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Towards the Implementation of the Sudan Interpolated Geoid Model (Khartoum State Case Study)

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ABSTRACT

The discussions between ellipsoid and geoid have invoked many researchers during the recent decades, especially during the GNSS technology era, which had witnessed a great deal of development but still geoid undulation requires more investigations. To figure out a solution for Sudan's local geoid, this research has tried to intake the possibility of determining the geoid model by following two approaches, gravimetric and geometrical geoid model determination, by making use of GNSS/leveling benchmarks at Khartoum state. The Benchmarks are well distributed in the study area, in which, the horizontal coordinates and the height above the ellipsoid have been observed by GNSS while orthometric heights were carried out using precise leveling. The Global Geopotential Model (GGM) represented in EGM2008 has been exploited to figure out the geoid undulation at the benchmarks in the study area. This is followed by a fitting process, that has been done to suit the geoid undulation data which has been computed using GNSS/leveling data and geoid undulation inspired by the EGM2008. Two geoid surfaces were created after the fitting process to ensure that they are identical and both of them could be counted for getting the same geoid undulation with an acceptable accuracy. In this respect, statistical operation played an important role in ensuring the consistency and integrity of the model by applying cross-validation techniques splitting the data into training and testing datasets for building the geoid model and testing its eligibility. The geometrical solution for geoid undulation computation has been utilized by applying straightforward equations that facilitate the calculation of the geoid undulation directly through applying statistical techniques for the GNSS/leveling data of the study area to get the common equation parameters values that could be utilized to calculate geoid undulation of any position in the study area within the claimed accuracy. Both systems were checked and proved eligible to be used within the study area with acceptable accuracy which may contribute to solving the geoid undulation problem in the Khartoum area, and be further generalized to determine the geoid model over the entire country, and this could be considered in the future, for regional and continental geoid model.

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KEYWORDS: GGM: Global Geopotential Model, GNSS: Global Navigation Satellite Systems, EGM2008, ITRF2005

1. INTRODUCTION

The main objective of the research is to create geoid model which represents the geoid undulation value that may facilitate the computation of the orthometric height from GNSS vertical height. Hence the derived geoid model could be either a continuous surface inspiring the geoid undulation at any position in the surface according to its coordinates (longitude and latitude) or by figuring out an equation that represents the geoid undulation geometrically which is defined here as a geometrical solution and also utilizing the longitude and latitude as main input factor. Both methods are used for geoid undulation computation for the specific limited area within Khartoum state to build a geoid model that offers a reasonable solution for the geoid undulation problem. Comparison between the two geoid computation methods has been carried out and showed suitable and durable statistical results for each model. The two models offer a quick and easy way of computing the geoid undulation values within acceptable accuracy and attempting to contribute to the practical use of GNSS ellipsoidal heights in the determination of orthometric heights in areas under consideration.

The study area is located within Khartoum state, composed of 44 benchmarks, observed by GNSS/leveling instruments, in which GNSS was carried out for the determination of the latitude and longitude of each benchmark, while precise leveling was carried out to obtain the orthometric height at each benchmark. The differences between the ellipsoidal and orthometric heights enable the calculation of the geoid undulation for the benchmarks, which would be as the norm for all geoid models that are going to be built later on. As stated, two methodologies for geoid model design were being used, gravimetric geoid model and geometrical geoid model. In the gravimetric geoid model, the GGM represented in EGM2008 was subjected to calculate geoid undulation for the benchmark positions in the study area using the three available grid size databases (1`*1`, 2.5`*2.5` and 10`*10`) offered by EGM2008. For the geometrical geoid model, a local deterministic interpolation was utilized to create a geoid undulation surface. For both GNSS/Leveling geoid undulation data and for their counterparts that are derived from the EGM2008, utilizing GIS capabilities, the two surfaces were then fitted to each other by predicting the general difference between the GNSS/Leveling geoid undulation and that computed from EGM2008. Hence the surface produced through EGM2008 has the same characteristics of that created by using GNSS/Leveling data and being tested by following cross-validation techniques to evaluate the validity of the geoid model and to obtain the required geoid undulation at the given location in the targeted area. The geometrical solution utilized the global interpolation to get the model plane equation which represents the whole targeted area GNSS/Leveling data, utilizing the least square solution to figure out model equation parameters (or coefficients) which would facilitate the geoid undulation calculation in any position using its coordinates (here UTM eastings and northings are used) as the main input in the equation. The geoid model was checked for validation using cross-validation techniques within the study area. Both methodologies were compared to each other statistically to figure out the eligibility of each model.

The two geoid models have been tested using two separate datasets one inherited from the main study area dataset as the result of splitting the data into a training dataset and testing dataset, while the other checking dataset was a separate ITRF2005 dataset and precise leveling for the other benchmarks group that part of them located within the study area while the others located beyond the study area. The tested result was recorded for both EGM2008 and geometrical geoid models to ensure the validity of each model. The EGM2008 geoid model was carried out by applying local deterministic interpolation techniques by using the checking dataset and ITRF dataset for evaluation and the RMSE was found to be 0.015m. The geometrical geoid model has also achieved an RMSE of 0.015m. For the geometrical geoid model utilization, the EGM2008 geoid undulation raw values were used for the parameters estimation in the geoid undulation computation for the sake of the geometrical geoid plane equation, and the result of the obtained dataset testing process achieved an RMSE of 0.035m.

2. Global Geopotential and Local Geometrical Geoid Models

The geoid is believed to be an equipotential surface which is considered to be relatively, close to the mean sea level, as the geoid is known as a constant potential surface which is not one of the main characteristics of the oceanic environment due to the tidal process of the ocean. The main role of the geoid is used as the reference for the vertical heights of points on the earth. Due to the similarity of the geoid shape to the earth shape and the perpendicularity of the plumb-line on the geoid so it is nominated to be suitable for applying all geometrical vertical height without significant error when compared to the ellipsoidal heights when used as vertical reference [4]. The relationship between the geoid and ellipsoid is known as geoid undulation (N), this value is very vital for the determination of all vertical observations because, it fixes the shape of the geoid and governs the relation between the three geodetic surfaces i.e. earth, geoid and ellipsoid surfaces. The geoid undulations range worldwide from −107 m to 85 m relative to the WGS 84 ellipsoid [7,9,13].

N can be calculated by equation (1) as follows:

$N = h - H$ -----------------------------(1) [7,9,13]

Where N: Geoid undulation, h: Ellipsoid height, and H: Orthometric height.

2.1. The global geopotential model (GGM):

The EGM2008 is used to calculate the geoid undulations considering further corrections being applied to ensure correctness of orthometric heights computations [5,6]. Since, GNSS observations offer only ellipsoidal heights, then for GNSS leveling, and according to Eq.1 the geoid undulation N is needed to calculate the orthometric height H. As it is commonly known geoid has three wavelengths, long, medium, and short. The GGM is used for calculating long wavelengths by applying well-known Earth Gravitational Models (EGM) which have several historical editions with different degrees and orders such as EGM84, EGM96, EGM2008, and XGM2019e. It is worth to be noticed that the degree and order of EGM express the harmonic coefficients where the more you increase them the more you get closer to the suitable values of the geoid undulation, N.

2.2. Creating continuous surface from point dataset: -

The study indicated that, the location of the study area dataset is with a high degree of importance, that the sample dataset order and amount have significant effects in the creation of continuous surface. Six types of sample datasets were considered in this study, namely regular sampling, random sampling, transect sampling, stratified random sampling, cluster sampling and contour sampling interpolation [4]. As well, the interpolation process is considered to be an essential factor in creating continuous surfaces. The process is categorized into two different methodologies, the global interpolation method and local deterministic method.

2.2.1. The Global interpolation

This method undertakes the concept of regression in order to create a model, either linear regression or using quadratic polynomial equation or any more degree according to the number of variables constituting the model. So least squares or any statistical methodologies could be used to figure out the model parameters, that, the excessive of data is obligatory to expand the scope of variables calculations and to minimize the residual errors as well [4]. The geoid undulation model equation using global method is given by.

$$N_g = \alpha_0 + \alpha_1 E_1 + \alpha_2 N_2 \text{ ------------------ } \quad (2)$$

Where α_0, α_1 and α_2 are the geoid model parameters. E_1 and N_2 are Eastings and Northings values of UTM projected coordinates, and Ng geoid undulation.

