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Sustainable Development of Dynamic Fisheries and Cost-Benefits Analysis with Mathematical Approach

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ABSTRACT

With Gambia's marine resources abundance, it is appropriate for Gambia's economic growth to gear towards the fisheries sector. Fishery management in The Gambia is still not operating optimally due to a lack of fisheries management infrastructure.

This study has uncovered important aspects of the fishing industry, especially in The Gambia. If the right policies and guidelines are put in place, the majority of waste and the depletion of renewable resources might be avoided. It is possible to maximize utilities without wasting resources. The solutions of the total cost (TC), total revenue (TR), and price functions of an operation are all given using differential equation.

The original equation of Schaefer's model forced numerous researchers to shed light on the spatial distribution of fish and fisheries. Using a cost operation model and an analysis of the rate of change over time, the author of this study found that fishing expenses can be decreased without compromising effectiveness or efficiency. The model was also expanded to include non-autonomous price and cost characteristics.

This study will inspire other academics and researchers to take similar actions to stop the decline and near-extinction of the critically important fish fishery, not only in The Gambia but also along the entire coast of West Africa, given that the fishes are highly migratory and a shared resource among the countries in the subregion.

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INTRODUCTION:

The analysis and synthesis of dynamic processes is a major focus of many current topics in applied mathematics that are connected to biology, chemistry, physics, economics, and so on. The parameters or structural constants present in the system of differential equations representing a dynamic system affect the stability of the system structurally. Significant portions of researchers have focused their attention on the management of natural resources in general and renewable resources in particular, (Goundry, 1960; Crutchfield, 1967; Wat, 1968; Garrod, 1973; Gulland, 1974). Coyle studied the dynamics of management system (Coyle, 1977) and of capital expenditure (Coyle, 1977). The policy framework for managing catch fisheries is often based on a straightforward model to match fishery production through harvest rates to the system's inherent productivity. According to total authorized catch programs, for example, fisheries management aims to exploit fish populations to their maximum sustained catch or to make the most money possible over the long term (e.g., catch share programs or pay and catch). It has been demonstrated in any fishing site that without entry restrictions, fishing activity tends to expand to levels that are beyond ecological viability, ceasing only when fishing expenses surpass the advantages (Beddington et al., 2007). Fishery assessments determine what level of fishing can be sustained at each of those target reference points for management. Fisheries policy establishes the priorities for the national fishery sector, department of fisheries in The Gambia, such as maximizing profitability of the sector, fish production, or maintaining as many fishing jobs as possible. However, there has been an increase in recent years in efforts to make national policies compatible with broader economic, social, political and conservation objectives. These fishery policies have historically been highly sectoral and poorly integrated with larger development initiatives (Thorpe et al., 2006). According to the scale of their resource, its ability to produce macroeconomic advantages, its relevance for both formal and informal employment, and the role fish plays in a nation's protein requirement, different countries may pursue vastly different policy objectives. Example, The Gambia, European Union (EU) and China fishing deals after the regime of Yahya A.J.J. Jammeh, 2017 in the Gambia.

Our goal in this research is to using revenue and cost analysis of fishing population dynamics in the Gambia, to demonstrate the highest optimal profitability point. We employ the cost function, which is our original work, to calculate the profit from fishing. The solutions to our cost and revenue functions were found using differential equations. The idea of technology being used in the fishing industry and included in Schaefer's model (E. Bittaye, 2023). A thorough research of natural resources, in particular the fishing industry, has been conducted to determine how technology may affect fishing's spatial distribution.

Table 1: The parameters and their definitions

Variables	Definition of variables
$G(x)$	Natural growth rate
$h(t)$	Rate of removal or harvesting
$n(t)$	The size of population at time (t)
Y	Sustainable yield
E	Effort per unit catch
MSY	Maximum sustainable yield
p	Constant price per unit of harvest
c	Constant cost per unit of catching effort
TR	Total revenue
TC	Total cost
SR	Sustainable economic rent
$E = E_\infty$	When $TR = TC$
N	Stock level
r	Intrinsic growth
k	Carrying capacity of the stock
q	Is the catchability
l	Labor cost
T	Trawlers cost
f	Fuel cost
v	Piecewise function of license status
$D(p)$	The positive linear demand function
a	The market capacity
s	The price speed adjustment

Considering this model that was originally developed by (Schaefer, 1957)

$$\dot{x} = G(x) - qEx \quad (1)$$

Has been expanded by (M. A. Shah, 2013), If $h(t)$ represents the rate of removal or harvesting, then the population growth with harvesting is described by the differential equation

$$\frac{dx}{dt} = G(x) - h(t) \quad (2)$$

where $x(t)$ denote the size of a fish population at time t. Whenever the harvest rate $h(t)$, exceeds the natural growth rate $G(x)$, equation (2) implies that the population level will decline as $\frac{dx}{dt}$ becomes negative. However, if $h(t) < G(x)$, then the population growth continue. If $h(t) = G(x)$, the population remains at a constant level. Thus, in this situation, the natural growth rate $G(x)$ becomes the 'sustainable yield' that can be harvested while maintaining the population at a fixed level. Symbolically, the sustainable yield Y will be given by:

$$Y = G(x) = En \ (3)$$

where E is the effort per-unit catch. For density dependent growth models degree $G(x) \geq 2$, therefore, if h is constant and $h < \max G(x)$, then equation (2) may possess two or more equilibriums. An explicit analysis of the model can be carried out only when $G(x)$ is given in explicit form. However, if $h(t) = h$, then equation (2) implies that a maximum sustainable yield (MSY) is given by

$$Y_{max} = h_{MSY} = \max_{n} G(x) \ (4)$$

with the property that any larger harvest rate will result into the depletion, and hence eventual extinction of the population. In order to achieve the maximum revenue return from fish harvesting and also to determine an optimal policy for fish harvesting, Pontryagin's maximum principle have applied in other works. Cost and revenue functions are applied in this direction, further, if we assume a constant price p, per-unit of harvested biomass, and a constant cost c, per-unit catching effort E, then the total sustainable revenue TR and total fishing cost TC are given by

$$TR = pY(E) \ (5a)$$

and

$$TC = cE \ (5b)$$

Our take before proceeding with the analysis of the model is that, depletion is taking place if the following assumptions hold.

