International Journal of Trend in Scientific Research and Development (IJTSRD)
Volume 9 Issue 2, Mar-Apr 2025 Available Online: www.ijtsrd.com e-ISSN: 2456 – 6470

Fractal Geometry and Its Iterative in the Design of Metal Pergolas Structures

Prof. Mohammad Abdullah Almandrawy

College of Art & Humanities, Jazan University, KSA

ABSTRACT

This research focuses on proposing a foundational list for designing metal pergola structures by applying the concept of generative design. This approach is based on the principle of repetition with fractal geometry, employing a recursive function system to create metal pergola structures. This research avoids the use of Euclidean geometric shapes—squares, triangles, circles, etc.—which have resulted in flat, inconsistent works. Instead, the use of fractal geometry is considered closer to the process of shape and transformation in nature. The concept and method of designing with fractal ideas is a design process implemented through experimental thinking, producing quality works. This new method is designed to address these limitations by providing improved control over the properties of fractals, including symmetry, scaling, and self-similarity. These are based on new concepts of self-similarity and infinite complexity, which are capable of understanding nature and its infinite scales. Unlike standard methods, it incorporates adaptive iteration and symmetry-preserving transformations, enabling the creation of complex patterns with consistent structural properties. This allows for the strategic design of typical configurations of these regular structures to provide sufficient strength and stability for the structures to withstand vertical, horizontal, and wind loads, resulting in lighter structures. This method also improves computational efficiency by reducing duplicate calculations and achieving faster convergence with fewer iterations. The research adopts a theoretical approach, delving into the fundamental theories related to fractal geometry, and uses an Iterated Function System (IFS) based on the concept of fractal geometry. To demonstrate the intersection of mathematics and design in both modern and traditional design process practices, the details of the design unit networks in the context of applying the concept of fractal geometry in the field of construction aim to activate a geometric system in design that is typically characterized by recursive self-similarity properties. A rule-based geometric system can be constructed using the recursive function system process, and based on fractal construction methods and algorithms for their visualization and practical applications, they achieve a high degree of visual and geometric balance, which makes them appropriate for high-precision applications. The design elements are formatted to obtain the best solution to the fundamental concepts of design problems.

How to cite this paper: Prof. Mohammad Abdullah Almandrawy "Fractal Geometry and Its Iterative in the Design of Metal Pergolas Structures" Published in International Journal of Trend in Scientific Research and Development (ijtsrd), ISSN: 2456-6470, Volume-9 | Issue-2, April 2025, pp.1322-1334, URL: www.ijtsrd.com/papers/ijtsrd79733.pdf

IJTSRD79733

KEYWORDS: *Fractal Geometry, Repetition, Steel Pergola Structures, Iterative Function Systems (IFS)*

INTRODUCTION

The geometric configurations of pergola structures are typically non-rigid repetitions of self-similar curved assemblies. In some lattice structures, the careful arrangement of members provides strength and rigidity to the structures. A metal pergola structure is a rigid assembly of interconnected primary and secondary members, which cannot be deformed without deforming at least one of the secondary members. A metal pergola structure can be made significantly lighter than a cylindrical pergola structure of identical strength. A metal pergola structure whose members" are themselves sub-beams is lighter. Mandelbrot's logical explanation for such a self-similar or repetitive geometric form was termed "fractalism. Thus, the potential application of fractal geometry could provide a more accurate geometry for such a metal pergola structure, which could be created using some rule-based generative processes, such as the iterated function system (IFS), and may offer better mechanical strength and an innovative aesthetic appearance. Regarding the mechanical properties of fractal-based structures, there is another geometric concept in nature and mathematics known as fractal geometry, first systematically defined by Benoit Mandelbrot in the late 1970s, usually with the property of self-similarity or self-affinity. In a fractal, the repetition of the original or complete shape is contained within its parts. Furthermore, fractal geometry can explain and design many complex shapes and networks in nature that have been difficult to explain and reproduce using other regular and traditional geometric systems. Shortly after its early development in the early 1980s, fractal geometry has been applied to understand and model nonlinear and complex shapes in a variety of disciplines. Because geometric structures are related to their shapes, we can predict that fractal geometry may be useful in the design of pergola structures. (1)

Mathematics seeks hidden truths, while pergola design applies this science. The motivation stems from the ability to determine the number of similar elements in a group of similar elements and develop a better understanding of the pergola's structure. (2) This is because fractal geometry is a branch of mathematics that studies the properties and behavior of fractions. In this context, fractal geometry is a mathematical study that discusses the shape of fractions or any shape that resembles itself. This means that fractals are a mathematical study that studies shapes that exhibit an infinite process of repetition. (3) Therefore, the designer must find the best ways to divide space, which relate to the precise repetition of shape or volume within it. Euclidean geometry is employed by the study of regular geometric shapes. Regular geometric shapes are those made up of familiar points, straight lines, and line segments: squares, rectangles, trapezoids, rhombuses, various triangles, and regular polygons on planes; cubes, etc., in space. Another type is geometric shapes composed of curves or surfaces, circles, ellipses, etc. in space. The dimensions of these points and lines are 0, 1, 2, and 3. Fractal geometry is more complex or more realistic, and its important feature is that it has no distinct length, and the lines or surfaces that form its shape are not smooth and non-differentiable. For example, clouds are not spherical, mountains are not conical, coastlines are not arcs, and tree bark is not smooth and even lightning does not cross the sky in a straight line. These irregular geometric shapes are difficult to describe with straight lines, smooth curves, and smooth curved surfaces in Euclidean geometry. (4) Although this type of object cannot be addressed by classical geometry, it does have some "good" properties. To facilitate research, important idealization assumptions are often made; that is, it is assumed to be self-similar. Traditional shapes differ from fractals. Classical geometry is concerned with regular shapes, but not with irregular shapes. In these, a point is drawn in zero dimensions and has no dimension, while a straight line has one dimension. (5) As for the fractional dimension, it is neither a number nor a value. A plane geometric figure has two dimensions, while a cube and a cylinder have three numerical dimensions. This is due to the complexity of fractal shapes. A straight line has only one dimension, length, while a coastline has more than one dimension and less than two dimensions. It is a line, but it is very winding and thus approaches the second dimension because its windings extend right, left, and backward. That is, it represents a complex shape. Its dimension is between one and two dimensions, and therefore it has what is called "fractal dimension." This clarifies the concept of fractal dimension. It is essential to understand what we mean by dimension in the first place. A line has one dimension, a plane has two dimensions, and a cube has three dimensions. Euclidean geometric shapes are regular and have integer dimensions (one, two, and three for the line, surface, and volume, respectively). A fractal has a dimension between one and two. Fractals are structures with two important properties: refraction and self-similarity. Any suitable part of a fractal line contains a small copy of the entire line; that is, it is no longer a Euclidean line but a kind of "thick line." Fractions can be geometric, algebraic, and arbitrary. Algebraic and geometric fractions are deterministic, that is, completely repeatable; they give identical images regardless of the number of iterations.

