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On a Coreflective Hull of a Zero-Dimensional Fuzzy Space in C-FTS

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ABSTRACT

In this paper, we determine the coreflective hull of the fuzzy topological space I_D, where $I_D = ([0,1], \delta_D)$ and $\delta_D =< \{id, 1 - id\} >$ in the category of constant-generated fuzzy topological spaces.

Keyword: *Coreflective subcategory; Coreflective hull, Constant-generated fuzzy topological spaces, zero-dimensional fuzzy topological space*

1. INTRODUCTION

It is well-known that the category of zero-dimensional T_0-topological spaces is the epireflective hull of 2_D in **TOP**, where 2_D denotes the two-point discrete space and the category of discrete spaces is the coreflective hull of 2_D in **TOP** (cf. [7]). It is also known that the category of zero-dimensional T_0-fuzzy topogical spaces is the epireflective hull of the fuzzy topological space I_D, where $I_D = (I, \delta_D)$ and $\delta_D =< \{id, 1 - id\} >$ in **FTS** (cf. [2])). We cannot expect that the coreflective hull of C-I_D in **C-FTS** would be the category of discrete fuzzy topological spaces as C-I_D is not a discrete fuzzy topological space and also in [12], it is shown that the coreflective hull of $L_D = (L, \delta_D)$ and $\delta_D =< \{id, 1 - id\} >$ in **L-TOP,** where L is any complete lattice, is not an L-valued discrete topological space. In this paper, we describe the coreflective hull of the space C-I_D and show that the objects of the coreflective hull of C-I_D are quite close to being discrete fuzzy topological spaces.

2. Preliminaries

For fuzzy topological concepts, we refer [3] but recall a few here, for convenience. Throughout, let I denote the interval [0, 1].

Let X be a non-empty set. A fuzzy set in X is a function from X to I=[0,1]. If $t \in I$, then $\underline{t}$ denotes the constant fuzzy set in X, which takes value t everywhere. In particular, $\underline{0}$ and $\underline{1}$ denote the constant fuzzy sets taking values 0 and 1 respectively.

The complement of μ is the fuzzy set $1 - \mu$, defined as $(1 - \mu)(x) = 1 - \mu(x), \forall x \in X$.

Definition (Chang [3]): A collection δ of fuzzy sets in X with $\underline{0}$ and $\underline{1}$, which is closed under finite meets and arbitrary joins is called a **fuzzy topology** on X and the pair (X,) a **fuzzy topological space**.

Definition (Lowen [8]): Let X be a non-empty set. A subset δ of I^X which is closed under arbitrary joins and finite meets and which contains all constant fuzzy sets, is called a fuzzy topology on X.

The members of δ are called open (or δ -open) fuzzy sets in X and their complements are called closed fuzzy sets in X. The smallest (resp. the largest) fuzzy topology on X is called the indiscrete (resp. discrete) fuzzy topology on X.

Definition: A mapping $f : (X, \delta) \rightarrow (X', \delta')$ between fuzzy topological spaces is called **fuzzy continuous** if

$f^{\leftarrow}(\mu) \in \delta, \forall \mu \in \delta'$ (where $f^{\leftarrow}(\mu) = \mu \circ f$).

Let (X, δ) be a fuzzy topological space, Y a set and f: $X \rightarrow Y$ a surjective mapping. Then

$\delta / f = \{\alpha \in I^Y : f^{\leftarrow}(\alpha) \in \delta\}$.

is clearly a fuzzy topology on Y, called the **quotient fuzzy topology** on Y with respect to f, while $(Y, \delta/f)$ is then called the quotient space of (X, δ) with respect to f. The resulting continuous mapping $f:(X, \delta) \rightarrow (Y, \delta/f)$ is called a **quotient space**.

Definition: A fuzzy topological space (X, δ) is said to be **zero-dimensional** if it has a basis of δ-clopen fuzzy sets (by clopen fuzzy set in (X, δ), we mean a fuzzy set, which is both δ-open and δ-closed).