1. If $h(t) > G(x)$.
2. Exceed amount of fishing vessels fleet in the waters.
3. If the demand $D(t)$ of fish from the population exceed the natural growth rate $G(x)$.

The net revenue, which is the difference TR and TC is called the 'sustainable economic rent'. Thus

$$SR = TR - TC = p.Y(E) - cE \ (5c)$$

The main finding of (Gordon, 1954) is that in open-access fisheries, effort tends to attain an equilibrium, the so-called bionomic equilibrium, at the level $E = E_\infty$, at which the sustainable economic rent is totally dissipated, that is

$$TR = TC \ (6)$$

If $E > E_\infty$ under Gordon's open-access fishing model, opportunity cost outweighs income, which leads to fishers leaving the fishery. Conversely, if $E < E_\infty$, then revenues are greater than opportunity costs, and as a result, efforts tend to be increased since fishing is now more lucrative than other jobs (Clark, 1990; Burghes and Graham, 1980). What is wrong with a system where fishers receive their exact opportunity cost from fishing at this point? A closer look reveals, firstly, that an excessive amount of effort is being put into using the fishing resource, which is capable of providing positive economic rent. The advantages that may arise when the fisheries were managed are not being reaped by either the fishers nor society at large. It is known as "economic overfishing" in this circumstance. Second, there is a chance that the fishery is experiencing "biological overfishing" because the sustained biomass output in this instance is lower than MSY.

As fishing is surging in The Gambian waters requires mathematical modeling to be done in order to maximize utility whiles preserve the natural resources that is being endowed on us. In order to investigate biological and ecological systems using a variety of analytical, numerical, and graphical techniques, mathematical modeling is a highly helpful tool. Several fields have made used of mathematical models. For instance, mathematical models are crucial to understanding population dynamics in ecology. The fishing chain and cost involvement are the top concerns in this study in order to maximize profit and reduce cost while maintaining an equilibrium fishing level.

The Gambia is a country located in West Africa with a population size approximately two million people. Its river is called the ”River Gambia” which is a fresh water that attracted lot of tourism and other commercial gains for both private and public. Agriculture is one of the backbone of the country’s economy and fishing plays a great part in her GDP. The sector contributes about 12.1% of the annual Gross Domestic Product (GDP) (GBoS, 2019) generating both direct and indirect employment for thousands of Gambian women and youth. Many local men and women earn their living through fishing. With the diversity in the fishery sector and the country’s policies entertain lot of foreign fishers with high sophisticated vessels in the river Gambia. This massive fishing has brought alarm to the high exploitation of fishes in The Gambia. What modeling strategy in the dynamic system of fishing should be employed to reduce overfishing in the world generally and The Gambia, particularly?

Traditional fishery management has far too frequently fallen short of adopting a preventative stance to uphold and safeguard sustainable fisheries, biodiversity, and the operation of marine ecosystems (Lauck et al., 1998).

One such fishery model uses ordinary differential equations (ODEs) by (Schaefer, 1991) in the form:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{k}\right) - qEN \ (7)$$

$$\frac{dE}{dt} = k(pqEN - cE) \ (8)$$

where N is the stock level, E is the level of fishing effort, r is the intrinsic growth rate of the stock, k is the carrying capacity of the stock, q is the catch ability, p is the price per fish, c is the cost per unit effort, and k is a proportionality constant. The first equation contains (logistic) growth term of the stock and harvesting term, which is proportional to both the stock level and the level of fishing effort. The second equation has economics interpretation rather than ecological, and it states that the rate of change of fishing effort is proportional to profit, with $pqEN$ as the revenue and cE as the costs (Kot, 2001).

Cost – Benefits

Systematic interventions that encourage economic growth is challenging because there is a lack of structured data on the structure of costs- benefits and added value for the operations of each business actor engaged. When it comes to high value-added operations, comprehensive knowledge of the value chain also points in the direction of investment potential. It is important to examine and ascertain the losses and gains those business actors in the sector face, whether it is related to financing or regulations that may not yet support the development of the economy. Since it appears that fishers and actors in the sector are not utilizing this investment opportunity, this analysis is necessary.

If landing catch is x fish and the price attach to each bundle of the catch is p, then we can write that the revenue gain from that transaction would be $p \cdot x = p_1x_1 + p_2x_2 + \cdots + p_nx_n$.

The cost of fishing varies depending on when the activities begin, how much it costs to catch the fish, and how often you are permitted to fish during a given season or year? We can then calculate the cost of each round by dividing the cost input by the number of rounds.

In our research, the cost items are as follows. The price of labor (l), the price of trawlers with nets (T), the price of fuel (f), and the price of licenses (v).

$$Total\ cost = l + T_{((n-1),n)} + f(l_i) + v.$$

If the money from the round catch plus any additional benefits exceeds the expenditure involved in the same round, then we can say that a profit or gain has been made. We refer to it as a loss if not.

Empirical data received from (DoF, 2022), which permits us to made analysis in some significant times, which we hope affected fishing in the Gambia both the Industrial, Artisanal and Aquaculture. There was a rise for amount catch comparing 2010 and 2011 both industrial and artisanal 0.81% and 1.03% respectively. 2015 and 2016 at the election period, the comparison between industrial and artisanal 5.37% increase and 5.22% decrease respectively. Post- election with new bilateral, industrial and artisanal fishing increased 6.15% and 18.37% respectively. 2019 and 2020 data shows that 9.94% increase of tons catch for industrial while artisanal 0% because of covid-19 restrictions.