For the quadratic polynomial equation illustration is given by [1,3,8] as:

$$N_g = b_0 + b_1\,E + b_2\,N + b_3\,E^2 + b_4\,E.N + b_4 \text{--- } (3)$$

Where b_0, b_1, b_2, b_3 and b_4 are model parameters and E and N are Eastings and Northings values of UTM projected coordinates.

2.2.2. Local deterministic interpolation: -

The local deterministic method could be nearest neighbor (thiessen polygon), Inverse Distance Weighting (IDW), Spline method and Kriging. It is worth to be mentioned that, no interpolation technique has privilege on the other types of techniques, that, each interpolation method has its own characteristics which makes it eligible for specific interpolation purpose, only the standard statistical norm such as cross validation can judge the eligibility of the nominated technique [4]. The IDW method is commonly used in interpolation to figure out missing cell's value counting on the principle that the surface that would be interpolated should be location wise dependent variable. IDW mainly leans on the algebraic inverse of a distance raised to a numerical power and this power reflects the magnitude of the significance of the cell that needed to be interpolated. Eq.4 expresses the way that IDW follows to compute unknown attribute cells [4] as:

$$j = \frac{X_1/d_1 + X_2/d_2 + X_3/d_3 + X_4/d_4 + X_5/d_5}{1/d_1^2 + 1/d_2^2 + 1/d_3^2 + 1/d_4^2 + 1/d_5^2} \text{---------------------} (4)$$

X1, X2, X3, X4 and X5: known attribute points (points dataset), d1, d2, d3, d4 and d5: distances between cell J and the other dataset points and the attribute value at J. The kriging interpolation method assumes that distances and directions between points will express the extent of spatial autocorrelation, which can be exploited to clarify the variation in the surface. kriging method could be formulated in Eq.5.

$$\hat{z}(S_0) = \sum_{i=1}^{N} \lambda_i\, Z(S_i) \text{-----------------------------} (5)$$

Z(Si): the measured value at location i, λi: weight for the measured value at location i (combines between distance between measured value and predicted value and overall spatial arrangement of measured points), $\hat{z}$(S0): the prediction location and N: number of measured values[4]

2.3. The local geoid model using local deterministic interpolation approach: -

As stated, the main objective of this study is to determine the geoid undulation (N) value, which slightly differs from one position to another due to the waving in the earth's potential force and hence the existence of continuous surface inspiring the geoid undulation. Exploiting the local deterministic interpolation is a very useful process especially the transformation from discrete object status to continuous attribute surface [4], regarding continuous geoid undulation surface needs to lean upon some sort of norms to evaluate and assess the results, so discrete GNSS/leveling benchmarks deployed in the targeted area are very important to figure out the general trends of the geoid undulations by creating a surface using one of the interpolation local deterministic methodologies like IDW or Spline method to form uninterrupted surface. The EGM surface is fitted to the surface made by using GNSS/leveling benchmarks, and this fitting would make the EGM surface more comprehensive and reliable.

2.4. The local geoid model using global interpolation approach: -

Global interpolation addresses straightforward model creation for geoid model by making use of GNSS/leveling benchmarks, but this trend in its widest status depends on the availability of the benchmarks which are not possibly available now and then and meanwhile is better to be hybrid and compared with that data inspired by EGM's software's to make sure that, the model is checked and verified. If it is assumed, that the existing raw data set consists of K's number of GNSS/leveling coordinates values accompanied by their equivalent geoid undulation N's values, which result from the algebraic subtraction of orthometric height (H) from ellipsoidal height (h) so the following equation can represent the model as follows:

$$N_g = \alpha_0 + \alpha_1 * E + \alpha_2 * N \text{ ------------------- } (6)$$

Ng: geoid undulation, E, N: are UTM Eastings and Northings projected ordinates, α_0**:** bias $\boldsymbol{\alpha_1, \alpha_2}$**:** model parameters. It is wide clear that for **Eq 6** the linear equation can be figured out by three unknown coefficient values (**α_0, α_1, and α_2**), which are to be estimated. Formulating **Eq 6** in the form of a matrix would facilitate the solution process as follows.

$$\underbrace{\begin{bmatrix} N_{g\_A} \\ N_{g\_B} \\ N_{g\_C} \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} 1 & E_A & N_A \\ 1 & E_B & N_B \\ 1 & E_C & N_C \end{bmatrix}}_{A} * \underbrace{\begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix}}_{\hat{x}} \text{ ------------------------------ } (7)$$

$$\hat{x} = A^{-1} * B$$

$$\begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 1 & E_A & N_A \\ 1 & E_B & N_B \\ 1 & E_C & N_C \end{bmatrix}^{-1} * \begin{bmatrix} N_{g\_A} \\ N_{g\_B} \\ N_{g\_C} \end{bmatrix} \text{ --------------------- } (8)$$

If the number of equations equal to the number of unknowns (K=3), the previous solution will lead to unique solution. But in case, the whole dataset (K>3) is used, then least squares adjustment can be applied to estimate the most probable values for the three coefficients, and hence the ($\hat{x}$) can be estimated as well equation 9 as:

$$\hat{x} = (A^T.\ w.\ A)^{-1}.\ (A^T.\ w.\ B) \text{ ---------------- } (9)$$

w: weight matrix

Eq .9 could be formulated as follows

$$B = A.\ \hat{x} + v \text{ ---------------------------------- } (10)$$

Where v: residual error

$$v = B - A.\ \hat{x} \text{ ------------------------------------ } (11)$$

From Eq 10, the normal matrix is given by

$$N = A^T.\ w.\ A \text{ ---------------------------------- } (12)$$

The variance covariance matrix [2,3,10] Cx for unknowns is given by

$$C_x = \sigma^2_0.\ N^{-1} \text{ ------------------------------------ } (13)$$

$\sigma^2 0$ is given by equation 14 as follows:

$$\sigma^2_0 = \frac{v^T .\ v}{n - m} \text{ ---------------------------------- } (14)$$

n: number of equations and m: number of unknowns.

The training and testing datasets for cross validation approach is mainly based on the statistical concept used for the cross-validation methods, namely, K fold cross validation, leave one out (jackknife), and random sample consensus [4]. MATLAB R2015 has been used for code generation regarding the jackknife cross-validation method.

3. Study area, Data set, and software used: -

The geometrical local geoid model mainly depends on the observed GNSS/leveling points which would form the backbone of the study to carry out all comparison processes and to ensure that the integrity and the approval constraints are not to be invaded. The targeted area of the study is located in Khartoum state, extending from latitude 15.43° N up to 15.62° N and from 32.47° E to 32.67°E covering about 310 km^2 area, containing 44 GNSS/leveling benchmarks that have been observed using GNSS and level instruments for the determination of ellipsoidal and the orthometric heights. Figure 1 shows a satellite image portraying the targeted area and the distribution of the observed benchmarks dataset. Other existing benchmarks in the area, have been used to check and validate both the EGM2008 and geometrical models. Some of these benchmarks are located north and southbound of the study area and the rest are within the study area the only difference between them and the main benchmarks study dataset they are referenced to ITRF2005.

3.1. Adopted and developed Geoid model software

3.1.1. EGM2008 undulation software: -

known as All Trans EGM2008 software, is an open source and tremendous software that is facilitated by using a software license agreement and allows the developers or the users to amend and redistribute it through legal publication. All Trans calculator version 1.2 has been used for generating geoid undulation N values in any position on the earth. The wisdom behind choosing this software based on several justifications appears in the great flexibility of the software which enables both generating geoid undulation for a single position or permitting the upload of files containing a bunch of positions to be handled simultaneously. It is worth noticing that the software generates the geoid undulation of a grid database counting on 10`.10`, 2.5`. 2.5` and 1`. 1` (2.5`. 2.5` and 1`. 1` grid database usually used as external file), moreover the software offers the results using four interpolation techniques, Bi-quadratic interpolation, Bi-linear interpolation, Nearest neighbor, and Triangulation. Hence the users have the freedom to choose the suitable interpolation technique that would satisfy their accuracy requirements.

