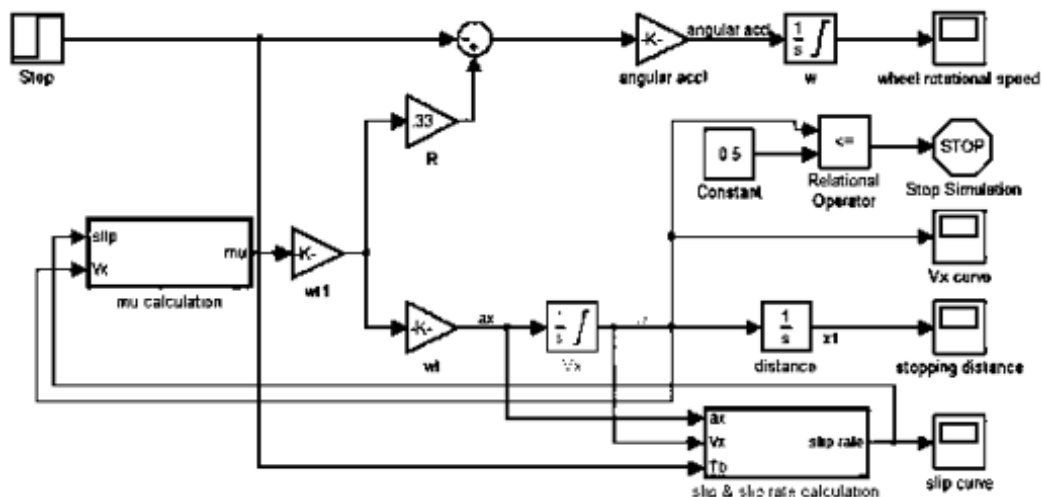
Simulink model of quarter vehicle

The quarter vehicle of ABS is modelled in Simulink environment incorporating the dynamic equations. Fig 4 shows the diagram of the Simulink model representing vehicle dynamics during straight line braking.



**Fig 3 Diagram Representing Dynamics of Equations**

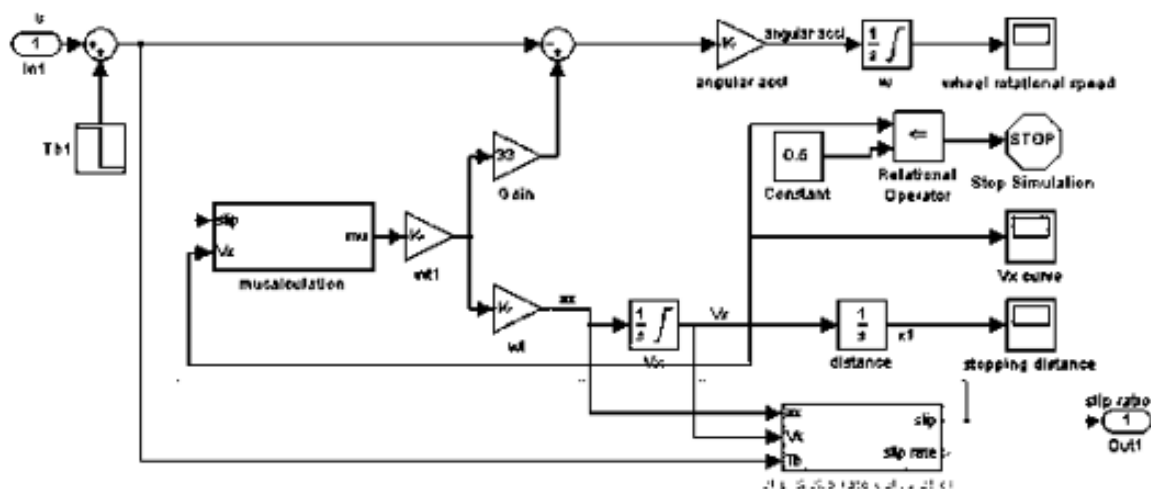
Simulink model of quarter vehicle during straight line braking without feedback control was obtained by the combination of the slip ratio and coefficient of friction.



**Fig 4 Vehicle Model Without Feedback Control**

The above feedback is modified to include SUM box between input terminal, which is control torque  $u$  and brake torque  $T_b$ .