Figure 1

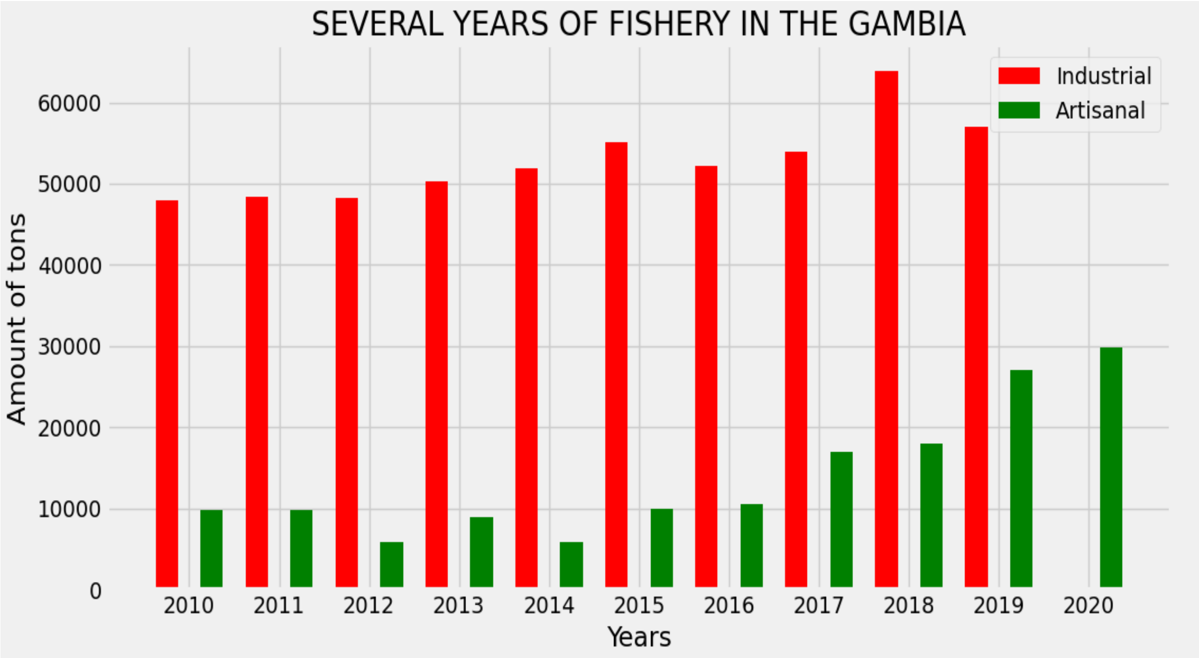

Figure 1: This figure shows the level of fish catch by the industrial and artisanal – aquaculture from 2010 to 2020. It shows that, the higher the cost of investing, the greater the chance of getting good number of fish.

Mathematical Modeling
Cost – Benefits Analysis

P(gain)=0.5 at the initial point of investment, according to probability theory. Gain or loss in the case of fishing products cannot be predicted with absolute certainty. It is now vital to advance our understanding of supply chain practices and costs due to the ongoing evolution of the business environment. The price of labor (l), the price of trawlers with nets (T), the price of fuel (f), and the price of licenses (v).

$$l = l_1 + l_2 + l_3 + \cdots + l_n$$

$$T = T_{((n-1),n)}$$

$$f = f(l_i)$$

$$v = \begin{cases} If\ pay = 1 \\ not\ pay = 0 \end{cases}$$

For a simple fishing investment cost, items attached these cost based on our study.

$$Total\ cost = l + T_{((n-1),n)} + f(l_i) + v$$

The variables of the cost function are detailed as follows;

l, can be 1 or more, that is why the number tend to n. It depends on the size of the trawler or traditional canoe. $\sum_1^n l$, where $n = 1,2,3, \dots \cdot$

T, is the owner of the trawler where he\she can own one or two, either he contract it to a second party.

$f(l_i)$, the litres of fuel it will consume for one fishing journey before it lands.

v, is a piecewise function simply because some trawlers gets in to the waters of territory where they are not licensed. $\sum_1^i l$, where $= 1, 2, 3, \dots, i$.

$$Cost\ of\ investment = \sum_1^n l + \sum T + \sum_1^i l + \sum v \ \ (9)$$

$$Total\ cost = l + T_{((n-1),n)} + f(l_i) + v \ (10)$$

Taking the derivative of the Total cost:

$$\frac{d}{dt} = \left[l + T_{((n-1),n)} + f(l_i) + v\right] e^{-\rho t}, t > 0. (11)$$

First, we simplify the equation by using the sum rule of differentiation and the constant multiple rule of differentiation:

$$\frac{d}{dt} = \left[l + T_{((n-1),n)} + f(l_i) + v\right] e^{-\rho t}$$

$$= \frac{d}{dt}[le^{-\rho t}] + \frac{d}{dt}\left[T_{((n-1),n)}e^{-\rho t}\right] + \frac{d}{dt}[f(l_i)e^{-\rho t}] + \frac{d}{dt}[ve^{-\rho t}]$$

Next, we apply the product rule differentiation to each of the terms on the right-hand side:

$$= -\rho le^{-\rho t} + \frac{d}{dt}le^{-\rho t} - \rho T_{((n-1),n)}e^{-\rho t} + \frac{d}{dt}\left[T_{((n-1),n)}\right] e^{-\rho t} - \rho f(l_i)e^{-\rho t} + \frac{d}{dt}[f(l_i)]e^{-\rho t} - \rho ve^{-\rho t} + \frac{d}{dt}[v]e^{-\rho t}$$

Since the derivative of a constant is zero, the last two terms simplify to:

$$= -\rho le^{-\rho t} + \frac{d}{dt}le^{-\rho t} - \rho T_{((n-1),n)}e^{-\rho t} + \frac{d}{dt}\left[T_{((n-1),n)}\right] e^{-\rho t} - \rho f(l_i)e^{-\rho t} + \frac{d}{dt}[f(l_i)]e^{-\rho t} - \rho ve^{-\rho t}.$$