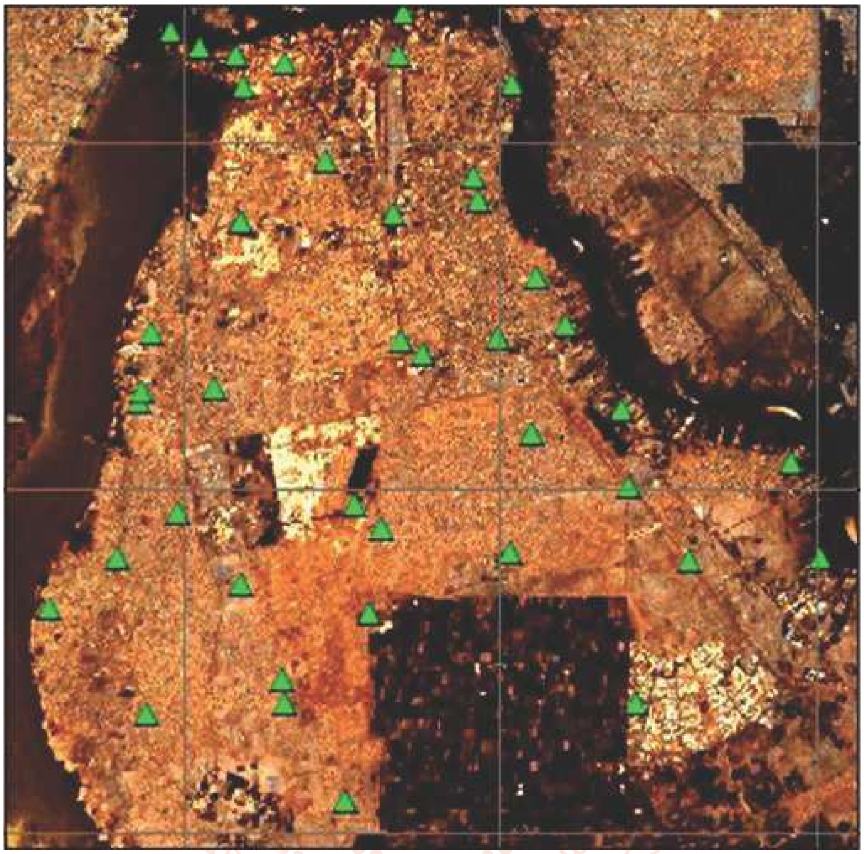

***Figure 1*. Study area and the location of 44 GNSS/Levelling benchmarks**

Here the dataset refining process - from a statistical point of view - consisted of geodetic coordinates obtained by GNSS, (Ø, λ) of benchmarks with their known orthometric heights, which have been processed into AllTrans software to get geoid undulation. Then these geoid undulation values are compared to the ones obtained from GNSS/leveling (N' values). So, the difference between N's values obtained from GNSS/leveling and the N values computed by EGM2008 software will represent the predicted values δN, which are to be algebraically added to EGM2008, N's values, so improved EGM2008 N's values will be prepared for the local deterministic interpolation utilizing both IDW and Kriging techniques to create both two surfaces one for improved N's EGM2008 values and the original GNSS/leveling dataset N's values. Contour lines will be generated and added to both surfaces to show the geoid undulation general trends and to clarify the similarity between the two surfaces. The improved EGM2008 surface has been tested using a testing data set to show to which extent that the surface achieves the main goal of the research study. GIS techniques have been utilized for creating surface models for both GNSS/Levelling N's values and their counterparts amended EGM2008 [1,3]

3.1.2. Geoid model designing using geometrical method approach: -

The geoid undulation calculation using the geometrical method depends mainly on Eq 6 for calculating the values of α0, α1, and α2 which enables the calculation of geoid undulation N value for any geodetic coordinates within the entire study area. Least squares adjustment has been utilized to calculate the parameters of Eq 8, to get the most probable values of these parameters. The covariance matrix has been obtained as given in equations Eq 9, 10, 11, 12, and 13. Finally, the geometrical undulation equation has been checked by using the testing dataset to prove the reliability and feasibility of the equation. Moreover, the equation has been tested by using another dataset (ITRF2005) located outbound of the targeted area to figure out how the geometrical equation would react towards outbound locations and to which geometrical extent the equation would be serviceable.

3.1.3. Geoid gravimetric surface model creation and analysis intended software:

Geoid model surface designing is considered to be essential for the avails that it could be inspired by the surface attribute in all over the research targeted area and also for comparison purposes especially when two surfaces are created and the difference between them considered to be significant. There are several software offering interpolation and surface design techniques and also comparisons and variation methodologies. For this purpose, ArcMap version 10.4 has been used in this study to take over surface designing and contour line graphing which is considered to be essential in presenting geoid model trends. Hence the use of GIS software is indispensable especially when commencing a gravimetric geoid model.

4. Result and analysis

Forty-four GNSS/leveling benchmarks within Khartoum State- **Figure.1-** have been observed in well-distributive positions containing the geodetic coordinates and their counterpart's universal transverse marketer (UTM) projected coordinates as well as the ellipsoidal and orthometric heights and the geoid undulation value. As given in Eq1, N represents the difference between ellipsoidal and orthometric heights, which is simply expressed as algebraic subtraction between the ellipsoidal and orthometric heights, central tendency measurements and statistical processes are used to estimate the standard deviation σ and average μ for N_{GNSS} as follows:

Undulation type	μ (m)	σ (m)
N_{GNSS}	2.551672727	0.12292724

σ: standard deviation

μ: mean value

4.1. EGM2008 data analysis: -

The EGM2008 represented in All Trans EGM2008 Calculator software with 1`*1`,2.5`*2.5` and 10`*10` grid sizes database have been used for calculating the geoid undulations of the forty-four benchmarks using their latitude (Ø) and longitude (λ) as an input data in the software. Four interpolation techniques, Bi-linear, Bi-quadratic, nearest neighbor, and triangulation have been used for calculating geoid undulation for 1`*1` grid size database in detail, while 2.5`*2.5` and 10`*10` grid size database have been involved just for comparison purposes to 1`*1` grid size database.

The four types of the AllTrans EGM2008 Calculator software interpolation output have been evaluated according to the central tendency measurements and statistical process that the four interpolation techniques have undergone for assessment to choose the suitable data that would be forwarded to the next refining methodology and the results were prepared as illustrated in the following tables.

The standard deviation, σ and the average, μ for the four interpolation techniques are tabulated from EGM2008 and δN (N_{GPS} - N_{EGM}) as follows.

Table 1. **Standard deviation σ and average μ for Bi-linear interpolation to obtain N_{EGM} and δN**

Interpolation type	μ for N_{EGM}	σ for N_{EGM}	μ for δN	σ for δN
Bi-linear 1`*1`	2.2605	0.126594288	0.291172727	0.072733631
Bi-linear 2.5`*2.5`	2.261136364	0.126521933	0.290536364	0.072701041
Bi-linear 10`*10`	2.270295455	0.123064576	0.281377273	0.071219031

Table 2. **Standard deviation σ and average μ for Bi-quadratic interpolation to obtain N_{EGM} and δN**

Interpolation type	μ for N_{EGM}	σ for N_{EGM}	μ for δN	σ for δN
Bi-quadratic 1`*1`	2.260340909	0.126686611	0.291331818	0.072788803
Bi-quadratic 2.5`*2.5`	2.260340909	0.126711904	0.291331818	0.072779904
Bi-quadratic 10`*10`	2.270295455	0.123064576	0.281377273	0.071219031

Table 3. **Standard deviation σ and average μ for Nearest neighbor interpolation to obtain N_{EGM}**

Interpolation type	μ for N_{EGM}	σ for N_{EGM}	μ for δN	σ for δN
Nearest Neighbor 1`*1`	2.258022727	0.130093077	0.29365	0.073885627
Nearest Neighbor 2.5`*2.5`	2.271159091	0.133718827	0.2926	0.074690615
Nearest Neighbor 10`*10`	2.27875	0.161114883	0.272922727	0.139613982

Table 4. **Standard deviation σ and average μ for Triangulation interpolation to obtain N_{EGM} and δN**

Interpolation type	μ for N_{EGM}	σ for N_{EGM}	μ for δN	σ for δN
Triangulation 1`*1`	2.259454545	0.128080726	0.292218182	0.073704467
Triangulation 2.5`*2.5`	2.257181818	0.127687166	0.294490909	0.073945446
Triangulation 10`*10`	2.26075	0.126160634	0.290922727	0.073180104

It is quite noticeable that Bi-linear, Bi-quadratic, Triangulation and the nearest neighbor interpolation methods are slightly akin to each other regarding the averages and the standard deviations for δN. Standard deviation values could be undertaken for differentiation between the four techniques which give a reasonable statistical norm for refining datasets. From the above tables, it is noticed that Bi-linear interpolation has the minimum standard deviation value regarding δN which would qualify it to be forwarded for further statistical process.