Fractals are structures with two important properties:

Refraction and self-similarity. Any suitable part of a fractal line contains a small copy of the entire line; that is, it is no longer a Euclidean line but a kind of "thick line." Fractions can be geometric, algebraic, and arbitrary. Algebraic and geometric fractions are deterministic, that is, completely repeatable; they give identical images regardless of the number of iterations. [6] Fractals are self-similar structures that exhibit complex geometric properties at various scales. Unlike conventional shapes, fractals have fractional dimensions. A fractal is also defined as a set of "complex" points in some simple space. (7) Since its introduction by Benoit Mandelbrot, the word "fractal" was first coined by Mandelbrot in 1975. Fractal geometry is a study in mathematics that discusses the shape of fractions or any shape that resembles itself. Fractal is a study in mathematics that studies shapes or geometries that exhibit infinite repetition. Fractals have been extensively studied for their ability to capture complex structures found in nature, such as coastlines, river networks, cloud formations, tree branches, and biological tissue. (8)

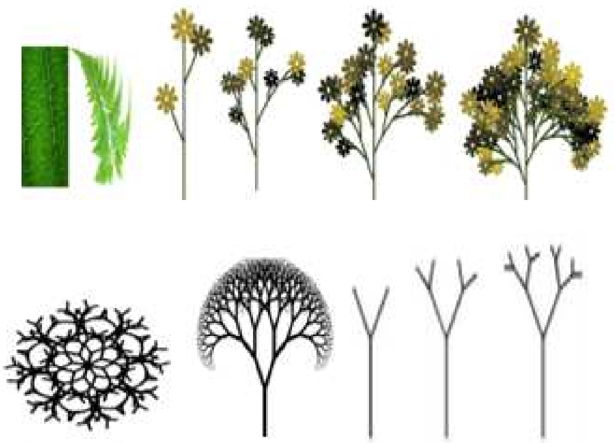

Figure (1) Fractal structure at different levels in nature

Three-dimensional fractals represent a fundamental step in the development of fractal geometry. The Mandelbulb shell, for example, is a three-dimensional fractal, an analogue of the Mandelbulb set, created by Daniel White and Paul Nylander using highly complex mathematics based on spherical coordinates. This explains the construction of the Eiffel Tower using only one type of metal rod, used in varying sizes throughout the tower to reduce the weight of the entire structure. This implementation can be called one of the first methods of applying fractal geometry to a metal structure. (9) From this perspective, the process of using fractal geometry to create structures capable of withstanding the required weight and stress is described. First, the diameter and thickness of a hollow metal beam are changed through re-engineering. Next, an iterative process is created to manufacture structural elements by replacing identical full-size versions. The resulting structures are lighter than those made of solid material. Therefore, the more iterations used, the lighter the fractal structure becomes. (10) Fractals can be classified in terms of geometry into three types according to three elements, according to the methods of their generation and the degree of self-similarity, which are: (11)

Similarity-self-identical: The strongest type of self-similarity is that the exact same shape appears at any scale of magnification, as in IFS systems.

Similarity-self: The fractal pattern appears identical at different scales. This property is often observed in natural formations, including vascular beds, ferns, and coastlines. Self-similarity is a prominent feature of fractals, as it refers to the similarity between the parts that make up the shape and the whole. That is, the part of the whole is very similar to the whole, and if a part of the shape to be studied is enlarged, its shape is the same as the whole. This means that any part of the whole is exactly like the whole. If we add an integral part of the fractal shape and then enlarge it several times, we will obtain the original shape. The overall pattern of the fractal shape is repeated similarly. This means that it is not random geometric shapes represented by irregular curves in nature but rather carries a hidden order lurking within the chaos.

Quasi-self-similarity: A loose type of self-similarity, where the fractals appear somewhat identical, but not identical, at different magnification scales. That is, the miniatures appear somewhat distorted. Statistical self-similarity: The weakest type of self-similarity, where the fractals appear non-identical

The properties of fractal geometry allow them to be used in scientific modeling, computer simulation, and artificial intelligence applications. Mathematically, fractals follow the rules of iterative generation, meaning that a simple initial structure is transformed through repeated operations. This was achieved through the development and analysis of various fractal generation techniques over time, including iterative function systems (IFS) and Lindenmayer systems (L-systems), which are applied and produce iterative models to generate and design complex patterns using fractal structures. A new method for iterative fractal modeling (RFM) is designed to generate complex fractal patterns with enhanced control over symmetry, scaling, and self-similarity. The iterative fractal modeling method builds on traditional fractal generation techniques but introduces adaptive iteration and symmetry-preserving transformations to produce fractals with design applications. (12) We used fractal modeling to create a mathematical model of fractal objects, allowing us to visualize polygonal shapes, Cauchy curves, Cayley trees, Serpin curves, Cantor sets, star shapes, cycloids, intersecting circles, and fractal trees. Using the proposed models, the fractal dimensions of these shapes were found, making it possible to create complex fractal patterns using a wide range of complex geometric shapes. (13) This emphasizes that, despite their mathematical elegance and practical utility, traditional fractal generation techniques often lack flexibility in parameter adjustments, symmetry control, and computational efficiency, limiting their ability to model highly detailed structures in a computationally feasible manner. Fractals are geometric shapes that result from applying a specific geometric pattern to a geometric shape multiple times and have fundamental properties that give them their unique structure. This group has some special properties, the most important of which are: [14]