Next, we substitute the original equation (11) for the total cost in to the derivative of the individual terms:

$$= -\rho\left[l + T_{((n-1),n)} + f(l_i) + v\right] e^{-\rho t} + \frac{d}{dt}\left[l + T_{((n-1),n)} + f(l_i) + v\right] e^{-\rho t}$$

$$= -\rho\left[l + T_{((n-1),n)} + f(l_i) + v\right] e^{-\rho t} + \left[\frac{d}{dt}(l) + \frac{d}{dt}\left(T_{((n-1),n)}\right) + \frac{d}{dt}\left(f(l_i)\right) + \frac{d}{dt}(v)\right]e^{-\rho t}.$$

Finally, we simplify the second term by using the sum of differentiation:

$$= -\rho\left[l + T_{((n-1),n)} + f(l_i) + v\right] e^{-\rho t} + [\frac{d}{dt}(l + T_{((n-1),n)} + f(l_i) + v)]\, e^{-\rho t}$$

$$= -\rho\left[l + T_{((n-1),n)} + f(l_i) + v\right] e^{-\rho t} + [\frac{d}{dt}(total\ cost)]\, e^{-\rho t}.$$

Thus, we end up with the differential equation relates the rate of change of the total cost to the total cost itself, as well as the exponential term with parameter.

Technology:

$$A = a^x, a > 1. (12)$$

By (E. Bittaye, 2023) extended the model of Schaefer where technology is captured as a variable in the fishing activities. The following points are vital as far as technology is concern in fishing.

1. Technique to identify the products.
2. Technique to get the right products.
3. Technique to process the products.
4. Technique to make end users to get the products.

The correlation of resources and the optimal fish catch is demonstrated mathematically as;

$$Y = f(n) = En$$

If $Y = En$ is the sustainable yield, then having, $AY = AEn$.

From the definition of $A = a^x$, which could increase the yield or decrease as a result of technology.

Transportation:

$transport_{(f,d,l,t)}$.

$(f = fuel, d = distance, l = labour\ and\ t = time)$. Transportation will take place within and outside territory of the country. After the landing of the catch, fish dealers within and outside the country transport the fishes to different sites for storage and sales.

Storage:

$store_{(l,t,s)}$.

$(l = labour, t = time\ and\ s = space)$. Big or small stores for the remaining fish and the first consignment and deploy all means of communication to ship the products with respect to the time it will spend in the store.

In the above discussion of the total cost of fishing, discount rate has to be considered, $e^{-\rho t}$.

Therefore, we write,

$$Total\ cost = \left[l + T_{((n-1),n)} + f(l_i) + v\right] e^{-\rho t}$$

Results

A cost and revenue model for fishing would be useful for estimating the profitability of a fishing operation. The model would take into account the costs associated with fishing, as well as the revenue generated by the sale of fish or other fishing-related activities. Here's an example of a cost and revenue model for fishing. (C. Jerry, N. Raïssi, 2010), they found that the stock change of fish is related to its own growth and the catch rate. Additionally, they found that the price change is affected by the market positive linear demand function and the catch rate. It has a positive correlation with the market positive linear demand function and a negative correlation with the catch rate. In this research, we found out that, the stock of fish depends mainly on the investment for harvesting. The model can show the effect of technology as it is part of the investment plan;

$$\begin{cases} \dot{x}(t) = rx(t)\left(1 - \frac{x(t)}{k}\right) - AY, x(0) = x_0 > 0\ (13) \\ \dot{p}(t) = s(D(p) - AY), p(0) = p_0 > 0.\ (14) \end{cases}$$

where the variables $x(t)$ and $p(t)$ denote the fish stock and the unit price of the stock at time t, respectively. $D(p)$ is the positive linear demand function such that $D(p) = a - p(t) \geq 0$, (J. T. Lafrance, 1985). The parameter r is the intrinsic growth rate of the biomass. k is the carrying capacity of the environment. Y is the catch rate. a is the market capacity. s is the price speed adjustment. r, k, Y, a and s are positive constants. The variable A as technology is attach to the catching rate variable which has a great effect on fishing particular the fish stock. The correlation matches as the technology gets as sophisticated as it should be the positivity counts, otherwise, it minimize the amount of fish.

$$\dot{x}(t) = rx(t)\left(1 - \frac{x(t)}{k}\right) - AY, x(0) = x_0 > 0$$

This is a non- linear ordinary differential equation known as the logistic equation with a linear feedback term.

Using the constants, we rewrite the differential equation as:

$$\dot{x}(t) = rx(t)\left(1 - \frac{x(t)}{k}\right) - AYx(t)$$

Where Y is a constant that represent the strength of the output effect on the catch rate.

Next, we rewrite the equation (13) in more standard form by factoring out $x(t)$ and defining a new constant $B = A * \frac{Y}{K}$

$$\dot{x}(t) = rx(t)\left(1 - \frac{x(t)}{k} - B\right)$$

This equation is separable, so we separate the variables and integrate:

$$\frac{dx}{dt} = x(t)\left(r - r\frac{x(t)}{k} - B\right)$$

$$\frac{dx}{[x(t)(r - r\frac{x(t)}{k} - B)]} = dt$$

Integrating both sides gives

$$\int \frac{dx}{[x(t)(r - r\frac{x(t)}{k} - B)]} = \int dt$$

The left – hand side can be integrated using partial fractions:

$$\int \frac{dx}{[x(t)(r - r\frac{x(t)}{k} - B)]} = \frac{A}{k}\int \frac{dx}{(x(t) - \frac{k}{B})} - \frac{A}{k}\int \frac{dx}{(x(t) * (\frac{r}{k} - x(t)) + B)}$$

Where $A = r - B$. This gives

$$\ln\left|x(t) - \frac{k}{B}\right| - \ln\left|x_0 - \frac{k}{B}\right| - \frac{A}{k}\ln\left|x(t)(\frac{r}{k} - x(t) + B\right| + \frac{A}{k}\ln\left|x_0(\frac{r}{k} - x_0) + B\right| = t * \frac{A}{k}$$

Where x_0 is the initial condition.

