International Journal of Trend in Scientific Research and Development (IJTSRD)
Special Issue on Innovative Strategies in Multidisciplinary
Research and Analysis | February 2024
Available Online: www.ijtsrd.com e-ISSN: 2456 – 6470

Moderation and Diffusion of Neutrons in the Reactor and its Multiplication Properties in the Field of Methodology

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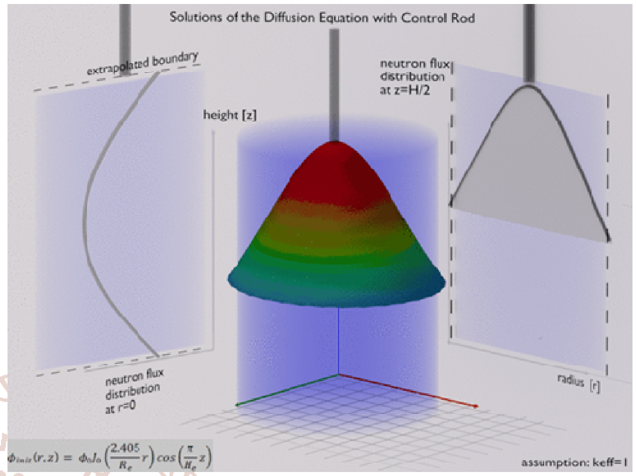
ABSTRACT

Conceptual study on diffusion and moderation of neutron in nonmultiplying and multiplying medium in nuclear reactor have been conducted by using mathematical explanation, experimental data and graphical representation. The corresponding governing equations have been explained. A brief discussion on neutron and criticality of a reactor is carried on. Necessity of a moderator medium in slow reactors has been discussed. Comparative analysis of usefulness and drawbacks as moderator for different medium (Graphite, Beryllium, light water, heavy water and Lithium Fluoride) have been studied in aspects of different features. Mathematical example of calculating numbers of collision required for thermalization process is shown. A primary starter source is introduced in the reactor to start the initial fission reaction. Once critical condition is reached chain reaction is established. Neutron is diffused from high to low concentration area according to Fick's law of diffusion. While going through the moderator medium fast neutron are slowed down. It is found that Fermi age and thermal diffusion length of neutron is smaller in light and heavy water than graphite or beryllium. Thus light and heavy water are effective moderator and are used conveniently in nuclear reactors.

KEYWORDS: *Diffusion, Moderation, Nonmultiplying Medium, Multiplying Medium, Fermi Age Equation.*

In the previous section, we dealt with the multiplication system and defined the **infinite and finite multiplication factors**. This section was about conditions for a **stable, self-sustained fission chain reaction** and maintaining such conditions. This problem contains no information about the **spatial distribution of neutrons** because it is a point geometry problem. We have characterized the effects of the global distribution of neutrons simply by a non-leakage probability (thermal or fast), which, as stated earlier, increases toward a value of one as the reactor core becomes larger.



Solution of the diffusion equation in a multiplying system with a control rod insertion. It is assumed that keff is equal to unity in every state.

To design a nuclear reactor properly, predicting how the **neutrons** will be **distributed** throughout the system is highly important. This is a very difficult problem because the neutrons interact differently with different environments (moderator, fuel, etc.) in a reactor core. Neutrons undergo various interactions when they migrate through the multiplying system. To a **first approximation,** the overall effect of these interactions is that the neutrons undergo a kind of **diffusion** in the reactor core, much like the diffusion of one gas in another. This approximation is usually known as the **diffusion approximation,** based on the **neutron diffusion theory**. This approximation allows solving such problems using **the diffusion equation**.

In this chapter, we will introduce the **neutron diffusion theory**. We will examine the **spatial migration of neutrons** to understand the relationships between **reactor size**, **shape**, and **criticality** and determine the spatial flux distributions within power reactors. The diffusion theory provides a theoretical basis for **neutron-physical computing** of nuclear cores. It must be added many neutron-physical codes are based on this theory.

First, we will analyze the spatial distributions of neutrons, and we will consider a **one-group diffusion theory** (**mono-energetic neutrons**) for a **uniform non-multiplying medium**. That means that the neutron flux and cross-sections have already been averaged over energy. Such a relatively simple model has the great advantage of illustrating many important features of the spatial distribution of neutrons without the complexity introduced by the treatment of effects associated with the neutron energy spectrum.

See also: Neutron Flux Spectra.