- A fractal set cannot be described in conventional geometric terms. It is neither a path of points that satisfy certain conditions nor a set of solutions to some simple equation.
- A fractal set possesses a certain form of self-similarity, which may be approximate or statistical.
- Most fractals involved in nature and various applied sciences are approximate.
- When the scale is reduced to the size of a molecule, fractalism disappears, and strict fractalism exists only in theoretical research.
- For a variety of different fractions, some may have all the above properties simultaneously, others may have most of them only, and some have exceptions to certain properties. However, this does not affect our calling this set a fractal.

Repetitive Structure in Fractal Geometry:

Repetition in pergola design is defined as the placement of similar elements in different locations within the design. It is the foundation of rhythm in all its forms. Repetition, in this sense, refers to the manifestations of extension and continuity associated with achieving movement across the design surface. Monotonous repetitions of units can tire the eye and induce boredom. However, when they are repeated, creating spaces of a new form, they add a rich variety to the design, enriching visual perception. Repetition is the simplest of decorative rules; any shape in nature can be transformed, adapted, and repeated. The diversity and different sizes of units help strengthen and beautify the design, alleviating the boredom that can arise from seeing all these units in a single size. [15] Fractal shapes are usually formed by starting with a simple geometric shape, then subjecting the shape to a mathematical equation, and then returning and applying the same equation (rule) to the resulting shape. Then we repeat it again on each shape resulting from each stage to infinity, noting that the shape becomes more complex with the increase in the number of repetitions, and therefore it is called "formation through repetition, because it is not through a simple repetition process for the sake of repetition, but rather a repetition of the process, the procedure, and the rule. So complexity is one of the features of the fractal shapes resulting from the repeated calculation of a mathematical fractional equation that is repeated to infinity of repetition. [16] Defining a fractal structure means defining the principle of constructing the shape and structure, considering their changes and dynamics. These components of the formation are based on the principle of unity in diversity. Two approaches can be used to construct fractions: [17]

The first approach: Involves repeating the same part without changing it, creating each of its parts. The next step in modeling the object depends on the development and formation of the previous object, but at a different development level.

The second approach: Involves developing dynamic systems using fractal geometry methods, which is interesting in structural design frameworks. In fractal pergola structures, the following principles are applied: self-similarity, dynamism and evolving ability, irregularity, repetition, and fractalism. Using fractal principles, it is possible to create stunning pergola structures, study the structural composition of structures, and design pergolas with space organization considering environmental requirements. Emphasizing that the fractal geometry approach is an effective method for analyzing already built structures, as well as a means of designing such design objects that can enrich the design. From a geometric perspective, randomness and systems of recursive functions are classified as follows: [18]

Iterated Function Systems (IFS) systems and L-systems: The basic idea of an L-system is to generate complex objects by substituting parts of an initial element according to certain rules. Other applications of fractals in products include L-systems, which allow for the creation of repetitive geometric structures. L-system-based fractal modeling has been used to generate pergola structural designs with high levels of self-similarity, improving the consistency and repeatability of pergola structures. Several recursive algorithms can be used to construct fractals, including the time-chase algorithm, the stochastic recursive algorithm, and the deterministic recursive method. These algorithms have been instrumental in improving the computational efficiency of fractal drawing and reducing the time complexity associated with repetitive calculations. Despite their extensive use, current fractal generation methods lack scalability, structural consistency, and computational efficiency. [19]

- **Self-similarity**: This is a process in which a part of an object changes in a specific way according to the fractal dimension, i.e., an increase or decrease, imposed on the entire object, creating a completely new structure. In addition to the property of self-similarity, fractal geometry has other properties:

- **Asymmetry and scalability:** While the first concept refers to the overlap between symmetry and asymmetry, the second concept refers to similarity with sliding. In this theory, Mandelbrot also considers the dynamic evolution of systems.- From this perspective, methods for creating fractal structures using iteration techniques can be defined as follows:
- **Iterative generation:** This involves repetitions of the generated element, replacing certain parts of the shape with a modified version, as in Koch curves and Sierpinski triangles. Formula-based fractal geometry, including the Mandelbrot set and Julia sets, also uses complex number mappings to create visually complex patterns. These iterative methods form the basis of modern fractal generation algorithms, which are applied in pattern recognition, scientific simulation, and design. - Micro gradient: Like self-similarity, gradient is the process of reducing and/or advancing elements. In a fractal set, the smaller subsets are the same as the larger sets that make up the entire fractal.

Repetition (balance): Fractal components and elements are repeated at different scales. This recurring rhythm in fractal shapes leads to balance.

Repetition and Mathematical Sequence:
Repetition can be thought of as the simple act of copying an element multiple time. In modular systems, repetition can become more interesting without requiring a repetition, because the repeated element can change while maintaining its basic structure for any number of variables, such as distance and time. Series and sequences are defined as the sum of all actions in a final sequence. A mathematical sequence is also based on an order of things (or events), which may be final. The order in which the actions appear is important, and the same action may appear multiple times in the sequence. (20)

Using this rule-based system, the designer can vary the repeating element according to its location, etc. Mathematically, repetition can be as simple as using the Fibonacci sequence, which states that, given the first two numbers 0 and 1, any subsequent number is the sum of the previous two numbers (0, 1, 1, 2, 3, 5, 8, 13, 21, etc.). In this way, repetition can become more interesting and perhaps more useful from a design perspective and help generate further branches of the repetition mechanism. (21) There are many types of numbers that are often found in mathematics. For example, real numbers, rational numbers, arithmetic numbers, natural numbers, complex numbers, etc. These numbers can form a pattern of number sequences and number series, whether geometric or arithmetic. The Fibonacci number sequence was invented by the Italian mathematician Leonardo Fibonacci around 1170. Some natural phenomena are known to represent the Fibonacci line. For example, some types of flowers: (22)

Figure (2). The golden ratio of Fibonacci flowers

Generative algorithms are a set of steps that lead to a certain shape, and any change that occurs to the inputs of this process leads to a difference in the shape according to the change to only three structures, which are as follows: (23)

First: Sequence an algorithm is a set of sequential instructions.