Definition([9]): A fuzzy topological space (X, δ) is said to be T_0 if for all distinct x, y ∈ X, ∃μ ∈ δ such that μ(x) ≠μ(y).

Definition: The fuzzy topology on I, generated by {id,1-id}, where id is the identity function, both in the sense of Chang and in the sense of Lowen, will be denoted by δ_D and C- δ_D respectively. The resulting fuzzy topological spaces will be denoted respectively by I_D =(I, δ_S) and C- I_D =(I, C-δ_S) respectively (hence it is clear that C-I_S is generated by { ($\underline{t}$ ∧ id) ∨ $\underline{r}$, ($\underline{p}$ ∧ id) ∨ $\underline{q}$: t, r, p, q ∈ I}). Clearly, I_D and C- I_D are zero-dimensional fuzzy topological spaces.

All category-theoretic notions and results used here, but not defined or explained, are fairly standard by now (and can be found in [1]). However, f o r convenience, we recall some of the categorical notions used in the sequel (subcategories are always assumed to be full and isomorphism-closed).

FTS shall denote the category of fuzzy topological spaces in Chang's sense and continuous functions. **C-FTS** will denote the category of fuzzy topological spaces in Lowen's sense and continuous functions and **C-FTS**$_0$ denotes the full subcategory of **C-FTS** containing all T_0-fuzzy topological spaces. Of course, **TOP** is just the category of usual topological spaces and continuous maps.

Definition: *A morphism f* : *X → Y in a category* **C** *is called* constant *if for each* **C**-*object Z and each pair of* **C**-*morphisms g, h* : *Z → X, f* ◦ *g≠ f* ◦ *h.*

It is known that the constant morphisms in **TOP** are precisely the constant maps (cf. [6]). We note that in **C-FTS**, like **TOP**, there is exactly one fuzzy topology on a single-point set. As a consequence of this, in **C- FTS** also, the constant maps are continuous. But, in contrast to **TOP** and **C-FTS**, in **FTS**, there can be many fuzzy topologies on a single-point set and hence constant maps need not be continuous in **FTS**.

Definition: *A category* **C** *is said to be* constant-generated *if for each pair* (*X, Y*) *of* **C**-*objects:* (*i*) **C**(*X, Y*) *≠∅ and* (*ii*) *for every distinct pair f, g* : *X → Y of* **C**-*morphisms, there exists a* **C**-*object Z and a constant* **C**-*morphism k* : *Z → X such that g* ◦ *k≠ f* ◦ *k.*

TOP is well-known to be constant-generated (cf. [6]). Like **TOP**, **C- FTS** is also constant-generated; the main reason being the continuity of constant maps in both the categories. We observe that for some pair (*X, Y*) of **FTS**-objects, we may have **FTS**(*X, Y*) =∅; in particular, if (*X, δ*) is an indiscrete fuzzy space and (*Y,* Δ) is a discrete fuzzy space in **FTS**, then there does not exist any continuous map from *X* to *Y* . So, **FTS** is not constant- generated.

Definition: A subcategory **U** of a category **C** is said to be coreflective in **C** if for each object X in **C**, ∃ an object X_U in **U** and an **X**-morphism $c_X : X_U \to X$ *such that for each object A in* **U** *and each* **X**-*morphism f* : *A → X, ∃ a unique* **A**- *morphism f'* : $A \to X_U$ *such that f* $= c_X \circ f'$.

The notions of reflective and coreflective subcategories have been studied by Herrlich and Strecker (([4], [5] and [6]).

We begin with a preliminary examination of the coreflective subcategories of **C-FTS**. We find that the characterization of coreflective subcategories of **C-FTS** **is** similar to that of the coreflective subcategories of **TOP** ([6]).

We now state the following results from [13] which will be used in the sequel.

Theorem:

1. A subcategory **U** of **C-FTS** is coreflective if and only it is closed under the formation of coproducts and quotients.
2. In **C-FTS**, the coreflective hull of any A∈ ob**C-FTS** always exists.