4.2. dataset 3D representation: -

For further analysis, the data is processed, regarding Bi-linear interpolation which has undergone three-dimension axis representation, to evaluate the extent of the approach, to which, common value nomination would represent the model. For the sake of graph representation, the UTM coordinates are used due to the simplicity of this projection in graphing aspects. The geodetic coordinates have been used in Alltrans N_{EGM}'s values computation so the UTM axes would be represented as X-axis: Eastings coordinates, Y-axis: Northings coordinates and Z-axis for δN values.

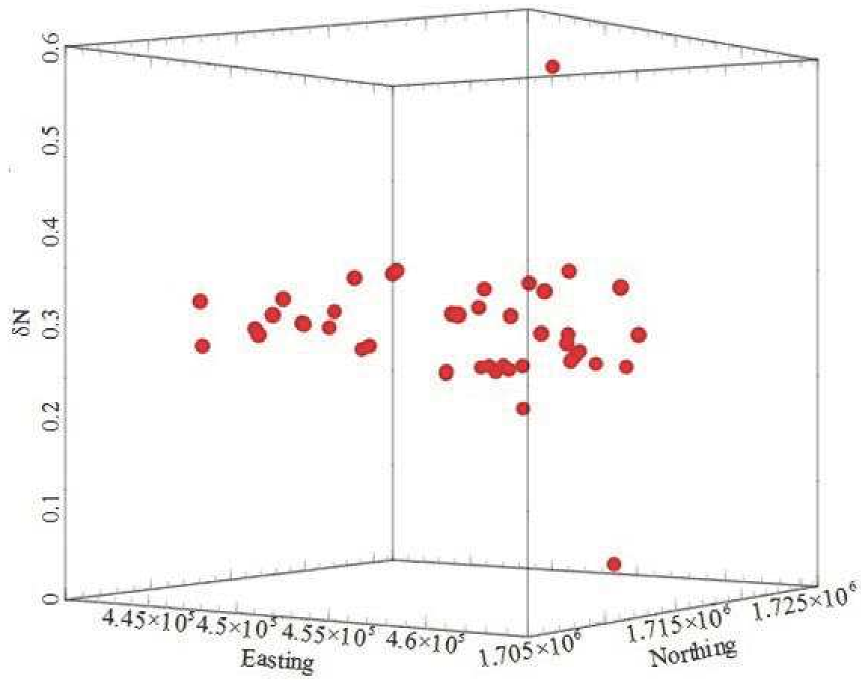

Figure 2. **Prototyping of plotting the UTM coordinates of the 44 benchmarks and their δN values**

4.3. N_{GNSS} and N_{EGM} suitability: -

According to **table.1**, the mean value of the difference between N_{GPS} and N_{EGM} (δN) is 0.290m so this value would be used as fitting factor (or surface corrector) between the N_{GNSS} and N_{EGM} surfaces.

Table 5. **RMSE, reaction towards the mean value 0.290**

δN predicted	μ	σ	RMSE for δN
0.290	0.291	0.074	0.073

RMSE: root mean square error

From **table.5** an essential notice that may arise is that **$\delta N_{predicted=}$ 0.290** has the best fit due to mediocrity and the lowest RMSE value which may qualify this value to be represent N_{EGM} differences from N_{GPS} values (**δN**) as common value or it would be used as fitting factor which may suit both N_{GPS} and N_{EGM} systems.

$$\text{Improved } N_{EGM} = N_{EGM} + 0.290 \text{ ---------------------------------- (15)}$$

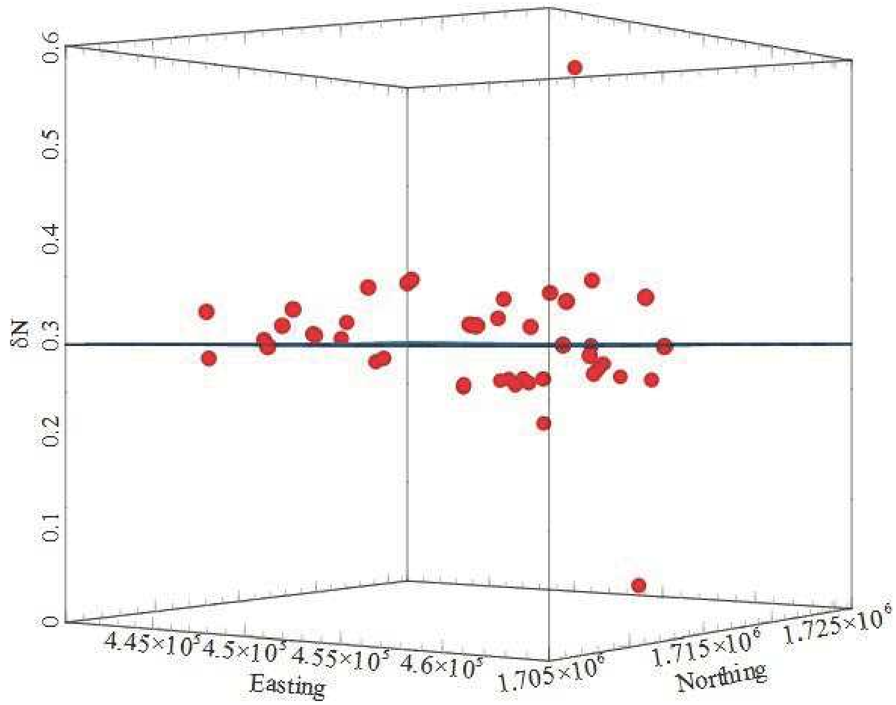

Figure 3. **3D plane passing the data using δN at 0.290 as mean value**

4.4. Sweeping Out outlier's ghost: -

For more improvement, the dataset's outliers should be swept out by applying traditional normal distribution at confidence level 95%. Here it can be figured out that, part of the dataset would be disappeared from the bundle due to its unsuitability with the specified confidence level. The calculated critical value which suites 95% confidence level, would be equal to 1.960*σ. This value would be added to the mean and subtracted from it as well to clarify the upper and lower bound of the dataset, which, may create a new dataset that improves the RMSE result.

For 95 % confidence the critical value is equal to 1.960, then:

Upper bound= mean value + critical value -------------------------------- (16)

Lower bound= mean value - critical value -------------------------------- (17)

From Table.5, $\mu_{\delta N}$= 0.291, and $\sigma_{\delta N}$= 0.074

Upper bound= $\mu_{\delta N} + 1.960 * \sigma_{\delta N}$ -- (18)

Lower bound= $\mu_{\delta N} - 1.960 * \sigma_{\delta N}$ -- (19)

Upper bound= 0.291+1.960 * 0.074= 0.436------------------------------ (20)

Lower bound= 0 .291- 1.960 * 0.074= 0.146----------------------------- (21)

$\mu_{\delta N}$: the mean value for δN

$\sigma_{\delta N}$: the standard deviation for δN

So, the dataset regarding δN should be in the range of 0.146 ≤ δ**N** ≤ 0.436. According to the 95% confidence level, and from the statistical analysis, it can be pointed out that two positions in the main dataset have δN values 0.602m and 0.020m consecutively, which would be out of range in according to the enumerated confidence level so the dataset will be diminished to 42 benchmark positions instead of the previous 44 positions. The statistical analysis would react differently in the new dataset by using **δ**$N_{predicted}$ 0.290, regarding the RMSE towards NDIFF which witnessed a substantial improvement to catch 0.036 m which would be considered as great RMSE enhancement.