Second: Choice some problems cannot be solved with a simple sequence of instructions. You may need to test certain conditions and observe the test results. If the result is correct, follow a path containing sequential instructions. If it is incorrect, follow a different path of instructions. This method is called decision-making or choice.

Third: Repetition when solving some problems, the same sequence of steps must be repeated several times. This is called repetition. It has been proven that no additional structures are needed. Using these three structures facilitates understanding the algorithm, detecting errors, and changing them. There are a few elements that restrict the types of operations that an algorithm can include and encompass:

- Clarity: The problem the algorithm addresses must be clear.
- Efficiency: This means that each step can be executed by anyone in a specific period of time.
- Limitations: This means that the algorithm must have a limited and specific number of operations and steps.
- Outputs: The algorithm must have one or more outputs and zero or more inputs.

Fractal Pattern Generation and Digital Systems:
The origins of mathematical fractal patterns date back to the end of the seventeenth century and later evolved into geometrically self-similar patterns that differ from traditional geometric patterns and their straight lines, called curves. Their smooth curves, with infinite circumference and finite space, led to the term "fractal" in the latter part of the twentieth century, fully described by the French mathematician Mandelbrot in 1975. Mandelbrot's work on replicating nature's creative code took a new turn using computer programs, making it possible to design and create self-similar natural forms in a way never achieved. Fractal geometry requires mathematics and simple equations repeated multiple times to demonstrate the fractal, like the self-similarity of a building, such as the structural elements of a pergola that repeat at different sizes, increasing or decreasing. This is one of the models expressing the fusion of mathematics and information technology. It is an important new modeling tool that expresses the repetitive pattern geometry to the farthest reaches of advanced technology. (24)

Digital tools in the current era are generative tools for extracting and transforming shapes using principles, such as the Sierpinski triangle, and mathematical equations that govern their structural construction. This enables architects to obtain multiple formative alternatives to achieve the optimal design. Among the most popular digital programs and applications currently used to assist in generating fractal patterns are Dynamo, included in Revit, a BIM application, and Grasshopper, included in Rhinoceros. Fractal structures are generated either by repeatedly growing them from a unit structure or by creating divisions into smaller units of the shape. The most important digital theories and systems using Grasshopper (GH) for generating fractals are explained, along with the codes used to create the pattern. (25) These algorithms are used to generate the final shape (output) from the given data (input).

Iterative Functional Systems (IFS):
The iterative function system is a fractal composition system and an important branch of fractal geometry. It is one of the most vital and promising areas of fractal design. It results from the development of a series of algorithmic rules and has been used in many aspects. The iterative function system has significant advantages in modeling a wide range of objects, especially in computer simulation of landscapes. It transforms two-dimensional fractals into three-dimensional fractals; the self-similar fractals it explores expand its scope of application. For the geometric transformation of original drawings, the linear transformation is extended to a nonlinear transformation; to discuss computer generation of landscapes, the modeling method is also expanded from two-dimensional to three-dimensional. (26) Fractals are formed by replicating the original shape after geometric transformations in the first step, then repeating this process iteratively in subsequent steps an infinite number of times. This process produces a fractal shape, which is often unpredictable. However, in 1981, based on the Hutchinson factor, Barnsley developed a system known as the iterated function system (IFS), which can predict the outcome of a fractal formation in a deterministic manner. In Barnsley's concept of an iterated function system, it is important to note that the outcome of the fractal shape is not determined by the initial shape. Instead, it is determined by affine transformations that can be considered the true "initial condition". (27) Structural formations are generated from recursive functional systems because of applying a geometric rule to each function. The resulting formations are characterized by the property of identical self-similarity, the most important of which are: (28)

Minkowski Curve:
Most classical curves, such as the Minkowski curve, rely on a geometric substitution rule for a specific mathematical rule. The recursive shape is a clear representation of the fractal, where the geometric shape is repeated according to the properties of the applied rule. These sets are called original recursive function systems.

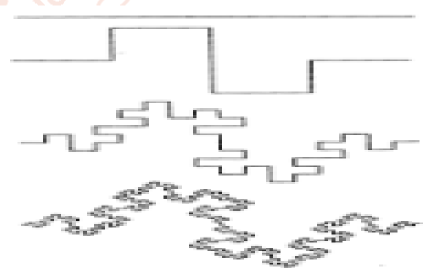

Figure (3). Minkowski curve

Sierpinski plane:
The Sierpinski plane is a planar fractal, a repetitive and symmetrical shape. It was discovered by Wadaw Sierpinski in 1916. Its construction begins with a square. Theoretically, the square is cut into nine identical subspaces in a 3×3 grid, and the central sub square is removed. The same procedure is then applied recursively to the remaining eight subspaces. The rule can be applied to shapes other than the square. At each iterative level, the plane consists of a shape: a square 1/3 the size of the plane at that level, placed as the center of the plane. The function then generates eight additional planes arranged around the base square. Both the iterative function and the data

structure reflect this arrangement. The number of times the iterative function calls itself at a single level is called the branching. Step 0 of the zero plane has one trim, while the level 2 plane has nine trims. The number grows very quickly. Thus, a plane of level 6 has: 1 + 8 + 64 + 512 + 4096 + 32768 + 262144 = 299593 decorative elements. Ultimately, the designer is free to access any level of this mechanism to limit the depth of recursion when facing high branching factors. [29].