Moreover, its objects are precisely the quotients of the coproducts of copies of A**.**

We proceed to give an internal description of the coproducts in **C-FTS** of copies of any fuzzy topological space, which we shall then use for our main results. Let (X, δ) ∈ ob**C-FTS** and J be some index set. Put X_j = X × {j}, j ∈J, and denote ⋃j∈JXj by X_j. For each μ ∈ δ, define μ_j: X_j → I as $\mu_j(x, j) = \mu(x)$ and put $\delta_j = \{\mu_j \mid \mu \in \delta\}$, j ∈ J}. Then δ_j is a fuzzy topology on Xj (and (Xj, δ_j) is homeomorphic to (X, δ)). Let $\delta^+ = \{\nu \in I^{X_j} \mid \nu|_{X_j} \in \delta_j, \forall j \in J\}$. It can be verified that (X_j, δ) is the coproduct of |J| copies of (X, δ) in **C-FTS**.

Let $[X]_{\text{C-FTS}}$ denote the coreflective hull in **C-FTS** of a **C-FTS**-object X.

Proposition 2.1([13]). Let X = (X, δ) be a **C-FTS**-object. Then Y = (Y, Δ) is an object of $[X]_{\text{C-FTS}}$ iff ∃ a family {(Y_j, Δ_j) | j ∈ J} of fuzzy subspaces of Y such that Y = $_j⋃_{∈J}Y_j$, each Y_j is a quotient of (X, δ), j ∈ J, and for each μ ∈ I^Y, μ is open in Y iff each $\mu|_Y$ is open in Y_j, j ∈ J.

3. The coreflective hull of C-I_D in C-FTS

As C-I_D is not a discrete fuzzy space, one cannot expect that the coreflective hull of C-I_D in **C-FTS** would be the category of discrete fuzzy spaces. Now, we describe the coreflective hull of the space C-I_D and show that the objects of the coreflective hull of C-I_D are quite close to being discrete fuzzy spaces.

In view of Proposition 2.1, it is clear that we can determine $[C\text{-}I_D]_{\mathbf{C\text{-}FTS}}$, if we can find all the quotients of C-I_D. This is done through the following result.

Proposition 3.1. A fuzzy topological space (X, δ) is a quotient of C-I_D iff

$|X| \leq |I|$ and $\delta =< \alpha, 1 - \alpha >$, where for some partition $\{X_1, X_2, X_3\}$ of X,

$\alpha|_{X_1} : X_1 \to [s, t]$ is bijective, $\alpha|_{X_2} = \underline{s}$ and $\alpha|_{X_3} = \underline{t}$.

Proof: Let (X, δ) be a quotient of C-I_D. Then there exists some quotient map $q : (I, C\text{-}\delta_D) \to (X, \delta)$ in **C-FTS**. As q is surjective, $|X| \leq |I|$.

We observe that each subbasic open fuzzy set in I is closed and so each open fuzzy set in I is closed also. If $\mu \in \delta$, then $q^{\leftarrow}(\mu) \in C\text{-}\delta_D$ and so $1 - (\mu \circ q) \in C\text{-}\delta_D$. But $1 - (\mu \circ q) = (1 - \mu) \circ q$ and so $q^{\leftarrow}(1 - \mu) \in C\text{-}\delta_D$, whereby $(1 - \mu) \in \delta$. Thus each $\mu \in \delta$ is closed also. Let $\mu \in \delta$. We note that:

1. If $q^{\leftarrow}(\mu) = \underline{t}$, then $\mu = \underline{t}$, $t \in I$.
2. If $q^{\leftarrow}(\mu) = id$, then q is a bijection (as $|X| \leq |I|$) and $\mu = q^{-1}$.
3. If $q^{\leftarrow}(\mu) = 1 - id$, then q is a bijection and $\mu = 1 - q^{-1}$.
4. If $q^{\leftarrow}(\mu) = (\underline{t} \wedge id) \vee \underline{s}$, for some $s, t \in (0, 1)$, then $\mu \circ q|_{[s,t]} = id|_{[s,t]}$, whereby $q|_{[s,t]}$ is injective and $\mu|_{q([s,t])}$ is injective such that $\mu(q([s, t])) = [s, t]$ and $\mu|_{q([0,s))} = \underline{s}$, $\mu|_{q((t,1])} = \underline{t}$.