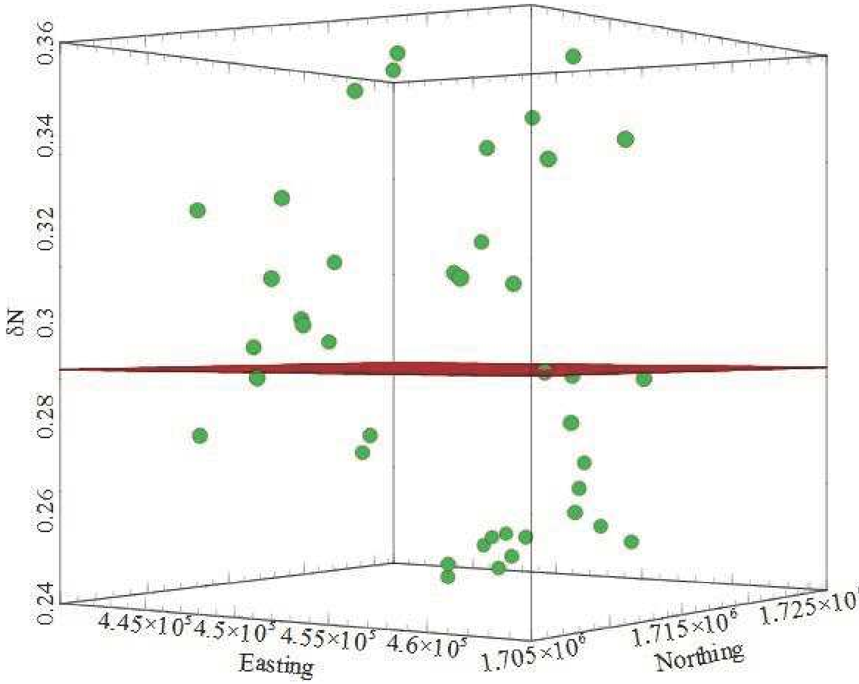

Figure 4. **95 % confidence level data set and the plane passing at δN=0.29**

4.5. Dataset cross validation process: -

The principle of cross validation is built on dividing the dataset into two datasets, one for the training dataset and the second for testing dataset. From previous processes, new dataset has been obtained due to the 95% confidence level containing 42 elements. If the whole dataset is denoted by N and the training dataset by K, the testing dataset would be (N-K), as given, that, the whole dataset N=42, the Training dataset K=38, and the Testing dataset N-K=4.

4.6. Local deterministic interpolation using AllTrans (1`*1`, 2.5`*2.5` and 10`*10`)

Inverse distance weighting (IDW) and Kriging have been used for the interpolation process for both the improved N_{EGM} and N_{GNSS} and continuous surfaces for both N_{GNSS} and the improved N_{EGM} have been created within the study area utilizing IDW and Kriging techniques. A clear similarity between the two surfaces has been noticed, which reflects, that the two surfaces are akin to each other, as well as the approach of attribute values in both surfaces. Contour lines with an interval of 0.020 m have been added to the four surfaces to show general trends of the geoid undulation regarding IDW and Kriging interpolated surfaces for both improved N_{EGM} and. compatibility of contour lines are well recognized between improved N_{EGM} and N_{GNSS} in both IDW and Kriging interpolation techniques [5,6,11,12].

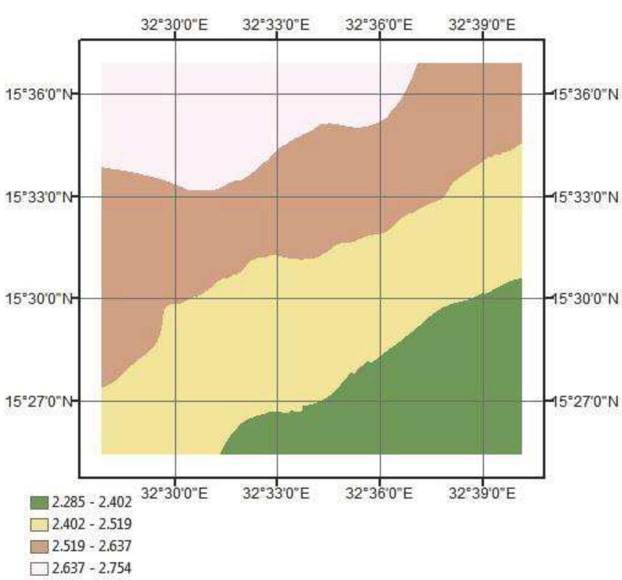

Figure 5. **Improved N_{EGM} IDW surface grid size 1`*1`**

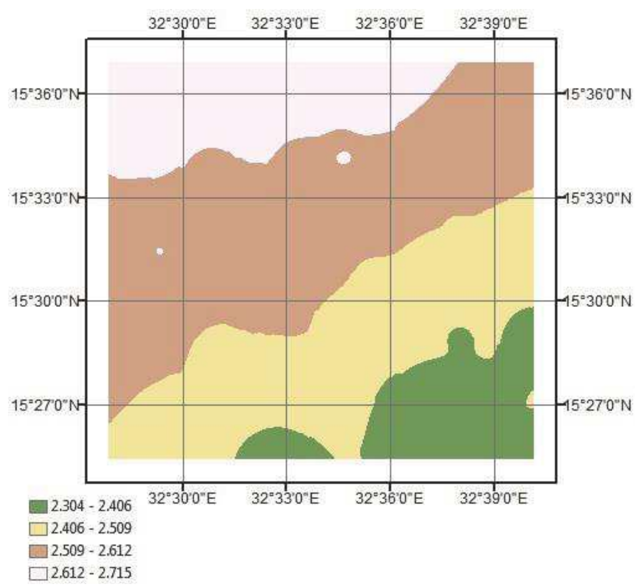

Figure 6. **N_{GPS} IDW surface**

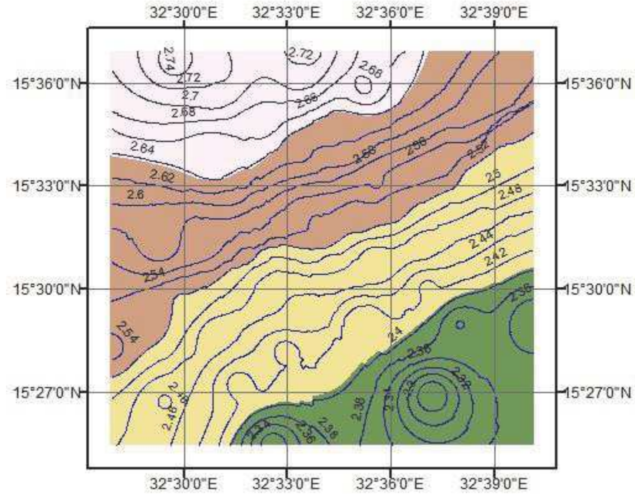

Figure 7. **Improved N_{EGM} IDW contour representation**

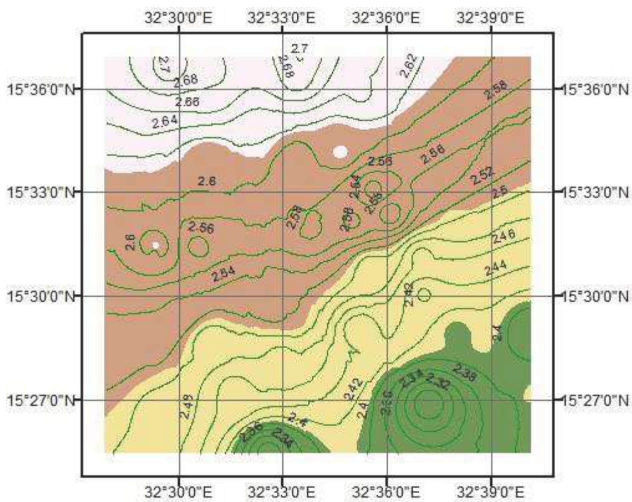

Figure 8. **N_{GNSS} IDW contour representation**
*** all contour lines are drawn at interval of 0.020 m**