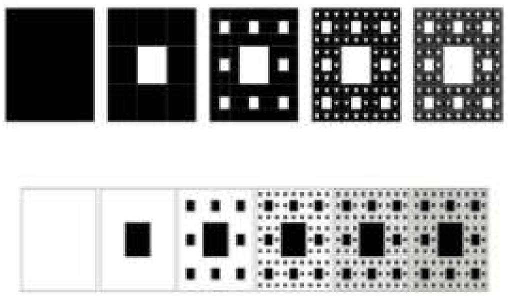

Figure (4). Sierpinski carpet

The Sierpinski Triangle is a fractal shape invented by the Dutch mathematician Waclaw Sierpinski in 1915. It is an equilateral triangle, divided by a central triangle into three triangles at the corners. When repeating the same process with the three resulting triangles, we arrive at a very complex shape. Another example is the Sierpinski Carpet, which results from applying the same process to a square plane. There is also the Square Menger sponge, which results from applying the same process to a cube. [30]

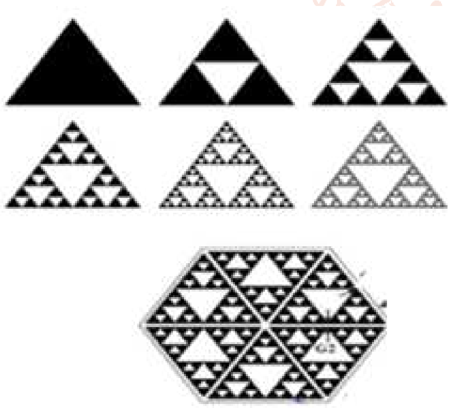

Figure (5) Sierpinski Triangle

Cantor Set:
Invented by the German scientist George Cantor in 1945 by applying a simple geometric rule to a straight line divided into three parts, with the middle part erased. The resulting shape is two lines with a space between them. The shape exhibits a complex feature when the same simple process is repeated on each resulting line. [31]

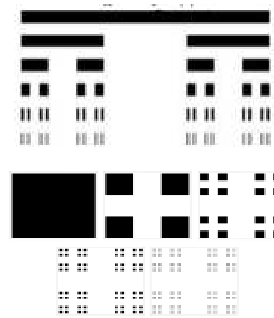

Figure (6) Cantor Set

Snowflake Koch Curve:
Introduced by Helge von Koch, it is a mathematical function with geometric content. It is the combination of a finite number of shapes and their triangular boundaries. The formation of the curve begins with a straight segment, then transforms into the first iteration by triangulating the straight segment and replacing the middle third with two equal legs, forming a shape of four repetitions of the basic shape. [32] In the next iteration, we transform each straight segment into the shape resulting from the first iteration, and with an infinite number of repetitions, we reach the curve. The perimeter of the shape expands until it reaches infinity, despite the space it occupies being finite. [33]

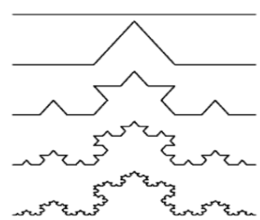

Figure (7) The process of forming the initial element in the Koch Snowflake curve.

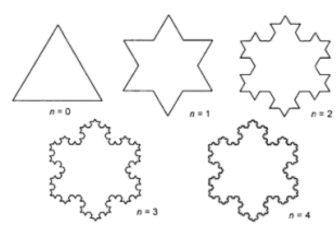

Figure (8) Four repetitions to create the "Koch Star" fractal

Recursive Pattern:
This tool works by repeatedly repeating the idea, incorporating a hierarchy of fractions into the design across all its parts. These parts replicate and transform the whole. This naturally leads to layers of information with a hierarchical structure, where the properties of each layer are immediately derived from the layer above it. [34]

A recursive geometric pattern is another classic example of repetition. The propagation of this type of pattern relies on the same procedure used for branching patterns. For example, if a pattern requires the creation of a second square whose side length is half the area of the original square, at the next iteration, the newly created square will be the original square. In nature, a recursive pattern is usually skewed and limited by the environment in which it occurs. When the pattern is repeated, it primarily considers the conditions of the parent pattern as well as the forces of nature acting upon it, as demonstrated by the Romanesco Brassica plant, a natural representative of such hierarchical structures. This tool is used when a designer needs to work with hierarchy in a design. (35)

Figure (9) Recursive geometric pattern

A recursive tool uses a generative copy rule to replicate the idea. It then calls itself at each replication, with the same replication rule. So, at each step, the recursive tool takes an existing idea, iterates over it, and applies it to itself. Each recursive function requires a termination condition to determine when to stop the process. This condition can be in the form of testing the size of the pattern to be generated (i.e., setting a minimum size), the total number of generated elements, or the level of recursion that has occurred so far (i.e., setting a maximum number of recursive iterations).

The effectiveness of iteration in pergola design:

The iterative function systems (IFS) and L-systems approach are extensible through custom modifications that enhance their ability to generate complex, high-resolution fractal patterns suitable for industrial applications. While these methods are typically used to generate fractals, iterative modifications allow the production of highly complex and culturally significant patterns, particularly useful in the design of metal pergola structures. Enhancements applied to IFS systems also include the incorporation of variable scaling factors and unique iterative transformations that control the arrangement and density of overlapping structures. (36) This allows us to generate fractals with greater symmetry and spatial accuracy, which is critical for patterns that require a high degree of visual regularity and complexity, such as star and circular designs. Similarly, our adaptation of the method includes adjustments to the boundary conditions and iteration parameters, enabling us to model circular fractals and intersecting patterns with a complex level of detail. These adjustments result in geometric configurations that maintain structural integrity across scales, enabling the creation of fractal models that can smoothly expand or contract for various design applications without loss of detail. (37)