We simplify the expression by taking exponentials on both sides

$$\frac{\left|x(t) - \frac{k}{B}\right|}{\left|x_0 - \frac{k}{B}\right|} * \left|x(t)(\frac{r}{k} - x(t) + B\right|^{\frac{A}{k}} = \exp(t\frac{A}{k})$$

Note the absolute values can be removed because $x(t) > 0$ for all t (as long as $x_0 > 0$).

Therefore, the solution to the logistic equation (13) with a linear feedback term is

$$x(t) = \frac{k}{B} + \left(x_0 - \frac{k}{B}\right) * [\frac{B}{\left(x_0\left(\frac{r}{k} - x_0\right) + B\right) * \exp\left(A\frac{t}{k}\right)} - \frac{B}{(k\left(\frac{r}{k} - x_0\right) + B)}]$$

Where $B = A * \frac{Y}{k}, A = r - B, and\ r, k, Y\ and\ x_0$ are given constants.

And also the given differential equation (14) of a price unit,

$$\dot{p}(t) = s(D(p) - AY), p(0) = p_0 > 0$$

Where $\dot{p}(t)$a vector function of time $t, p(0)$ is the initial condition, $D(p)$ is a diagonal matrix with elements being the components of vector p, Y is a constant vector, A is a constant matrix, and s is a positive constant.

To solve this differential equation, we can start by expanding the derivative term using the chain rule.

$$\dot{p}(t) = \frac{dp(t)}{dt} = \frac{dp}{dD} \cdot \frac{dD}{dp} \cdot \frac{dp}{dt}$$

Where $\frac{dp}{dD}$ denotes the derivative of vector p with respect to the diagonal matrix $D(p)$. Since $D(p)$ is a diagonal matrix, its derivative with respect to p is also a diagonal matrix with the same diagonal elements as $D(p)$:

$$\frac{dD}{dp} = diag(p_1^{(-2)}, p_2^{(-2)}, \ldots, p_n^{(-2)})$$

Where p_i is the $i - th$ component of vector p.

The derivative of vector p with respect to time can be written as:

$$\frac{dp}{dt} = daig(\dot{p_1}(t), \dot{p_2}(t), \ldots, \dot{p_n}(t))$$

Where $\dot{p_\iota}(t)$ is the derivative of the $i - th$ component of vector p with respect to time.

Substituting these expressions into the given differential equation, we get

$$\frac{dp}{dD} * \frac{dD}{dp} * \frac{dp}{dt} = s(D(p) - AY)$$

Multiplying both sides by $\frac{dD}{dp}$ and integrating with respect to p, we obtain:

$$\int \frac{dp}{dD} * \frac{dp}{dt} dp = s\int (D(p) - AY)\frac{dD}{dp} dp$$

Integrating the left-hand side with respect to p, we get:

$$\int \frac{dp}{dD} * \frac{dp}{dt} dp = \int \frac{d}{dt}\left(\frac{p^2}{2}\right) dp = \frac{p^2}{2} + C_1$$

Where C_1 is an integration constant:

Integrating the right-hand side with respect to p, we get:

$$\int (D(p) - AY)\frac{dD}{dp}dp = \int diag\left(p_1^{(-2)}, p_2^{(-2)}, \dots, p_n^{(-2)}\right) daig\left(\dot{p_1}(t), \dot{p_2}(t), \dots, \dot{p_n}(t)\right)dp - AYp + C_2.$$

Where C_2 is another integration constant.

Since the diagonal matrices commute, we rearrange the integrand or the right-hand side as:

$$\int diag\left(p_1^{(-2)}, p_2^{(-2)}, \dots, p_n^{(-2)}\right) daig\left(\dot{p_1}(t), \dot{p_2}(t), \dots, \dot{p_n}(t)\right)dp$$

$$= \int diag\frac{\dot{p_1}(t)}{p_1}, \frac{\dot{p_2}(t)}{p_2}, \dots, \frac{\dot{p_n}(t)}{p_n})dp.$$

Using the substitution, $u_i = \ln p_i$ we can convert this integral in to simpler form:

$$\int diag\frac{\dot{p_1}(t)}{p_1}, \frac{\dot{p_2}(t)}{p_2}, \dots, \frac{\dot{p_n}(t)}{p_n})dp$$

$$= \int diag(\frac{du_1}{dt}, \frac{du_2}{dt}, \dots, \frac{du_n}{dt})\exp(u_1 + u_2 + \dots + u_n)\, du_1, du_2, \dots, du_n$$

$$= \exp(u_1 + u_2 + \dots + u_n) + C_3$$

Where C_3 is another integration constant.

Substituting these expressions back into the original differential equation (14), we get:

$$\frac{p^2}{2} + C_1 = s(\exp(u_1 + u_2 + \dots + u_n) - AYp + C_2) + C_3$$

Simplifying and solving for p, we get:

$$p(t) = \sqrt{2s(\exp(u_1 + u_2 + \dots + u_n) - AYp + C_2 + C_3) - 2C_1)}$$

Using the initial condition $p(0) = p_0$ we can solve for constraints $C_1, C_2\ and\ C_3$:

$$C_1 = \frac{p_0^2}{2}$$

$$C_2 = AYp_0 - \exp\left(u_1(0) + u_2(0) + \dots + u_n(0)\right)\frac{s}{2} - AYp_0 + C_1$$

Where $u_i(0) = \ln\left(p_i(0)\right)$.