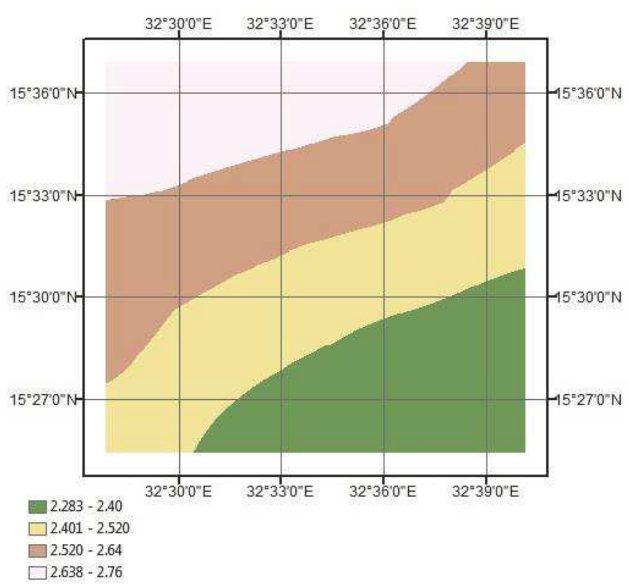

Figure 9. **Improved N_{EGM} Kriging surface 1`*1 grid size**

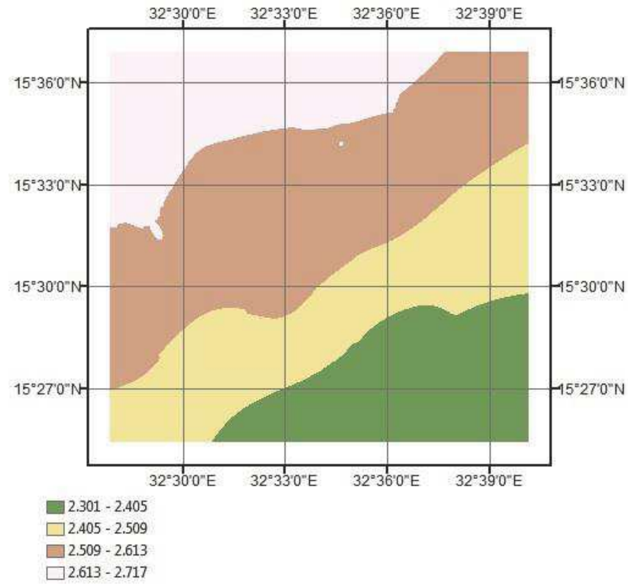

Figure 10. **N_{GPS} Kriging surface**

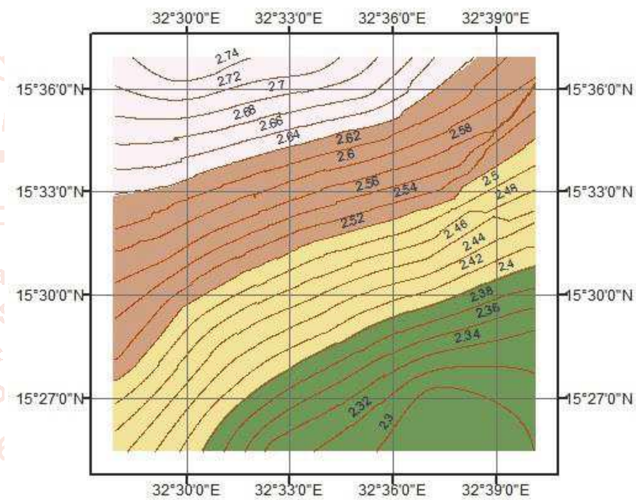

Figure 11. **N_{EGM} Kriging contour representation**

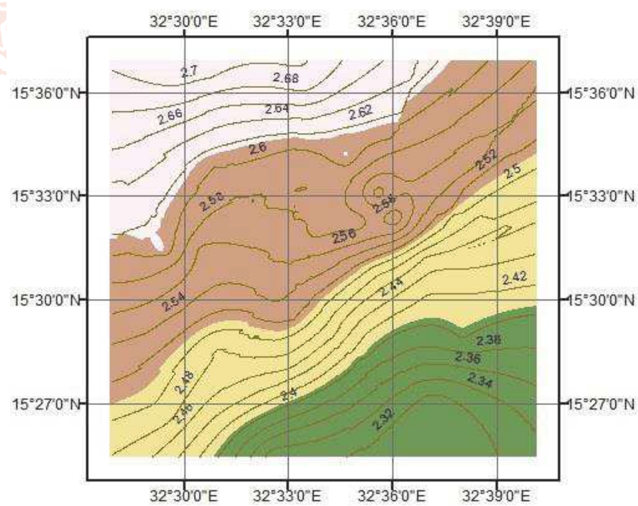

Figure 12. **N_{GNSS} Kriging contour representation**
*** all contour lines are drawn at an interval of 0.020 m**

4.6.1. Surfaces verification check using testing dataset: -

The mechanism followed in the testing process was that the attribute value (N_{EGM}) of the improved N_{EGM} surfaces that were being created using either IDW or Kriging interpolation techniques were extracted from the surface for the four points of the testing dataset and the other five points of the ITRF2005 benchmarks that existed within the study area.

4.6.1.1. Improved N_{EGM} IDW surface (1`*1` grid size) suitability check:

All testing tables for the N_{EGM} obtained from either IDW or Kriging surfaces that are based upon AlltTrans 1`*1` grid size resolution dataset for checking purposes with a common table header that consists of Point ID (point identification), latitude, longitude, N_{GPS} from GNSS/leveling, N_{EGM} value inspired from either N_{EGM} IDW or Kriging surfaces and difference which is algebraic subtraction between N_{GPS} and N_{EGM} IDW or N_{EGM} Kriging surfaces

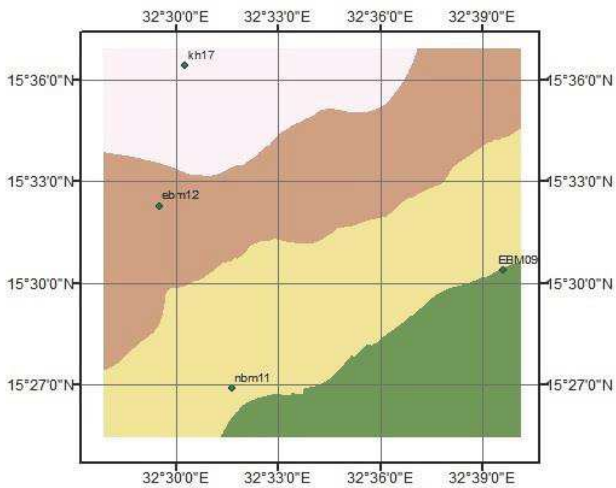

Figure 13. **Testing dataset locations screenshot**

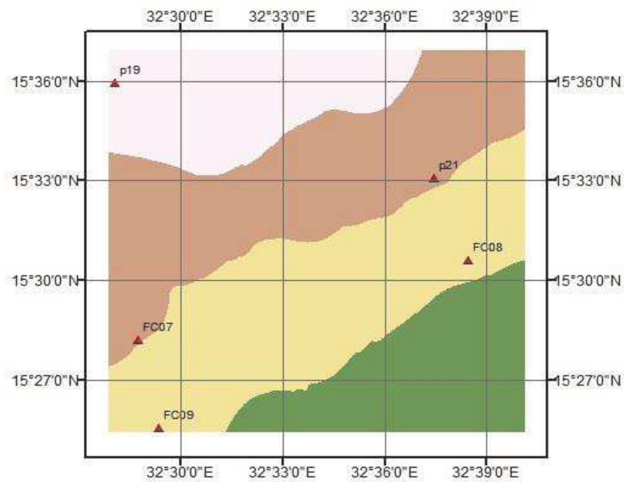

Figure 14. **ITRF2005 benchmarks within the study area**

Table 6. **N_{EGM} IDW surface (1`*1` grid size) check using testing dataset**

Point ID	Lat	Long	N_{GPS}	N_{EGM}_IDW	DIFF
EBM09	15.506115	32.660443	2.4035	2.399526	0.003974
ebm12	15.537832	32.491423	2.5972	2.589437	0.007763
kh17	15.606981	32.504353	2.7043	2.733429	-0.029129
nbm11	15.44784	32.527045	2.4193	2.41776	0.00154
Standard deviation for N_{EGM}_IDW_surface			**0.136**	**RMSE**	**0.015**