Consider the case when q is bijective. Then $q^{-1} \in \delta$ (as q is a quotient map and $id \in C\text{-}\delta_D$). Put $\alpha = q^{-1}$. Then α is bijective. By the above argument, $1 - \alpha \in \delta$. We show that $\delta =< \{\alpha, 1 - \alpha\} >$. As for every subbasic open fuzzy set μ in C-I_D, $\exists$ some $\nu \in< \{\alpha, 1 - \alpha\} >$ such that $q^{\leftarrow}(\nu) = \mu$ and as $q^{\leftarrow}$ is arbitrary join- and arbitrary meet- preserving, for each $\mu \in C\text{-}\delta_D$, $\exists \nu \in< \{\alpha, 1 - \alpha\} >$ such that $q^{\leftarrow}(\nu) = \mu$. Hence $\delta =< \{\alpha, 1 - \alpha\} >$, where α is a bijection on X.

Consider now the case when q is not bijective. Then q cannot be injective.

We have the following cases:

(A) For some pair $s, t \in (0, 1)$, $q|_{[s,t]}$ is injective (then $q|_{[1-s,1-t]}$ is also injective, as q is a quotient map and every open fuzzy set in C-I_D is closed also).

In this case, by (iv), $\exists \mu \in \delta$ such that $\mu \circ q = (\underline{t} \wedge id) \vee \underline{s}$, implying that $\mu \circ q|_{[s,t]} = id|_{[s,t]}$ and so $\mu|_{q([s,t])}$ is injective, $\mu(q([s, t])) = [s, t]$, $\mu|_{q([0,s))} = \underline{s}$, $\mu|_{q((t,1])} = \underline{t}$, whereby $\exists$ a partition $\{X_1, X_2, X_3\}$ of X such that $\mu|_{X_1} : X_1 \to [s, t]$ is a bijection, $\mu|_{X_2} = \underline{s}$ and $\mu|_{X_3} = \underline{t}$. As $1 - \mu \circ q = 1 - (\underline{t} \wedge id) \vee \underline{s}$, i.e., $(1 - \mu) \circ q = (1 - \underline{t} \vee (1 - id)) \vee (1 - \underline{s}) \in C\text{-}\delta_D$, $1 - \mu \in \delta$. We show that $\delta =< \mu, 1 - \mu >$. Let $\beta \in \delta$. Then $\beta \circ q = ((\underline{v} \wedge id) \vee \underline{u}) \wedge ((1 - \underline{v}) \vee (1 - id)) \wedge (1 - \underline{u})$. Now we consider the following cases:

If $s < 1/2 \leq t$, then $\beta|_{q([u,1/2])} : q([u, 1/2]) \to [u, 1/2]$ is a bijection for $u \geq s$, $\beta|_{q([0,u))} = \underline{u}$ and $\beta|_{(1-v,1]} = 1 - \underline{v}$, $v \leq t$ $\beta|_{q[1/2,1-v]} : q[1/2, 1 - v] \to [1 - v, 1/2]$, whereby $\beta \in< \mu, 1 - \mu >$.

If $t < 1/2$, then $\beta|_{q([u,v])} : q([u, v] \to [u, v]$, $\beta|_{[0,u)} = \underline{u}$ and $\beta|_{q((v,1])} = \underline{v}$, whereby $\beta \in< \mu, 1 - \mu >$.

If $1/2 < s$, then $\beta|_{q([1-u,1-v])}$: $q([1 - u, 1 - v] \to [1 - v, 1 - u]$, $\beta|_{[0,1-u)} = 1 - \underline{u}$ and $\beta|_{q((1-v,1])} = 1 - \underline{v}$, whereby $\beta \in< \mu, 1 - \mu >$. Similarly the case can be considered if $\beta \circ q = ((\underline{v} \wedge id) \vee \underline{u}) \vee ((1 - \underline{v}) \vee (1 - id)) \wedge (1 - \underline{u})$.