Hence total torque input  $T$  to wheel is  $T = \mu + T_b$ . This results in figure 5



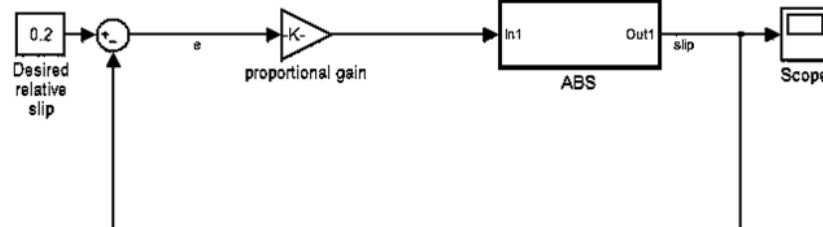
**Fig 5 Modified Vehicle Model Without Feedback Control**

With figure 4 and 5 we can have a simplified feedback control system with input and output as shown in figure 6.



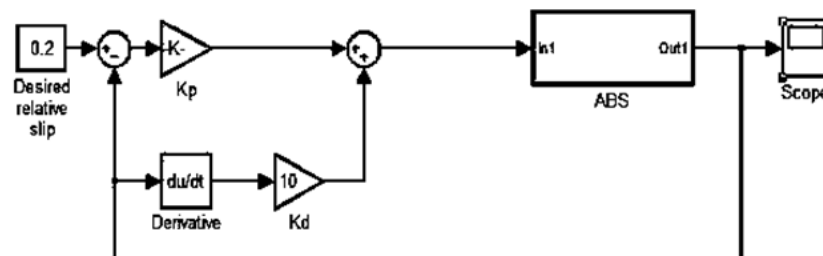
**Fig 6. A simplified feedback control system**

Integrating proportional feedback control with proportional gain  $K_p$  gives Fig 7.



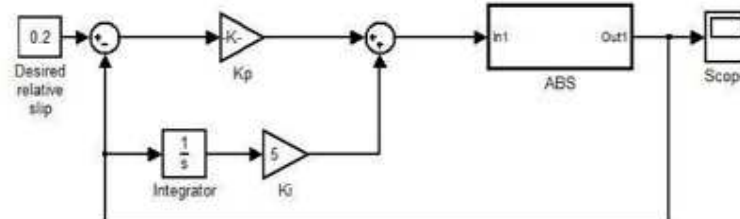
**Fig 7. Integrating proportional feedback control**

Also, the system is fed with proportional differential feedback control. Where  $K_p$  is proportional gain and  $K_d$  is differential gain. This system is shown in Fig 8.



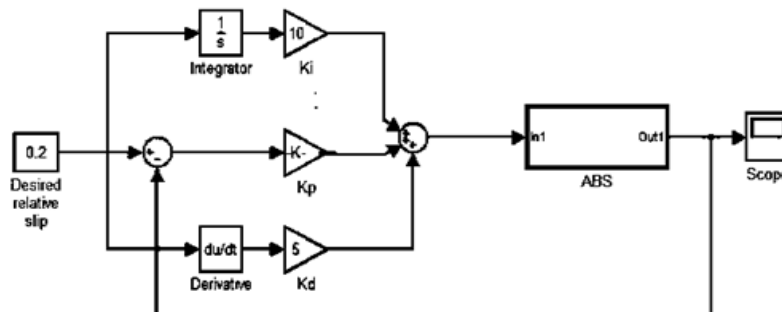
**Fig 8 Proportional differential feedback control**

ABS System is fed with proportional integral feedback control where  $K_p$  is proportional gain and  $K_i$  is integral gain. This system is shown in Fig 9.



**Fig 9 proportional integral feedback control**

Simulink model of ABS using proportional integral differential feedback control system where  $k_p$ , is proportional gain  $k_d$  is differential gain and  $k_i$  is integral gain are used. This system is shown in Fig 10.



**Fig 10 proportional integral differential feedback control**

## 5. RESULTS

Various vehicle performance in straight line braking conditions were simulated under different parameters, i.e with or without feedback.