Moreover, mathematical methods used to analyze a **one-group diffusion equation** are the same as those applied in more sophisticated and accurate methods such as **multi-group diffusion theory**. Subsequently, the one-group diffusion theory will be applied in simple geometries on a uniform multiplying medium (a homogeneous "nuclear reactor"). Finally, the multi-group diffusion theory will be applied in simple geometries on a non-uniform multiplying medium (a heterogenous "nuclear reactor").

Fick's Law

The derivation of the diffusion equation depends on **Fick's law**, which states that solute diffuses (**neutron current**) from high concentration (high flux) to low concentration. As can be seen, we have to investigate the relationship between the **flux (φ)** and the **current (J)**. This relationship between the flux (φ) and the current (J) is identical in form to the law (**Fick's law**) used in the study of physical diffusion in liquids and gases.

In chemistry, Fick's law states that:
Suppose the concentration of a solute in one region is greater than in another of a solution. In that case, the solute diffuses from the region of higher concentration to the region of lower concentration, with a magnitude that is proportional to the concentration gradient.

In one (spatial) dimension, the law is:

$$J = -D\frac{\partial \phi}{\partial x}$$

where:
- J is the diffusion flux,
- D is the **diffusion coefficient,**
- φ (for ideal mixtures) is the concentration.

The use of this law in **nuclear reactor theory** leads to the **diffusion approximation**.

Fick's law in reactor theory stated that:
The current density vector J is proportional to the negative of the gradient of the neutron flux. The proportionality constant is called the diffusion coefficient and is denoted by the symbol D.

In one (spatial) dimension, the law is:

$$J = -D\frac{\partial \phi}{\partial x}$$

where:
- ***J* is the neutron current density** (neutrons.cm^{-2}.s^{-1}) along the x-direction, the net flow of neutrons that pass per unit of time through a unit area perpendicular to the x-direction.
- D is the **diffusion coefficient,** it has the unit of cm, and it is given by:

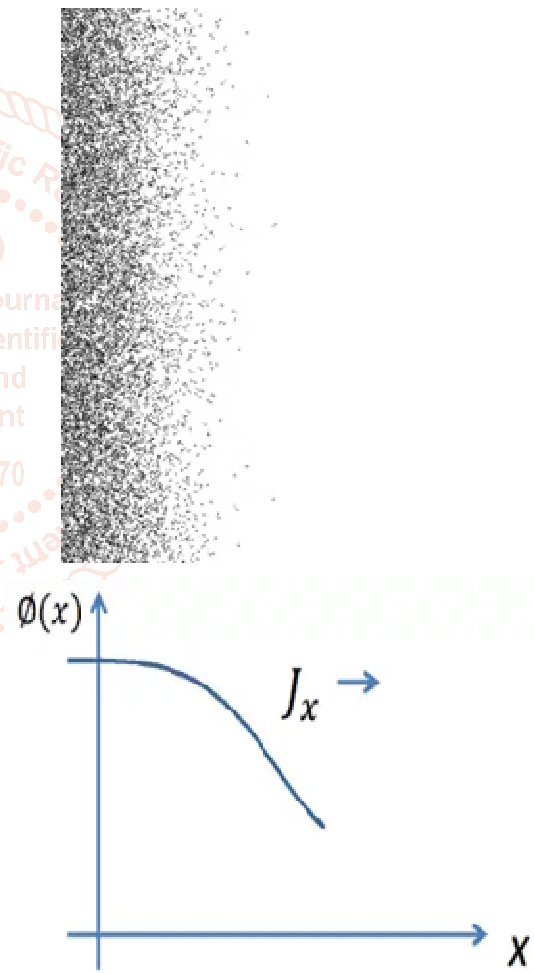
$$D = \frac{1}{3\Sigma_s(1-\bar{\mu})} = \frac{1}{3\Sigma_{tr}} = \frac{\lambda_{tr}}{3}$$

- φ is the neutron flux, the number of neutrons crossing through some arbitrary cross-sectional unit area in **all directions** per unit time.

The generalized Fick's law (in three dimension) is:

$$J = -D\nabla\phi$$

where **J** denotes the **diffusion flux vector**. Note that the gradient operator turns the neutron flux, which is a **scalar quantity** into the neutron current, which is a **vector quantity**.



The physical interpretation is similar to the fluxes of gases. The neutrons exhibit a net flow in the direction of least density. This is a natural consequence of **greater collision densities** at positions of **greater neutron densities**.

Consider neutrons passing through the plane at x=0 from left to right due to collisions to the plane's left. Since the concentration of neutrons and the flux is larger for negative values of x, there are **more collisions per cubic centimeter on the left**. Therefore more neutrons are scattered from left to right, then the other way around. Thus the neutrons naturally diffuse toward the right.