In addition to these mathematical adjustments, a specially designed visualization algorithm was developed to automate the fractal generation process. This allows for the generation of high-resolution patterns, facilitating the efficient design of complex fractal structures for pergola structures. The software also supports customization of user-defined parameters, enhancing its utility in pattern design and visualization. (38) This concept is important in fractal geometry and is used to define the analytical equations of geometric shapes. This system is compatible with every part of the pergola structure, ensuring the quality and accuracy of its performance. Using these systems, it is possible to create an implicit form for the boundary equations of complex objects based on known equations for simple spheres. Methods for constructing systems of domain geometry equations provide a solid technical basis for automating the process of structuring these equations. In fact, only the process of constructing the predicate equations requires automation. The transition from these equations to simple elementary domain geometry equations is accomplished by replacing the symbols of the logical function with the corresponding symbols of the systems. However, the domain symbols are not equivalent to their corresponding left-hand parts. Therefore, the input data for the systems are as follows: (39) . (40)

- The appearance of the standard primitives used: straight line, circle, ellipse, rectangle, triangle, convex polygon, circle, regular polygon, etc.
- The geometric parameters that define the size and position of the standard primitives. While the dimensions of classical fractals, such as Koch curves, Cayley trees, and Cantor sets, are widely known, we used the method of iterative functional systems to systematically derive these dimensions.
- This method allows us to calculate the dimensions of fractions by combining geometric modeling with analytic logical functions. Unlike traditional methods, the iterative functional systems framework provides a powerful mechanism for iteratively modeling and proving the self-similarity properties of fractions.

Applied Study:

Metal pergola structures are frame structures, and a basic structural system widely used for building lightweight, free-form structures. The reason for choosing a structural steel pergola is its lightweight lattice structure, high structural rigidity, and excellent mechanical behavior against both compression and tension. The mechanical behavior of the structure generally depends on the connection patterns of the various members within the main outer frame. Most of these patterns follow regular, connected geometric shapes, often inspired in some way by nonlinear mathematical concepts and unconventional geometry. In a fractal shape, repetitions of the original or complete shape are contained within its parts. This is because geometric structures are linked to their geometric shapes. Therefore, we can predict that fractal geometry may be useful in the design of structural shapes, characterized by a set of features and properties, the most important of which are:

- Recursive Logic: This is a logic based on the repetition of inputs that represent determinants of the standard process that follows. The most important feature of fractal design is the conversion of the design into digital information so that the computer can understand and manipulate it, as it can only understand it. The logic of information repetition stems from this concept, relying on variables of values specified by the designer to influence the structural modeling. Whenever the values of the modeling parameters change in the repetitive logic, the structural logic of the process changes.
- Logical sequential processes: These are successive steps according to mathematical logic and a sequential geometric basis influenced by the nature of the information inputs and outputs or data. Each step includes a programming space for one or more specific calculations. These sequential processes are built according to formal design concepts and based on a specific goal.
- Fractals can change continuously.
- They can create new and highly diverse forms from a pre-defined initial structure.
- Fractals are models of dynamic systems capable of evolution and change.
- They contribute to the creation of each of the components, making the design unique and completely different from the initial structure.
- The fractal system has two important properties that are useful for structural design.

The first property is that a fractal system can be expressed using a simple algorithmic function. This property is useful for easily programming a fractal model on a computer to design structures.

The second property is that a fractal system is a collection of self-similar unit elements; therefore, it forms a hierarchical or modular system within the structure. This property can be useful for constructing long-span structures where modular units are required to strengthen and reinforce the structure. Stages of generating a fractal pattern for pergola structures:

The first stage: Designing a specific shape: This is the initial design or starting shape.

The design shape is constructed through a recursive relationship, resulting from increasing numbers, resulting from applying a mathematical equation multiple times, resulting in a fractal shape. The complexity lies in repetition. This is prepared using several computer software systems, classified into three main groups:

- Elementary and Generator Systems: A simple, initial system
- Recursive Functions (IFS): A common system, it results from combining rescaled versions of the original with the original being scaled down infinitely in different directions
- L-System: The most common system in contemporary software, the L-System is a set of methods for describing proposed plants and trees from a plant morphological perspective. Initially, it focuses solely on the topological structure of plants, i.e., the adjacent relationships between plant components.

The second stage: Choosing a generation rule for how to change this shape: replacing each copy of the initializer with a smaller copy of the generator, which is a set of scaled copies of the initializer and can vary in direction. This depends on determining the following:

- The idea of the fractal pattern: This relies on two mathematical methods: "algorithms" to represent growth and "fractals" to represent the nature of the forest through tree branches, using computational and parametric techniques on the computer; the Renault program includes the Grasshopper.
- **The pattern and its impact on the design:** Trunks are used in the design to form the pergola structure, while the branch networks provide shade and a sense of shelter, while the curved shapes express the meaning of nature. The

development of the three-dimensional fractal pattern is associated with the fractal pattern in the horizontal projection. The fractal growth pattern of the pergola and the evolution of the generations of the original unit—repetition with a Srbenski triangle curve. An abstract sketch of the idea of black holes as a natural fractal pattern. Gradual scale. Source:

- **The pergola structure design:** Relies on the use of a fractal hexagonal grid containing a flower shape. The common use of the repetitive geometric pattern is that the small parts of the structure are like the larger parts, and in the form of a regular hexagonal pattern divided into irregular hexagonal polygons, successively graduated in scale, repeated throughout the roof area, while continuing to simulate natural growth. The use of computer techniques, standard calculations, algorithms, and fractal growth to build the structure. The stress and flexural strength behavior of the flower as a natural fractal pattern are evaluated, and the appropriate grid is selected to develop the fractal growth pattern and detail the design of the horizontal projection pattern of the pergola. The roof resembles an umbrella designed with a fractal pattern. The roof extends horizontally, relying on columns that align with the shape to resemble the rest of the elements. A fabric-like facade with passive cross-ventilation. The perforated facade reduces the effects of sun and wind by designing spaces that help adapt to the local climate and provide natural ventilation for learning spaces from all sides. This improves environmental performance using an innovative concept that achieves spatial dynamism. The facade's surface pattern is based on the primitive metaphor of the infinite dimension of the universe, starting with the initial plan and applying it to surfaces using the Minkowski curve in various forms. The pergola's shape expresses its function and climate protection, stimulating the relationship between humans and nature.
- The role of pattern and its impact on design: The pergola's structural design is characterized by the use of fractal geometry to attract and enigmatically achieve the required aesthetic and formative values by placing a standard building to achieve ventilation, natural lighting, and protection from sunlight according to the climate. The roof's shape is characterized by design flexibility, control over lighting distribution, and the aesthetic appeal of an unconventional design that connects the pergola's elements (columns and roofs) by achieving continuity of form.