The input parameters used are;  $r = 5.0$  m,  $m = 568$  kg,  $J_w = 1.13$  kgm<sup>2</sup>,  $g = 9.81$  m/s<sup>2</sup>, Max braking torque = 2000Nm, Initial linear velocity = 33.33m/s = 120 km/h, Initial rotational speed =  $33.33/5.0 = 6.67$  rad/s,  $\lambda_d = 0.2$ ,  $k_p = 250$ ,  $k_a = 5$ ,  $K_i = 10$ .

The behavior of vehicle parameters during straight line braking without any controller. Figure 11 a and b shows plot of vehicle angular velocity, stopping distance.

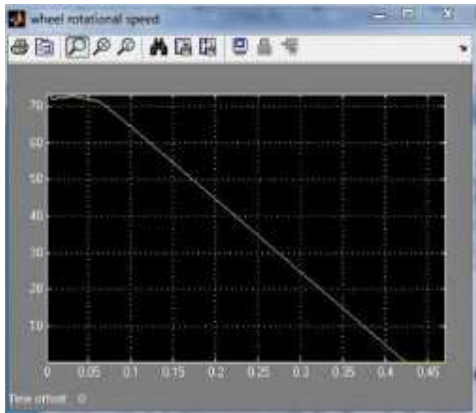
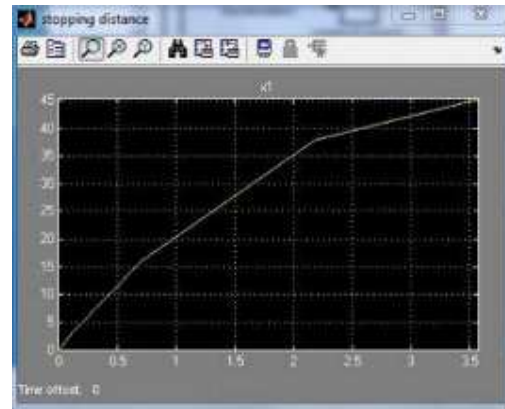


Fig. 11 a) angular speed against time;



b) stopping distance against time

Figure 12 a and b shows vehicle linear velocity and slip ratio respectively versus time.

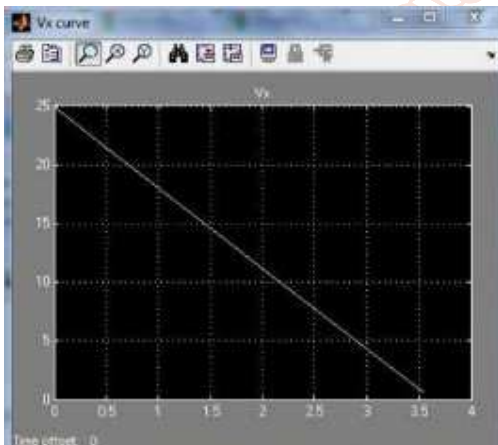
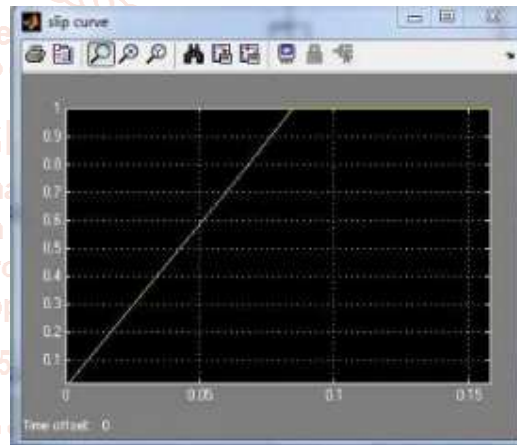


Fig 12 a) linear velocity against time;



12 b) slip ratio against time

From the plots it is observed that vehicle with its slip ratio and coefficient of friction not monitored with ABS, moving at a speed 70m/s stopped in 3.6 secs, at a stopping distance of 45m indicating wheel lock before vehicle comes to stop. The slip ratio has been varied between 0 to 1 on application of brakes to the wheels. It is also observed that braking steerability is lost at 0.42 seconds due to locking of wheel.