Hence $\delta =< \{\mu, 1 - \mu\} >$, where for some partition $\{X_1, X_2, X_3\}$ of X and for some pair $s, t \in (0, 1)$, $\mu|_{X_1} : X_1 \to [s, t]$ is a bijection, $\mu|_{X_2} = \underline{s}$ and $\mu|_{X_3} = \underline{t}$.

(B) For any pair $s, t \in I$, $q|_{[s,t]}$ is not injective.

In this case, $\nexists\ \beta \in \delta$ such that $\beta \circ q = ((\underline{t} \wedge id) \vee \underline{s}) \wedge ((1 - \underline{t}) \vee (1 - id)) \wedge (1 - \underline{t})$, unless $s = t$. Hence, $\delta = \{\underline{t} \mid t \in I\}$.

So if (X, δ) is a quotient of C-I_D, then δ is $< \alpha, 1 - \alpha >$, where for some partition $\{X_1, X_2, X_3\}$ of X, $\alpha|_{X_1} : X_1 \to [s, t]$ is bijective, $\alpha|_{X_2} = \underline{s}$ and $\alpha|_{X_3} = \underline{t}$.

Conversely, let $(X, \delta) \in ob$**C-FTS** such that $|X| \leq |I|$ and $\delta =< \alpha, 1 - \alpha >$, where for some partition $\{X_1, X_2, X_3\}$ of X, $\alpha|_{X_1} : X_1 \to [s, t]$ is bijective, $\alpha|_{X_2} = \underline{s}$ and $\alpha|_{X_3} = \underline{t}$. Then clearly for some partition $\{X_4, X_5, X_6\}$ of X, $(1 - \alpha)|_{X_4} : X_4 \to [1 - t, 1 - s]$ is bijective, $(1 - \alpha)|_{X_5} = 1 - \underline{t}$ and $(1 - \alpha)|_{X_6} = 1 - \underline{s}$.

Let $q : (I, C\text{-}\delta_D) \to (X, \delta)$ be a map such that $q|_{[s,t]}$ and $q|_{[1-t,1-s]}$ are injective, $q([s, t]) = X_1$ and $q([0, s)) = X_2$, $q((t, 1]) = X_3$ and $q([1-s, 1-t]) = X_4$, $q((1-t, 1]) = X_5$, $q([0, 1-s)) = X_6$. As $q^{\leftarrow}$ is arbitrary join- and arbitrary meet- preserving, it is sufficient to show that for any subbasic open fuzzy set μ in X, $q^{\leftarrow}(\mu) \in C\text{-}\delta_D$. Then for $\mu = \alpha$, $q^{\leftarrow}(\mu)|_{[s,t]} = id|_{[s,t]}$ and $q^{\leftarrow}(\mu)|_{[0,s)} = \underline{s}$, $q^{\leftarrow}(\mu)|_{(t,1]} = \underline{t}$ and so $q^{\leftarrow}(\mu) = (\underline{t} \wedge id) \vee \underline{s}$. For $\mu = \underline{t}$, $t \in I$, $q^{\leftarrow}(\mu) = \underline{t}$, $t \in I$. For $\mu = 1 - \alpha$, $q^{\leftarrow}(\mu)|_{[1-s,1-t]} = 1 - id|_{[1-s,1-t]}$ and

$q^{\leftarrow}(\mu)|_{(1-t,1]} = 1 - \underline{t}$, $q^{\leftarrow}(\mu)|_{[0,1-s)} = 1 - \underline{s}$ and so $q^{\leftarrow}(\mu) = ((1 - \underline{s}) \wedge (1 - id)) \vee (1 - \underline{t})$. For $\mu = \underline{t}$, $t \in I$, $q^{\leftarrow}(\mu) = \underline{t}$, $t \in I$. Hence $q^{\leftarrow}(\mu) \in C\text{-}\delta_D$ for each $\mu \in \delta$.