Therefore, the solution to the given differential equation for unit price is;

$$p(t) = \sqrt{2s\left(\exp(u_1(t) + u_2(t) + \dots + u_n(t)) - AYp_0 + AYp(t) + \frac{p_0^2}{2}\right)}$$

Total Revenue

To modify the total revenue function of (Rui Wu et al, 2014) as we have;

$$TR = (p - c)AYe^{-\rho t} \quad (15)$$

where p is the revenue per unit, c is the cost per unit, and ρ is the instantaneous social rate of discount.

The revenue function $(p - c)AYe^{-\rho t}$ represents the total revenue generated by the amount of fish harvested, where:

To solve the revenue function, we need to make the derivative of the function with respect to the price p:

$$\frac{d}{dp}(p - c)AYe^{-\rho t} = AYe^{-\rho t} - \frac{ACYe^{-\rho t}}{dp}$$

Simplifying the expression, we get:

$$\frac{d}{dp}(p - c)AYe^{-\rho t} = AYe^{-\rho t} - ACYe^{-\rho t}$$

$$\frac{d}{dp}(p-c)AYe^{-\rho t} = AYe^{-\rho t(1-c)}$$

This expression tell us how much the revenue changes as we change the price of the fish. If we set this expression equal to zero, we can find the optimal price that maximize revenue:

$$AYe^{-\rho t(1-c)} = 0$$

Since $e^{-\rho t}$ is always positive, we can simplify this expression to

$$1 - c = 0$$

$$c = 1$$

This means that the optimal price that maximizes revenue is equal to the variable cost per unit plus markup.

Now, to find the profit we use solutions of equation (11) and (15). Therefore, we have;

$$\boldsymbol{profit = Total\ Revenue - Total\ Cost}$$

Models

By (DoF, 2021), fishing data of The Gambia from 2010 to 2020 shows interesting results about the landing on the river sites. The artisanal, industrial and aquaculture subsectors make up The Gambia's fisheries sector. The primary activities of the artisanal subsector are relatively low- to medium-input fishing and processing (DoF. Frame Survey, 2019). Regarding food security, socioeconomic activity, revenue production, and employment, it is the most significant sub-sector. The highest catch from 2010 to 2020 was 84,173.846 metric tons in 2019 compare to the lowest in 2020, which was 29,782.72 metric tons. (DoF, 2021) reported that, it was as a result of the covid-19 pandemic where the industrial was not in function.

Considering the components constitute fishing in the Gambia, one can say that foreign vessels have more advantage of the spatial of fish due to technology that we introduced in our model as a new variable in the Schaefer's model.

The official fish catch and the actual amount does not tally simply because most of the time foreign vessels are caught in the waters fishing illegally. There are no mechanism to stop or properly regulate the illegal fishing in the Gambia.

Figure 2.

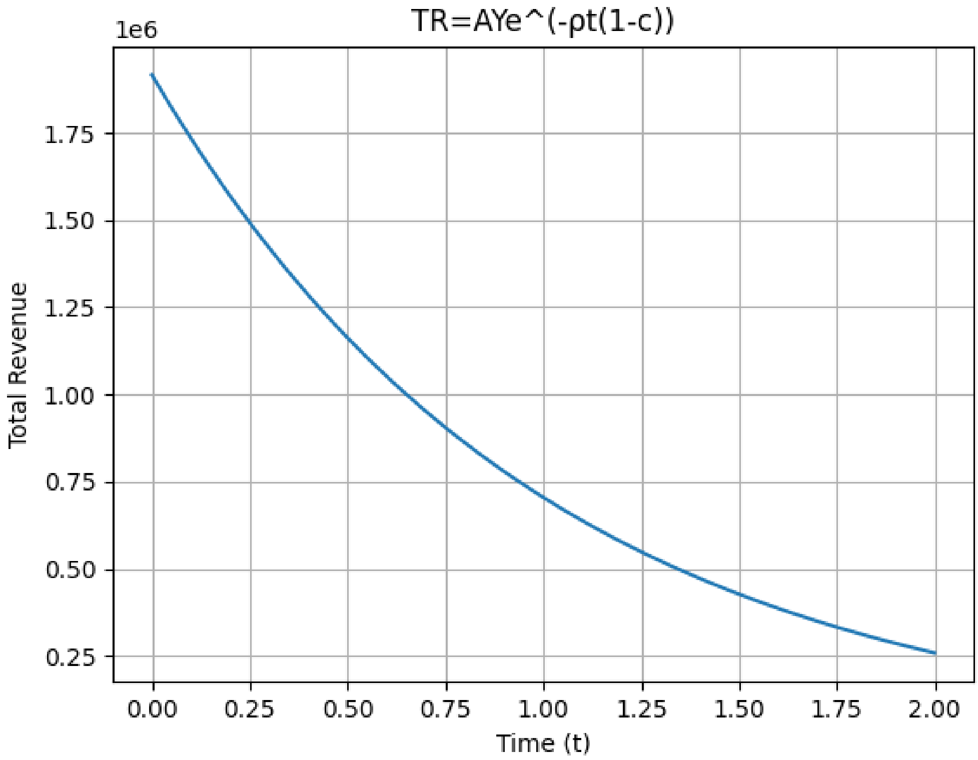

Figure 2: In a revenue function where the annual catch $Y = 84173.846$ metric tons, $A > 1$ and the cost $c > 1$ has demonstrated the graph of revenue that could be obtained. The amount of fish used is obtained from (DoF, 2019) as an empirical data from the Gambia. In a period of several years that year was the highest catch from 2010 to 2020.