To maintain constant slip ratio a feedback control is incorporated in the system. simple linear model called Proportional Feedback Control with a constant gain  $k_p$  is first incorporated.

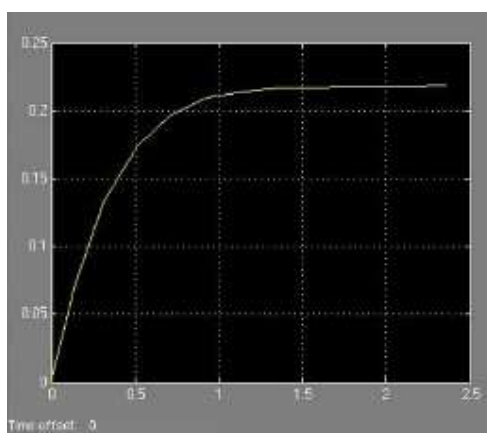
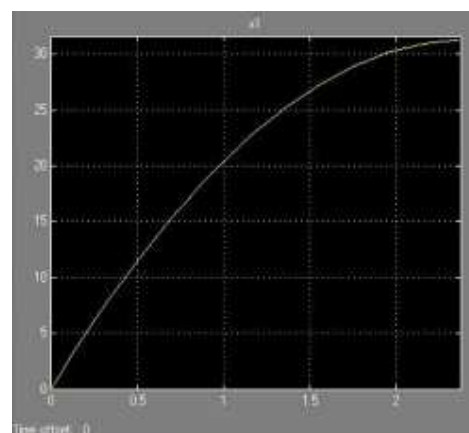


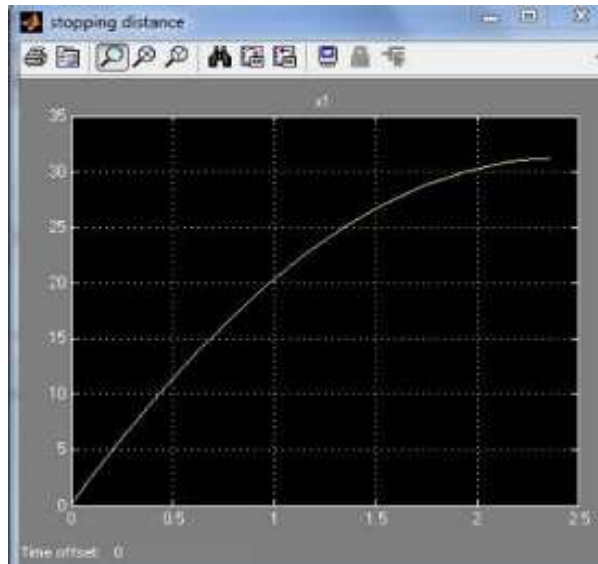
Fig 13 a) slip ratio v/s time ( $k_p = 250$ )



13b) stopping distance v/s time

From figure 13, the stopping distance was reduced 33 m because a P – controller supplies a control force and maintain slip ratio with 0.01.

When proportional derivative (PD type) feedback control incorporated, the stopping distance was further reduced to 32 m at stopping  $y$ =time of 24seconds as shown figure 14.



**Fig 14 stopping distance v/s time**

Finally, when proportional integral derivative control was incorporated, the plots shows that both stopping time and stopping distance are slightly reduced. Stopping time is 2.3 seconds and stopping distance is 31 m.

## DISCUSSION

From figure 12-14, it is observed that ABS improve efficiency of the braking system. The graph of slip ratio against time for different control feedback indicate that a Proportional Feedback Control will reduce the braking time. Proportional Integral Feedback Control will eliminate the steady state error, but it may make the transient response worse. Proportional Integral Derivative Feedback Control will increase the stability of the system, reduce the overshoot, and improve the transient response. The results of the three-feedback control system on a closed-loop system are summarized in the table 1.

**Table 3.1 Braking Performance Results**

ABS Controller	Stopping distance (meters)	Stopping time (seconds)
Without controller	45	3.6
Proportional Feedback Control	33	3
Proportional Integral Feedback Control	32	2.4
Proportional Integral Derivative Feedback Control	31.2	2.3

## 6. CONCLUSION

The work has shown the results of different application of various type of feedback controller used for antilock braking systems. The modeled quarter vehicle dynamics and differential equation of motion was formulated were applied using feedback control system. The slip ratio evaluation was the major aspect of the work as frictional force and normal reaction are function of slip ratio.