Table 7. **checking improved N_{EGM} Kriging surface (AllTrans 1`*1` grid size testing dataset**

Point ID	Lat	Long	N_{GPS}	N_{EGM}_ Krig	DIFF
EBM09	15.506115	32.660443	2.4035	2.389474	0.014026
ebm12	15.537832	32.491423	2.5972	2.612265	-0.015065
kh17	15.606981	32.504353	2.7043	2.743536	-0.039236
nbm11	15.44784	32.527045	2.4193	2.399717	0.019583
Standard deviation for N_{EGM}_ Kriging _surface			**0.149**	**RMSE**	**0.024**

4.6.1.2. Five ITRF 2005 referenced benchmarks dataset: -

Five ITRF2005 referenced and precise levelling measured benchmarks lays within the study area that were being used for checking the surfaces for both IDW and kriging N_{EGM} surfaces.

Table 8. **Improved N_{EGM} IDW surface check using ITRF2005 dataset (1`*1` grid resolution)**

Point ID	Lat	Long	N_{GPS}	N_{EGM}_IDW	DIFF
p19	15.59973086	32.46729877	2.692	2.697	-0.005
p21	15.55176939	32.62417031	2.517	2.528	-0.011
FC07	15.47064469	32.47885226	2.466	2.52478	-0.05878
FC08	15.51023597	32.64095474	2.451	2.42928	0.02172
FC09	15.42593284	32.48917495	2.391	2.463279	-0.072279
Standard deviation for N_{EGM}_IDW_surface			**0.092**	**RMSE**	**0.043**

Table 9. **Improved N_{EGM} Kriging surface check using ITRF2005 dataset (1`*1` grid resolution)**

Point ID	Lat	Long	N_{GPS}	N_{EGM}_ Krig	DIFF
p19	15.59973086	32.46729877	2.692	2.719	-0.027
p21	15.55176939	32.62417031	2.517	2.541	-0.024
FC07	15.47064469	32.47885226	2.466	2.520841	-0.054841
FC08	15.51023597	32.64095474	2.451	2.415495	0.035505
FC09	15.42593284	32.48917495	2.391	2.434291	-0.043291
Standard deviation for NEGM_ Krig _surface			**0.109**	**RMSE**	**0.039**

For the analytical aspect of the gravimetric geoid model which is counting on EGM2008, we figure out that the results which have been obtained above, using the 1`*1` grid size resolution database surface, it could be noticed that when using the testing dataset, the 1`*1` grid scored the least RMSE which is 0.015m for IDW and 0.024m for Kriging interpolation techniques. When applying the ITRF2005 dataset, the RMSE was found to be 0.043m and 0.039m for IDW and Kriging respectively. When applying the testing dataset for the 2.5`*2.5` grid size resolution the RMSE was found to be 0.016m and 0.025m respectively for both IDW and Kriging interpolation techniques. When applying the ITRF2005 dataset in the same grid size resolution, the RMSE scored 0.045m and 0.039m for IDW and Kriging consecutively. Using the same assessment process, when applying the testing dataset for the 10`*10` grid size resolution, the RMSE was found to be 0.019m and 0.026m for the IDW and Kriging interpolation techniques consecutively, and for the ITRF2005 dataset, the RMSE was being 0.048m and 0.042m for IDW and Kriging. Overall, the results showed that, the smaller the grid size, the better the RMSE values.

In comparing, the local deterministic interpolation IDW and kriging, it can be observed that the IDW presented well-countable results, i.e. most of the results expressed standard deviations in IDW lesser than kriging in the testing dataset, although Kriging interpolation showed some improvement in ITRF2005 in comparison to IDW. Here it can be concluded that the IDW, 1`*1` N_{EGM} surface is countable and could be used for predicting geoid undulation values within the study area with reasonable RMSE range from 0.015m to 0.043m.

4.7. Geoid undulation using GPS/leveling geometrical solution approach: -

Regarding geometrical solution Eq.6 considered as the main guidance in the geometrical solution. The 38 benchmark training datasets were being used to compute the three parameters α0, α_1, and α_2 within the targeted area. Jackknife's statistical cross-validation technique has been utilized for improving the solution. MATLAB code has been used for a jackknife solution in which the code will leave one point out and calculate the model, then substitute the left point into the model equation to get the geometrical error of the left point so an iterative solution would be followed to get geometrical errors from which the least one would be chosen and its equivalent dataset would be authorized as main training dataset and its accompanied parameter α_0, α_1, and α_2 would be undertaken. In the analysis, the geometrical error obtained by every point that was left out when substituting it in the model equation was the least geometrical error that was being scored at benchmark **kh11** and was found to be 0.002 m. hence the α0, α1, and α2 values that led to this result is as follows

$\alpha_{0=}$ -19.4365713901061, $\alpha_{1=}$ -7.58018069521756e-06, $\alpha_{2=}$ 1.48000742463961e-05

then the geometrical undulation equation is given by

$\mathbf{N_{geometric=}}$ -19.4365713901061**-** 7.58018069521756e-06. **E +** 1.48000742463961e-05. **N ---(22)**

$\mathbf{N_{geometric}}$: Geoid undulation calculated using a geometrical solution

Table.10 illustrates the comparison between $\mathbf{N_{GNSS}}$ and $\mathbf{N_{geometric}}$ when using the testing dataset

Table 10. **GPS/leveling geometrical model verification check using the testing dataset**

Point ID	Easting	Northing	N_{GNSS}	$N_{geometric}$	DIFF
EBM09	463582.193	1714334.621	2.4035	2.422	-0.018
ebm12	445462.636	1717878.904	2.5972	2.611	-0.014
kh17	446866.913	1725524.405	2.7043	2.714	-0.010
nbm11	449260.729	1707915.884	2.4193	2.435	-0.016
Standard deviation for $N_{geometric}$			**0.1227**	**RMSE**	**0.015**

Table 11. **Geometrical model verification using ITRF2005 dataset that lays within the study area**

Point ID	Lat	Long	N_{GNSS}	$N_{geometric}$	DIFF
p19	442892.6695	1724732.044	2.692	2.732384453	-0.040384453
p21	459700.7235	1719391.065	2.517	2.525929481	-0.008929481
FC07	444096.486	1710450.287	2.466	2.511888242	-0.045888242
FC08	461492.7881	1714793.895	2.451	2.44430685	0.00669315
FC09	445192.081	1705501.931	2.391	2.430347398	-0.039347398
Standard deviation for $N_{geometric}$			**0.108**	**RMSE**	**0.033**

The GNSS/leveling geoid undulation and geometrical model verification using the ITRF2005 dataset was also made and the results are shown in Table 19.