Figure 3:

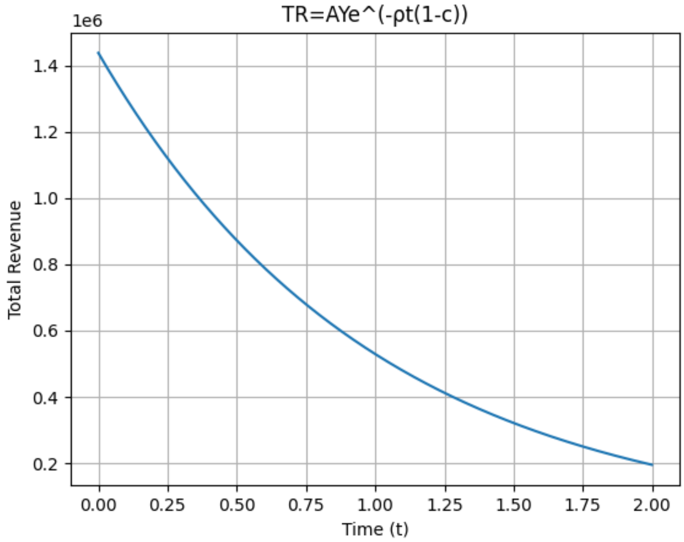

Figure 3: In a revenue function, where the annual catch $Y = 29782.72$ tons, $A > 1$ and the cost $c > 1$ has demonstrated the graph of revenue that could be obtained. The amount of fish used is obtained from (DoF 2020) as an empirical data from the Gambia. In a period of several years that year was the lowest catch from 2010 to 2020.

Figure 4:

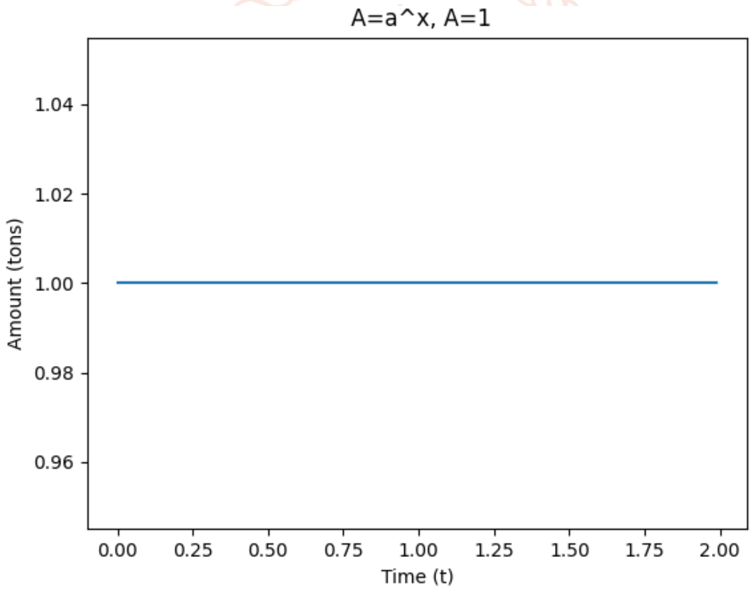

Figure 4: In an event where technology is not considered when fishing, the likelihood of catching high number is low.

Figure 5:

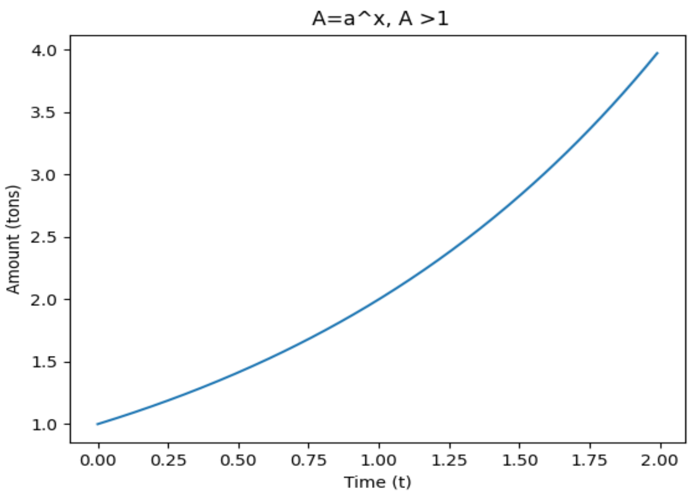

Figure 5: When technology is considered in fishing increases the chances of having high amount of fish.

Figure 6

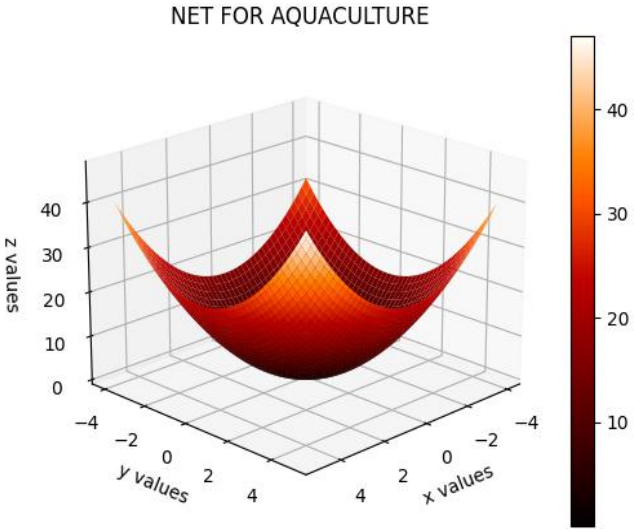

Figure 6: The netting suggested for the aquaculture is a half of a unit circle that get the targeted fish only.

Figure 7

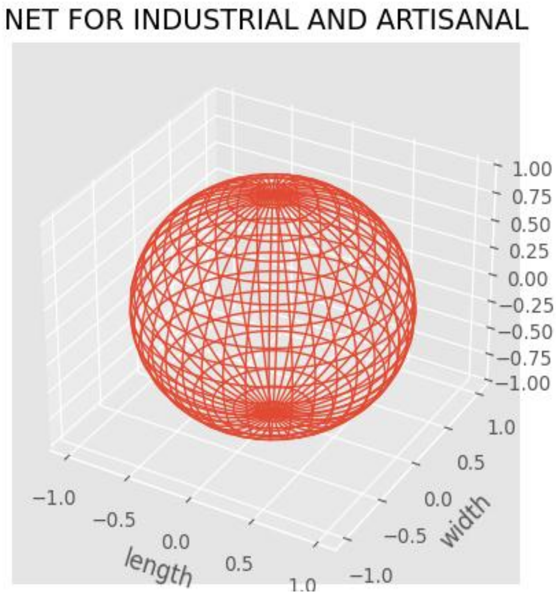

Figure 7: These are recommend mesh net for industrial fishing by authorities to avoid hazardous fishery.