The third stage: The rule is applied to the shape an infinite number of times, thus producing the shape using fractal geometry. This process is used to create a horizontal plan, facade, or three-dimensional form. When designing a pergola structure by assembling columns, beams, and supports within the main frame, the primary goal is always structural integrity, managing the forces acting on the members to achieve maximum stability and high rigidity while maintaining aesthetic appeal. Repetition here aims to increase the rigidity of the metal pergola structure by reducing the slenderness of the members. In general, the greater the number of overlapping repetitions within the metal pergola structure, the greater the rigidity of the metal pergola structure. There are various methods for designing a simple algorithmic function that can generate a fractal model. Thus, infinite structural configurations can be achieved for designing metal pergola structures with unconventional relationships between their components by employing three schemes: fractal design principles, fractal construction steps, and repetition patterns.

Research results:

After examining the basic concepts of fractal theory, its properties, areas of application, and methods for calculating fractal dimensions, it was found that fractal patterns with a fractal structure can be used to organize the design processes of modern and classic designs in the manufacture of lightweight pergola structures, and have economic benefits.

Fractal geometry is defined as a branch of mathematics that studies shapes that possess self-similarity and infinite repetitions, but the fractal design is a design of shapes that exhibit the same details in every part, possess self-similarity across different scales, and are composed through repetition. It is characterized by complexity, dynamism, abstraction, chaos, and irregularity. These designs are the product of a fusion of mathematics and design relying on computing power to repeatedly calculate mathematical formulas.

A fractal is an approximate or fragmented geometric shape that can be divided into parts, each of which is (at least approximately) a smaller version. The forms of innovation are linked together by irregularity and fragmentation, by constantly repeating the same shape on a smaller scale each time it is repeated using iterative shape generation systems.

The choice of the initial value for the first iteration has a greater impact on the final calculation result. If the initial value for the iteration is not chosen correctly, the iterations will not converge or will converge to the wrong result, which will affect the accuracy of the fractal design.

Fractal geometry has similarities to each other's shapes (self-similarity), and in the order of multiplication, it is not restricted by a rule of thumb; rather, it tends to twist varying sizes, ranging from small to large.

These fractals are found in nature, such as the patterns on leaves and tree branches, broccoli, white cloud clusters, ripples in waves, the details we can see on snowflakes, and much more when we try to pay close attention to our surroundings.

Iterated Functional Systems (IFS) and Mandelbrot sets are used to create these designs. Designs and patterns are created using computer programs that support the creation of fractal designs.

The importance of creating a set of new fractal structural pergola structures, partly as a geometric improvement over more traditional schemes, and partly as truly new designs. This is interesting from an architectural perspective, as such new solutions can exhibit aesthetic values and an innovative appearance that make them suitable for real design applications.

Structural analysis shows that the self-similar fractal branching of the pergola structure is suitable for carrying nearly distributed loads, both on the inclined upper pergola structure and on the lower chord. The internal force flows depend largely on the applied load condition and indicate that new forms can be derived from fractal patterns.

A more complete evaluation of the structural application require the complete design of the fractal-based pergola structure, including optimal member sizing, as a new engineering concept in the design and development of innovative structural systems.

Based on this research, it is recommended to create a set of new fractal structural pergola structures, partly as a geometric improvement over more traditional schemes, and partly as truly new designs. This is interesting from an architectural perspective, as such new solutions can exhibit aesthetic values and an innovative appearance that make them suitable for real design applications.

References:

[1] Irina Mayatskaya, Irina Kashina, et: Fractal geometry and design of modern structures- E3S Web of Conferences (2021) p1:8

[2] Golib Rashidovich Berdiev: Designing Fractal Buildings Using Iterative Function Systems- Central Asian Journal of Education and Computer Sciences- volume 1, issue 2, april 2022 P21:28

[3] Yifei Guan, Simi Liu , Yiwei Zhang: The origin, development, and application of fractal geometry theory in architecture-Proceedings of the 3rd International Conference on Computing Innovation and Applied Physics2024, (https://creativecommons.org/licenses/by/4.0/ P42:49

[4] D.A. Juraev and N.M. Mammadzada: Fractals and its applications-Karshi Multidisciplinary International Scientific Journal-University of Economics and Pedagogy-UEP 2024- Vol. 1(1) (2024), P27–38

[5] Md. Nurujjaman , Ahammad Hossain ,. Payer Ahmed: Mathematical Approach-Science Journal of Applied Mathematics and Statistics-Science Journal of Applied Mathematics and Statistics 2017; 5(3): 98-105 - http://www.sciencepublishinggroup.com/j/sjam s . P88-97

[6] Anton Vrdoljak, Kristina Miletić: Principles of fractal geometry and applications in architecture and civil engineering- Number 17, June 2019.P40:50

[7] Özcan Geli¸sgen, Temel Ermis: Inversions and Fractal Patterns in Alpha Plane- international electronic journal of geometry volume 16 no. 1 (2023) p 398–411

[8] Kenneth Falconer: Fractal Geometry Mathematical Foundations and Applications-Second Edition Kenneth Falconer John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, England 2003

[9] Joori Suh , William Stalker , Steen Pedersen, and Assaf Harel : Three-Dimensionalities Mediates the Subjective Experience of Fractal Interior Spaces-Architecture 2024- https://www.mdpi.com/journal/architecture P651:665

[10] Yanguang Chen: Fractal Modeling and Fractal Dimension Description of Urban Morphology-Entropy 2020, 22, 961; doi:10.3390/e22090961 -P1:21

[11] Wojciech Jan Krzysztofik: Fractals in Antennas and Metamaterials Applications- 28 February 2017 Published: 14 June 2017 DOI: 10.5772/intechopen.68188

[12] Ostwald, M.J., Vaughan, J.: Fractal Dimensions in Architecture: Measuring the Characteristic Complexity of Buildings, In: Sriraman B. (eds) Handbook of the Mathematics of the Arts and Sciences, Springer, Cham, 2018.