Three state space equations were used to obtain the block diagram representing dynamics of equations that was applied in the Simulink software that generated the result of the stopping distance, stopping time and slip factor variation. To achieve this, different feedback control like - proportional

feedback control, proportional integral feedback control, proportional integral derivative feedback control was implemented and the result obtained shows that feedback control increases the stability of the system, reduce the overshoot, and improve the transient response.

## References

- [1] P. M. Hart, [2003] "Review of Heavy Vehicle Braking Systems Requirements (PBS Requirements)," Draft Report,
- [2] R. Fling and R. Fenton, [1981] "A Describing-Function Approach to Antiskid Design," IEEE Transactions on Vehicular Technology, Vol. VT-30, No. 3, pp. 134- 144. doi:10.1109/T-VT.1981.23895

- [3] S. Yoneda, Y. Naitoh and H. Kigoshi, [1983] "Rear Brake Lock-Up Control System of Mitsubishi Starion," SAE Paper, Washington.
- [4] T. Tabo, N. Ohka, H. Kuraoka and M. Ohba, [1985] "Automotive Antiskid System Using Modern Control Theory," IEEE Proceedings, San Francisco, , pp. 390-395.
- [5] H. Takahashi and Y. Ishikawa, [1989] "Anti-Skid Braking Control System Based on Fuzzy Inference," U.S. Patent No. 4842342.
- [6] R. Guntur and H. Ouwerkerk, [1972] "Adaptive Brake Control System," Proceedings of the Institution of Mechanical Engineers, Vol. 186, No. 68, 855-880. doi:10.1243/PIME\_PROC\_1972\_186\_102\_02
- [7] Sahil Jitesh, [2014]. International Journal of Mechanical Engineering and Robotics Research. Vol. 3, No. 4, pp. 253-259.
- [8] Li Zhou, Lu Xiong, Zhuoping Yu, [2013]. Proceedings of the 2nd International Conference on Computer Science and Electronics Engineering pp. 1999-2003
- [9] G. F. Mauer, "A Fuzzy Logic Controller for an ABS Braking System," [1995], IEEE Transactions on Fuzzy Systems, Vol. 3, No. 4, pp. 381-388. doi:10.1109/91.481947
- [10] W. K. Lennon and K. M. Passino, [1999] "Intelligent Control for Brake Systems," IEEE Transactions on Control Systems Technology, Vol. 7, No. 2, pp. 188-202.
- [11] B. Lojko and P. Fuchs, [2002] "The Control of ASR System in a Car Based on the TMS320F243 DSP," Diploma Thesis, Dept. of Radio & Electronics, Bratislava.
- [12] National Semiconductor Inc., [1974], "Adaptive Braking Systems (ABS)," US Patent No. 3825305.
- [13] C. K. Huang & H. C. Shih, [2010], 'Design of a hydraulic ABS for a motorcycle', J Mech Science Technology, vol. 24, pp. 1141-1149,
- [14] Q. Ming, [1997], "Sliding Mode Controller Design for ABS System," MSc Thesis, Virginia Polytechnic Institute and State University.
- [15] M. Stan, R.-E. Precup and A. S. Paul, [2007], "Analysis of Fuzzy Control Solutions for Anti-Lock Braking Systems," Journal of Control Engineering and Applied Informatics, Vol. 9, No. 2, pp. 11-22.
- [16] V. Ivanov, D. Savitski, and B. Shyrokau, [2015]. A Survey of Traction Control and Anti-lock Braking Systems of Full Electric Vehicles with Individually- Controlled Electric Motors, IEEE Transactions on Vehicular Technology, vol. 64, issue: 9, pp. 3878- 3896,
- [17] J. A. Cabrera, J. J. Castillo, E. Carabias, and A. Ortiz, [2015]. Evolutionary Optimization of a Motorcycle Traction Control System Based on Fuzzy Logic, IEEE Transactions on Fuzzy Systems, vol. 23, no. 5, pp. 1594-1607.