The covariance matrix C_{x_EGM} is calculated according to Eq.9 up Eq.14 and is given by

$$C_{x\_EGM} = \begin{bmatrix} 0.43452282817471 & -9.30066159779651e-08 & -2.28627336223442e-07 \\ -9.30066159779651e-08 & 1.72406514647727e-13 & 8.80407588846624e-15 \\ -2.28627336223442e-07 & 8.80407588846624e-15 & 1.30856658896956e-13 \end{bmatrix}$$

4.8. Geometrical Geoid undulation solution utilizing EGM2008 instead of GNSS/Leveling:

In many cases the availability of GPS/leveling data is not always in our hands hence Instead of using N_{GPS} we can use N_{EGM} directly obtained from AllTrans software and follow the same procedure by processing the original N_{EGM} into MATLAB code to estimate the parameters, α_0, α_1, and α_2 values and the data would be subjected to jackknife refining as well. The α_0, α_1, and α_2 that equivalent to the least geometrical error obtained by applying a jackknife and it was being scored by point nbm12 (0.00036938) as follows

$\alpha_{0=}$ -26.5865279624704

$\alpha_{1=}$ -9.51494528177166e-06

$\alpha_{2=}$ 1.93043110635754e-05

$N_{geometric=}$ -26.5865279624704 **-** 9.51494528177166e-06. **E +** 1.93043110635754e-05. **N---------------------** (23)

When the δN value (0.290m) is added to Eq.15, the result would be geoid undulation value (N) hence Eq.19 could be reformulated as follows

$N_{geometric=}$ -26.5865279624704 **-** 9.51494528177166e-06. **E +** 1.93043110635754e-05. **N+** δN **------** (24)

Table 12. **Geoid undulation geometrical model using raw EGM2008 undulation data verification check using testing dataset.**

Point ID	Easting	Northing	N_{GNSS}	$N_{geometric}$	DIFF
EBM09	463582.193	1714334.621	2.4035	2.386561628	0.016938372
ebm12	445462.636	1717878.904	2.5972	2.627388163	-0.030188163
kh17	446866.913	1725524.405	2.7043	2.761617674	-0.057317674
nbm11	449260.729	1707915.884	2.4193	2.398920279	0.020379721
Standard deviation for $N_{geometric}$			**0.158**	**RMSE**	**0.034**

Table 13. **Geoid undulation geometrical model using raw EGM2008 undulation data verification check utilizing ITRF2005 dataset that lays within the study area**

Point ID	Lat	Long	N_{GNSS}	$N_{geometric}$	DIFF
p19	442892.6695	1724732.044	2.692	2.7841364	-0.0921364
p21	459700.7235	1719391.065	2.517	2.521104766	-0.004104766
FC07	444096.486	1710450.287	2.466	2.496982672	-0.030982672
FC08	461492.7881	1714793.895	2.451	2.41530817	0.03569183
FC09	445192.081	1705501.931	2.391	2.391033542	-3.35425E-05
The standard deviation for $N_{geometric}$			**0.140**	**RMSE**	**0.046**

4.9. Gravimetric or geometrical geoid model privilege: -

The application of the gravimetric or geometrical geoid models depends upon the required accuracy. From the implementation of both methods, it could be seen that the RMSE values are almost identical when used for the testing dataset for both gravimetric surface and geometrical geoid model, in which, the RMSE is equal to 0.015m. However, still, the geometrical solution is believed to be more practical and its process is easier to implement in comparison to the gravimetric model. But in most cases, the GNSS/Leveling benchmark values are not available, so the alternative method is to adopt and merge the two methodologies by using EGM2008 undulation values and process the data by applying geometrical solution using this hybrid to compute the geoid undulation without carrying out any leveling observations and it is very vital to be noticed that the general difference between geoid undulation that given by GNSS/Levelling benchmarks and that inspired from EGM2008 (δN) in the study area should be known as general trend for training area to facilitate the practicability of the geometrical equation.

5. Conclusions: -

In recent years the determination of the gravimetric and geometrical geoid model became essential in most geomatics and engineering applications. So, in this study, the two methods of geoid determination were applied, assessed, and compared for the sake of exposing some sort of enlightenment to this dilemma but still, this difference between geoid and ellipsoid needs more effort to be exerted to achieve improvement. One of the future challenges that how to develop this model to be built in the GNSS receivers to directly calculate the geometrical heights with an acceptable tolerance which would be considered very convenient within the boundary of the of the study area and the geoid undulation geometrical model methodology would be more nominated to be utilized for the GNSS instruments application due to the mathematical nature of this method and most of the equations could be programmed by any suitable programming language to suit GNSS receivers operating systems.

The geoid model utilizes both GNSS/leveling benchmarks which have been considered as the norm for comparison. The EGM2008 geoid model was used to obtain the geoid undulations in the in-training area and more geometrical solutions have contributed to geoid undulation computation. The obtained geoid undulation values from the two methods were compared and the final result was being as a continuous surface that offers geoid undulation at any position in the area under consideration, with RMSE from 0.015m up to 0.043m when utilizing Alltrans software EGM2008 1`*1` grid size resolution. The geometrical model is applied by using the GNSS/leveling benchmarks for creating a mathematical model for the study area to facilitate geoid undulation computation, which resulted in the determination of geometrical geoid undulation solution with RMSE from 0.015m up to 0.033m. Here it can be stated that, for many applications, the absence of orthometric heights of the benchmarks may not hinder the obstinance of geoid undulation with reasonable accuracy when using raw EGM2008 geoid undulation data and to be processed as input values to solve geoid undulation geometrical solution parameters with the knowledge of δN value of the targeted area and that achieved RMSE in the study that extend from 0.034m up to 0.046m. The ambiguous value of the geoid undulation will remain as the world of geodesy major interest that needs more deep learning and more research to be fully demystified particularly within the gigantic development in the field of computer and engineering modern capabilities.

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References: -

[1] Abdalla, K. A (2006). Uncertainties of Al Ain Geometrical Geoid. Conference: Map Middle East 2008 Conference, Volume: Proceedings of Map Middle East 2008 Conference, Dubai World Trade Center. https://www.researchgate.net/publication/330566688- Uncertainties of Al Ain Geometrical Geoid.

[2] Abdalla, K. A. and Hassan H. Fashir (1997). Space Geodesy for Monitoring Deformations and Datum Transformation in Sudan-1997. Sudan Engineering Society Journal.

[3] Ahmed A. (2019). A Research paper submitted as the partial fulfillment of the requirement for the award of the degree of Master of Science in Geomatics Engineering, University of Khartoum, Sudan, academic year 2019.

[4] Burroughm McD. (1998). principles of geographical information systems. Oxford University Press; 333 p.

[5] Chymyrov A. . 2016. Analysis of EGM2008 as a Geoid Model for Kyrgyzstan.p 1-9.

[6] Dawod G, Mohamed H, Ismail S. (2009). Evaluation and adaptation of the EGM2008 geopotential model along the Northern Nile Valley, Egypt: Case study. J Surv [Internet]. FEBRUARY):2008–12. Available from: http://ascelibrary.org/doi/pdf/10.1061/(ASCE)SU.1943-5428.00000024

[7] Hofmann-wellenhof B. (2005). Physical Geodesy. Austria: Springer-Verlag Wien. 396 p.

[8] Featherstone, W. E, Denfith, M. C and Kirby, J. F (1998). Strategies for the Accurate Determination of orthometric Heights from GPS. Survey review, 34, 267.

[9] Hofmann, B. LH. (2008).GNSS – Global Navigation Satellite Systems GPS, GLONASS, Galileo, and more. Springer-Verlag Wien. 516 p.

[10] Oluyori1, P. D., Ono, M. N2. and Eteje SO3. (2018). Comparison of Two Polynomial Geoid Models of GNSS / Leveling Geoid Development for Orthometric Heights in FCT, Abuja. ;(10). p 1–9.

[11] Sami K. A. and Aborida A. M. a:(2022). On the Evaluation of the Neural Network Khartoum Geoid Model. . *American Journal of Engineering and Technology Management*. Vol. 7, No.4, 2022, pp.147-151.

[12] Sami K. A. and Aborida A. M. b:(2023). On the Evaluation of the Derived Khartoum Geoid Models. *American Journal of Engineering and Technology Management*. Vol. 8, No. 2, 2023, pp.21 – 26.

[13] Seeber G. (2003). Satellite Geodesy. Austria: Springer-Verlag Wien. 589.