[13] Jonathan Calkins, Dr. Ed Aboufadel: Fractal Geometry and its Correlation to the Efficiency of Biological Structures-GVSU Honors Senior Thesis Apr. 22nd, 2013.

[14] Xiaoshu Lu, Derek Clements-Croome , Martti Viljanen: Fractal Geometry and Architecture Design- Chaotic Modeling and Simulation (CMSIM) 2:, 2012 p311-322

[15] Oleksandr Tomchuk: Review Models for Simulation of Fractal-like Particle Clusters with Prescribed Fractal Dimension-Fractal Fract. 2023, 7, 866. P4:25

[16] Sharif Ullah , Doriana Marilena D'Addona, Yusuke Seto, Shota Yonehara , Akihiko Kubo: Utilizing Fractals for Modeling and 3D Printing of Porous Structures- Fractal Fract. 2021, p5, 40

[17] Ahammad Hossain, Rita Akter , Payer Ahmed: Fractals Generating Techniques- Sonargaon University Journal Volume 1, Number 2, December 2016 -p43-54

[18] Renu Chugh, Ashish: Fractals Generated by Various Iterative Procedures – A Survey- Mathematical Journal of Interdisciplinary Sciences Vol. 2, No. 2, March 2014 pp. 195–208

[19] Anna Alfieri: L-system Fractals: an educational approach by new technologies- Conference Paper · July 2015

[20] S.I. Denisov: Fractal Dimension of Random Processes- Elsevier Science Ltd- Solitons Fractals Vol.1998.p1491:1494

[21] RETNANINGSIH: Fractal Geometry, Fibonacci Numbers, Golden Ratios, and Pascal Triangles as Designs- The Journal of Academic Science- Vol 1 No 1 2024- P51:65

[22] Nikoletta Minarova: The fibonacci sequence: nature's little secret the fibonacci sequence: nature's little secret- Cris bulletin 2014/01 P7:15

[23] Muhammad Zahid 1 , Fahim Ud Din 1 , Mudasir Younis: Mathematical Modeling of Fractals via Proximal F-Iterated Function Systems-Axioms 2024, 13, 881.

[24] Denny Gulick: The Beauty of Fractals-The Mathematical Association of America-United States of America-2010 P10:11

[25] Abror Shavkatovich Buriboev, Djamshid Sultanov , Zulaykho Ibrohimova , Heung Seok Jeon : Mathematical Modeling and Recursive Algorithms for Constructing Complex Fractal Patterns- Mathematics 2025, p13, 646

[26] Bulusu Rama, Jibitesh Mishra: Developing fractals using iterated function systems- ARPN Journal of Engineering and Applied Sciences- VOL. 13, NO. 4, FEBRUARY 2018 P1402: 1408

[27] Iasef Md Rian, Mario Sassone: Fractal-Based Generative Design of Structural Trusses Using Iterated Function System- International Journal of Space Structures Volume 29 . Number 4 . 2014 P181:203

[28] Minhyeok Lee , Soyeon Lee : Persistent Homology Analysis of AI-Generated Fractal Patterns: A Mathematical Framework for Evaluating Geometric Authenticity- Fractal Fract. 2024, p8, 731

[29] David Ettestad, Joaquin Carbonara: The Sierpinski Triangle Plane- Fractals, Vol. 26, No. 1 (2018) p11

[30] Miguel-Ángel Martínez-Cruz, Julián Patiño-Ortiz, Miguel Patiño-Ortiz and Alexander S. Balankin: Some Insights into the Sierpi ´nski Triangle Paradox- Fractal Fract. 2024, p8, 655 Choudhury, Arun Ma

[31] Gautam hanta, Hemanta Kr. Sarmah, Ranu Paul: Cantor Set as a Fractal and Its Application in Detecting Chaotic Nature of Piecewise Linear Maps- The National Academy of Sciences, Section A · June 2019

[32] Ipek Ebru Karaçay , Salim Yüce: Applications of Cantor Set to Fractal Geometry- International electronic journal of geometry volume 17 No. 2-2024, P 712–726

[33] Mamta Rani, Riaz Ul Haq, Deepak Kumar Verma: Variants of Koch curve-National Conference on Development of Reliable Information Systems, Techniques and Related Issues (DRISTI) 2012- P20:25

[34] Subhadha Battina: Application of Fractal Growth Patterns in Housing Layout Design - Creative Space Vol-3, No-2, January 2016 p185–207

[35] Retnaningsih: Fractal geometry, fibonacci numbers, golden ratios, and pascal triangles as designs- The Journal of academic science- Vol 1 No 1 2024 p51:66

[36] Iasef Md Rian, Mario Sassone: Fractal-Based Generative Design of Structural Trusses Using Iterated Function System- International Journal of Space Structures Volume 29 . Number 4 . 2014 P181:203

[37] Akhlaq Husain, Manikyala Navaneeth Nanda , Movva Sitaram Chowdary, Mohammad Sajid: Fractals An Eclectic Survey, Part II -Fractal and Fractional · July 2022

[38] Andrew Vince: Real Projective Iterated Function Systems- arXiv:1003.3473v2 [math.DS] 24 Mar 2011, P2:31

[39] Anil. D. Chavan, Chetan. V. Suryawanshi: Correlation of Fibonacci Sequence and Golden Ratio With its Applications in Engineering and Science-Volume-10, Issue-3 (June 2020) P31:36

[40] Tihomir Dovramadjiev, Mariana Stoeva, Violeta Bozhikova, Rozalina Dimova: Digital Parametric Design of Fractal Geometric Koch Snowflake Patterns-Applied Mathematics, Mechanics, and Engineering Vol. 64, Issue Special I, January, 2021