Validity of Fick's Law

It must be emphasized that Fick's law is an approximation and was derived under the following conditions:

1. **Infinite medium.** This assumption is necessary to allow integration of overall space but flux contributions are negligible beyond a few mean free paths (about three mean free paths) from boundaries of the diffusive medium.
2. **Sources or sinks.** Derivation of Fick's law assumes that the contribution to the flux is mostly from elastic scattering reactions. Source neutrons contribute to the flux if they are more than a few mean free paths from a source.
3. **Uniform medium.** Derivation of Fick's law assumes that a uniform medium was used. There are different scattering properties at the boundary (interface) between the two media.
4. **Isotropic scattering**. Isotropic scattering occurs at low energies but is not true in general. Anisotropic scattering can be corrected by modification of the diffusion coefficient (based on transport theory).
5. **Low absorbing medium**. Fick's law derivation assumes (an expansion in Taylor's series) that the neutron flux, *φ,* is slowly varying. Large variations in φ occur when Σ_a (neutron absorption) is large (compared to Σ_s). $\mathbf{\Sigma_a << \Sigma_s}$
6. **Time-independent flux.** Derivation of Fick's law assumes that the neutron flux is independent of time.

To some extent, these limitations are valid in every practical reactor. Nevertheless, Fick's law gives a reasonable approximation. For more detailed calculations, higher-order methods are available.

Neutron Balance – Continuity Equation

The mathematical formulation of **neutron diffusion theory** is based on the **balance of neutrons** in a differential volume element. Since neutrons do not disappear (β decay is neglected), the following neutron balance must be valid in an arbitrary volume V.

rate of change of neutron density = production rate – absorption rate – leakage rate

where

$$\text{Neutron change rate} = \int_V \frac{\partial n}{\partial t} dV$$

$$\text{Production rate} = \int_V s\,dV$$

$$\text{Absorption rate} = \int_V \Sigma_a \phi\, dV$$

$$\text{Leakage rate} = \int_V \nabla \mathbf{J}\, dV$$

Substituting for the different terms in the balanced equation and by dropping the integral over (because the volume V is arbitrary), we obtain:

$$\frac{\partial n}{\partial t} = s - \Sigma_a \phi - \nabla . J$$

where

- **n** is the **density of neutrons**,
- **s** is the rate at which neutrons are emitted from sources per cm³ (either from external sources (S) or from fission ($\mathbf{\nu.\Sigma_f.\Phi}$)),
- **J** is the neutron current density vector
- **Φ** is the scalar neutron flux
- $\mathbf{\Sigma_a}$ is the macroscopic absorption cross-section

In steady-state, when n is not a function of time:

$$\text{div} J + \Sigma_a \phi - s = 0$$

The Diffusion Equation

In previous chapters, we introduced **two bases for the derivation** of the diffusion equation:

Fick's law:

$$\boldsymbol{J = -D\nabla\phi}$$

which states that neutrons diffuse from high concentration (high flux) to low concentration.

Continuity equation:

$$\frac{\partial n}{\partial t} = s - \Sigma_a \phi - \nabla . J$$

which states that rate of change of neutron density = production rate – absorption rate – leakage rate.

We return now to the neutron balance equation and **substitute** the neutron current density vector by **J = -D∇Φ**. Assuming that ∇.∇ = ∇² = Δ (therefore **div J =** -D div (∇Φ) = **-DΔΦ**) we obtain the **diffusion equation**.

$$D\Delta\phi - \Sigma_a \phi + S = \frac{1}{v}\frac{\partial \phi}{\partial t}$$

or (D and Σ_a are not dependent of the position and time – homogeneous system)