Next, let $q^{\leftarrow}(\mu) \in C\text{-}\delta_D$, for some $\mu \in I^X$. We wish to show that $\mu \in \delta$. As $q^{\leftarrow}(\mu) \in C\text{-}\delta_D$, $\mu \circ q = ((\underline{v} \wedge id) \vee \underline{u}) \wedge ((1 - \underline{v}) \vee (1 - id)) \wedge (1 - \underline{u})$. Now we consider the following cases:

If $s < 1/2 \leq t$, then $\mu|_{q([u,1/2])} : q([u, 1/2]) \rightarrow [u, 1/2]$ is a bijection for $u \geq s$, $\mu|_{q([0,u))} = \underline{u}$ and $\mu|_{(1-v,1]} = 1 - \underline{v}$, $v \leq t$ $\mu|_{q[1/2,1-v]} : q[1/2, 1 - v] \rightarrow [1 - v, 1/2]$, whereby $\mu \in \delta$.

If $t < 1/2$, then $\mu|_{q([u,v])} : q([u, v] \rightarrow [u, v]$, $\mu|_{[0,u)} = \underline{u}$ and $\mu|_{q((v,1])} = \underline{v}$, whereby $\mu \in \delta$.

If $1/2 < s$, then $\mu|_{q([1-u,1-v])} : q([1-u, 1-v] \rightarrow [1-v, 1-u]$, $\mu|_{[0,1-u)} = 1-\underline{u}$ and $\mu|_{q((1-v,1])} = 1 - \underline{v}$, whereby $\mu \in \delta$. Similarly the case can be considered if $\mu \circ q = ((\underline{v} \wedge id) \vee \underline{u}) \vee ((1 - \underline{v}) \vee (1 - id)) \wedge (1 - \underline{u})$.

We have thus shown that $\mu \in \delta \Leftrightarrow q^{\leftarrow}(\mu) \in C\text{-}\delta_D$, which in turn shows that (X, δ) is a quotient of $(I, C\text{-}\delta_D)$.

We now characterize $[C\text{-}I_D]_{\mathbf{C\text{-}FTS}}$, the coreflective hull in **C-FTS** of $C\text{-}I_D$.

Theorem 3.1. A fuzzy topological space (X, δ) is an object of $[C\text{-}I_D]_{\mathbf{C\text{-}FTS}}$

If and only if it satisfies the following two conditions:

A. $X = \bigcup_{j\in J} X_j$ for some index set J such that for each $j \in J$, $|X_j| \leq |I|$ and the subspace fuzzy topology δ_j on X_j is $\delta =< \alpha, 1 - \alpha >$, where for some partition $\{X_1, X_2, X_3\}$ of X_j, $\alpha|_{X_1}: X_1 \rightarrow [s, t]$ is bijective, $\alpha|_{X_2} = \underline{s}$ and $\alpha|_{X_3} = \underline{t}$.

B. for each $\mu \in I^X$, $\mu \in \delta$ iff $\mu|_X \in \delta_j$, for each $j \in J$.

Proof: The proof directly follows from Propositions 2.1 and 3.1.

Remark 3.1. In view of the proof of Proposition 3.1 and Theorem 3.1, if $(X, \delta) \in ob[C\text{-}I_D]_{\mathbf{C\text{-}FTS}}$, then every open fuzzy set in X is closed also.

Theorem 3.2. A topological space (X, T) is an object of the coreflective hull of 2_D in **TOP** if and only if it can be written as the disjoint union of single-point spaces, i.e., iff it is a discrete space.

Proof: If we replace I by 2 = {0, 1}, then Theorem 3.1 takes the following form.

A topological space (X, T) $\in ob[2_D]_{\mathbf{TOP}}$ if and only if (X, T) satisfies the following properties:

1. $X = \bigcup_{j\in J} X_j$, for some index set J such that for each $j \in J$, $|X_j| \leq 2$ and the subspace (X_j, T_j) is either a single-point space or a discrete space,
2. for each $U \subseteq X$, $U \in T$ if and only if $U \cap X_j \in T_j$, for each $j \in J$.

Since any discrete space can be uniquely written as a coproduct of its single- point subspaces, (X, T) $\in ob[2_D]_{\mathbf{TOP}}$ iff it can be written as the disjoint union of single-point spaces, i.e., if and only if it is a discrete space.

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