Figure 8

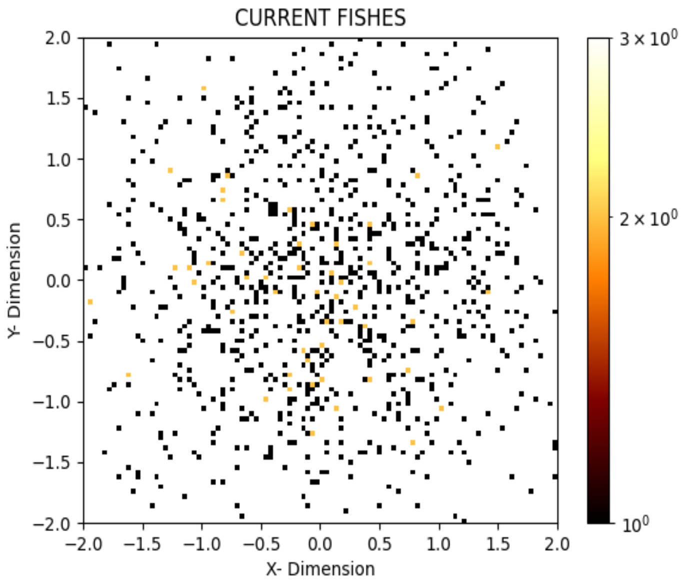

Figure 8: This show the level of fishes in our waters. The level of depletion is towards certain species because of technology and profitability that the fishers concentrated.

Figure 9

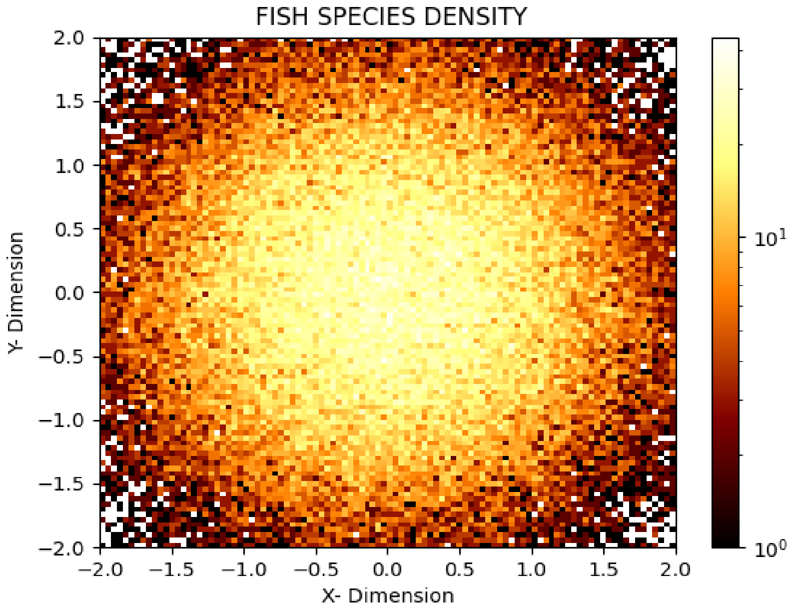

Figure 9: Areas that are secured for economic reasons as the 9 nautical miles are dense with fish. Fish migrate as the fishing surge more.

Conclusion

The research has shown how overfishing can have negative effects and how management regulations can be utilized to guide fishing. The chain of fisheries as a sector avoided the Gambia's prohibitions on illegal fishing. Every fisheries industry has as its top priority the maximum sustainability of profit, which is why this research's assessment of cost-benefit analysis was warranted. In The Gambia, the infrastructure for fisheries management is lacking, which hinders effective fisheries management. Recent technological developments require marine management to be more creative and give high-quality fish more of a priority (Setiawan, E.A, 2021). To identify solutions that will maximize revenue and profit while minimizing cost in the fishing-related space, differential equation method was employed.

Profitability is greatly influenced by price. According to (DoF, 2023), a fish basket can cost anywhere from \$17 and \$33 depending on the species and market availability. Fish stocks may be depleted as a result of the price's enthusiasm. According to our assumptions, depletion occurs when the harvest exceeds the stock, $\left(h(x) > G(x)\right)$. Research has showed us that under Scheafer's model $\dot{x} = G(x) - qEx$, the amount of effort expended would influence the amount of fish gained, which suggests that income depends on the availability of the fish to the fishers while the government's revenue depends on the availability of the vessels and fishermen. If the tons caught are high, then revenue increases; otherwise, it is low. With an estimated annual catch of 53, 000 tons and a basket price of \$25, the revenue is \$37, 857,150.

The price of investing in fishing varies depending on the type of fishing industry. The Gambia produces three different sectors of fish: aquaculture, artisanal, and industrial. Technology was largely responsible for the industrial sector's dominance. That was extensively covered in our prior paper.

Given the availability of the country's maritime resources, The Gambia's economic development should focus on the fishing industry with all the efforts to get solution on the cost-benefit of the finishing industry. However, there is a gap in the cost function that we think can be improved in other researches. On the other hand, current European Union fishery policy is aimed at gradually eliminating fish discards. It forces fishing vessels to land all catches of regulated commercial species. The unwanted catches landed that cannot be directly sold for human consumption, due to the lack of a market, are considered as by-products (European Commission, 2020). Overall, the current study highlights the critical role that cost –benefits knowledge plays in the sustainable use of natural resources, which must be taken into consideration prior to increased exploitation. Hence, in future research on the spatial fish as well as in the development of conservation policy, it is necessary to consider the knowledge of all types of fishers.

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