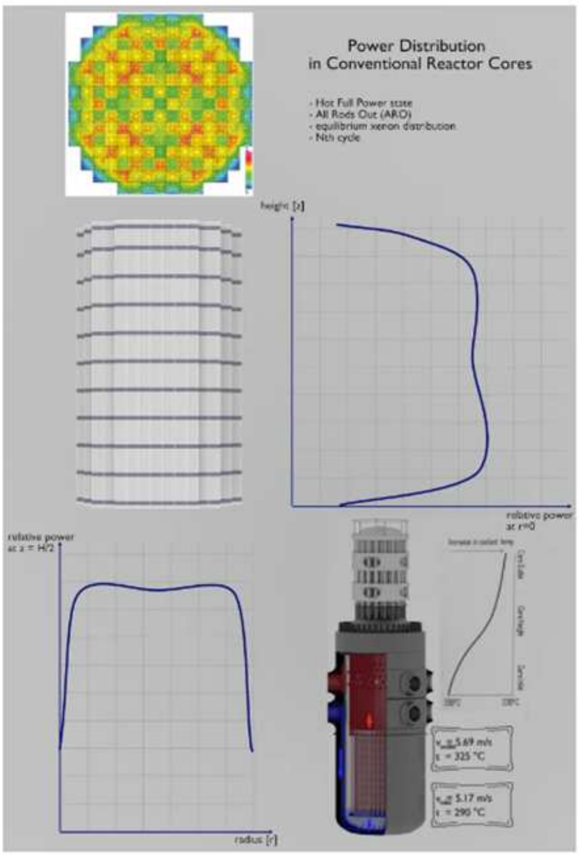
$$D\Delta\phi(\vec{r},t) - \Sigma_a \phi(\vec{r},t) + S(\vec{r},t) = \frac{1}{v}\frac{\partial \phi(\vec{r},t)}{\partial t}$$

The derivation of the diffusion equation is based on Fick's law which is derived under many assumptions. Therefore, the diffusion equation cannot be exact or valid at places with strongly differing diffusion coefficients or in strongly absorbing media. This implies that the diffusion theory may show deviations from a more accurate solution of the transport equation in the proximity of external neutron sinks, sources, and media interfaces.

Material	Σa (1/cm)	λa (cm)	D (cm)	L (cm)
H_2O	0.022	45.5	0.142	2.54
D_2O	3.3 E-5	30300	0.840	160
Be	1.24 E-3	806	0.416	18.3
C	3.2 E-4	3120	0.916	53.5

Diffusion parameters for thermal neutrons of 0.025 eV in some materials

Power Distribution in Conventional Reactor Cores



In commercial reactor cores, the flux distribution is significantly influenced by many factors. Simply, there is no cosine and J0 in the commercial power reactor at power operation.

In commercial reactor cores, the flux distribution is significantly influenced by:

Heterogeneity of fuel-moderator assembly. The core's geometry strongly influences the **spatial and energy self-shielding** that takes place primarily in heterogeneous reactor cores. In short, the neutron flux **is not constant** due to the heterogeneous geometry of the unit cell. The flux will be different in the **fuel cell** (lower) than in the **moderator cell** due to the high absorption cross-sections of fuel nuclei. This phenomenon causes a significant increase in **the resonance escape probability** ("p" from the four-factor formula) compared to homogeneous cores.

Reactivity Feedbacks. At power operation (i.e., above 1% of rated power), the reactivity feedbacks cause the **flattening** of the flux distribution because the feedbacks acts **stronger** on positions where the **flux is higher**. The neutron flux distribution in commercial power reactors depends on many other factors such as the **fuel loading pattern**, control rods position, and it may also oscillate within short periods (e.g.,, due to the spatial distribution of xenon nuclei). Simply, there is no cosine and J_0 in the commercial power reactor at power operation.

Fuel Loading Pattern. The key feature of **PWRs fuel cycles** is that there are **many fuel assemblies** in the core. These assemblies have **different multiplying properties** because they may have **different enrichment** and **different burnup**. Generally, a common fuel assembly contains energy for approximately **4 years of operation at full power**. Once loaded, the fuel stays in the core for 4 years, depending on the design of the operating cycle. During these 4 years, the reactor core has to be refueled. During refueling, every 12 to 18 months, some of the fuel – usually **one-third or one-quarter of the core** – is removed to the **spent fuel pool**. At the same time, the remainder is rearranged to a location in the core better suited to its remaining level of enrichment. The removed fuel (one-third or one-quarter of the core, i.e., 40 assemblies) must be replaced by a fresh fuel assembly.

To summarize, neutron diffusion theory works well outside of fuel pellets outside of fuel pellets

- Neutron diffusion originates from Boltzmann's equation
- Neutron diffusion models: neutron sources, sinks, leakage
- at periphery
- General approach is to develop energy group dependent energy group dependent
- constants constants for: D, Σt , Σa
- Diffusion theory is pretty good explaining general reactor
- neutron flux profiles
- Reactor physicists know how to correct known weaknesses known weaknesses

of diffusion theory via adjustments adjustments

This article No. AM-PZ-2019062031 was written on the basis of a pedagogical analysis of materials prepared within the framework of the innovative project "Nuclear Energy", "Nuclear Medicine and Technology", "Radiation Medicine and Technology" to create multimedia textbooks for bachelors and masters.lib, we express thanks to the authors of the textbooks.

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