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An Existence of Optimal Control in the Fishing Sector Using Individual Model

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ABSTRACT

In this study, we looked at an optimum control model that has been around for a while and can be modified thanks to technology. To demonstrate that the fishing industry can be in an ideal control by employing a single product (species), the individual model to optimize utility is clearly discussed. It has been demonstrated that fish growth can match with pricing to close the "for others" gap.

To maximize the utility of the consumer, a nonlinear convex idea is implemented utilizing an individual model of consumer behavior. In doing so, it has demonstrated the best possible control of the fishing boat as it navigated the river in search of fish. Consequently, in the fishing area. The model used will demonstrate mathematical analysis of the best possible control strategies used by the fishing industry.

There are also given the necessary and sufficient requirements for the supplied vector field to be an individual demand function. The findings in this research significantly outperform earlier findings by a number of researchers in this field.

Models that demonstrated the effect of technology on the fishery sector has been shown.

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INTRODUCTION

The issue of optimal control has been studied for a long time (Clark, C.W. and Munro, G.R. (1975)), (Craven, R.D. (1995)) and in recent years, the best method for collecting particular species has received a lot of attention by many authors (Zhang, R., Sun, J., & Yang, H. (2007), Hongying, L. U., & Ke, W. A. N. G. (2010) and Liu, Y. P., Luo, Z. X., & Lin, L. (2008)). The optimal harvesting policy given using Pontryagin principle of the maximum by many authors (Sun, J. F., & Gou, X. K. (2011)) and (Zhang, L., & Huang, Q. (2015)) They consider the following fishing model:

$$\dot{x} = G(x) - qEx$$

Where x is the biomass of the fish population $G(x)$, is biological net growth rate, q is the catchability ratio, and E is fishing effort. This model was originally developed by (Schaefer, M. B. (1957)), as a management tool for eastern tropical Pacific tuna fishing. In Schaeffer's original model, $G(x)$ was specified in "logistic" form

$$G(x) = rx(1-\frac{x}{k})$$

Where r and k are positive parameters called "intrinsic growth rate" and "carrying capacity", given the term

$$h = qEx$$

Represents the introduced fishery's mortality rate, or the catch rate in relation to the specified fishing "effort" E. To define max is their goal $h = qEx$

Studying predator-prey patterns with prey as well attracted the attention of researchers; see, e.g., (Bardi, M., & Dolcetta, I. C. (1997)) and (Brauer, F., & Soudack, A. C. (1981)., Beddington, J. R., & Cooke, J. G. (1982)). Maybe even extensive and unregulated fishing lead to a decrease in total fishing. Some related papers demonstrated that the ideal start the sum of investment of x is the ideal value for ensuring the ideal total amount of fishing by (Zhang, L., & Huang, Q. (2015)), but they did not provide the ideal speed of each fishing strategy. More importantly, is it possible to determine the ideal fishing strategy if the initial investment the number of fish is not x?

Using the aforementioned model as a guide, we propose that modern "technology

(A) be added to the model in order to expand it and decouple it from fishing effort(E) which was not done by other authors.

$$\dot{x} = G(x) - [qEx + A]$$

The model's expansion has shown us how important technology has been to the fishing industry. The speed (s), chemical (c), and sensor (d) used to find fish are collectively referred to as;.$A = scd = a^x$.

Lemma: (Minkowski Inequality) (Charles E. (2014))

Let $f, g \in L^p\left(\Omega\right)$, then $\|f+g\|_{L^p} \leq \|f\|_{L^p} + \|g\|_{L^p}$

Let $p \in [1, +\infty[$, the dual of $L^p\left(\Omega\right)$ is defined as the following.

$[L^p\left(\Omega\right)]^* = f: L^p\left(\Omega\right) \to R$, Linear boundary.

Theorem:

Let $p \in [1, +\infty[$, let q it's conjugate, that is $\frac{1}{p} + \frac{1}{q} = 1$, then if $L \in \left[L^p\left(\Omega\right)\right]^*$, there exist a unique function $g \in L^p\left(\Omega\right)$ such that;

$(i)\ \forall\ f \in L^p\left(\Omega\right), l(f) = \int_\Omega f(x)g(x)dx$

$(ii)\ \|f\|_{[L^p]^*} = \|g\|_{L^p\left(\Omega\right)}.$

This result allows one to identifies the elements of $\left[L^p\left(\Omega\right)\right]^*$ as element of $L^q\left(\Omega\right)$.

Remark

If $\frac{1}{p} + \frac{1}{q} = 1, 1 \leq p \leq \infty$.

$$\left[L^p\left(\Omega\right)\right]^* = L^q\left(\Omega\right).$$

The theorem is shown to justify that technology has a great role in fishing nowadays. Because it is in a space that is continuous.

Given the impact on the expected revenue of the effective utilization rate for renewable resources, (Wu, R., Shen, Z., & Liao, F. (2015)) they introduce the concept of effective utilization into a renewable resources development model and we propose an optimal control model to ensure that it approximates the actual situation. Reduced productivity of ecological systems is an important economic impact of environmental pollution (Freeman, A. M., Herriges, M., & Kling, C. (1993)). A typical example had been given by (Olsen, J. R., & Shortle, J. S. (1996)) the effect of pollution on fisheries. Some species are in danger of going extinct. The age-old issues of overfishing and bycatch waste are major contributors to diminishing populations. Water contamination is a significant additional factor. Fish can suffer immediate injury from pollution exposure. For instance, The Gambia has had multiple fish kills as a result of acute pesticide exposure. (Department of fisheries, 2022).

In this paper, we present discrete optimum distribution control method and optimum fishing strategy control method to satisfy Walras Law. In contrast to existing optimal approaches like the conventional variational method, Pontryagin's principle maximum, and discrete dynamic programming work by (Zhang, L., & Li, C. Y. (2014)), this approach is novel and effective for the discrete optimal control problem. As a typical logistical function that is tied to the individual model, model building. The design of each fishing strategy's optimal control was done using discrete optimal control and the optimal fishing strategy approach; the core of our work is a rigorous mathematical study of the optimal control problem utilizing a specific model. Analysis enables us to determine an individual's and the industry's overall optimal initial investment amounts. Additionally, we learn that fishing operations shouldn't be begun for a few years to make a total quantity of fishing optimal if the beginning investment amount is lower or higher than the optimum value and the intrinsic growth rate of the fish R is also too low. Finally, a few common instances are provided to highlight the outcomes. In a mathematical analysis of natural resources, optimal control is crucial. Continuous-time production, under the premise of discrete-time demand, offers for more flexibility by taking into account a dual supply policy (Yi, F., Baojun, B., & Jizhou, Z. (2013)). The amount of fish caught in the Gambia is enough to feed all of the country's inhabitants, but on the other hand, neither the locals nor the fishermen are able to purchase fish from the market recently. One of the reasons for this research is to find ways to change that trend. Methods to curb that trajectory is among the reason of this research. Natural resources are not evenly distributed particularly fish, that's an important aspect of the research to see that method that can be used to see that fish are supplied equally to curb the high prices in some area where it is lacking. We think, having what we call "for others" as an approach in equally distributing the resource might bridge the gap.

If the quantity to satisfy the area where is harvested is x then "for other" would be $x + 1$. Therefore, price of x is equal to price of $x + 1$. The variation parameters here should not make a difference because government has to interfere in mitigating the gap through that idea of optimal control.

Methodology of models

Several years data of fish catch in the Gambia has been use as empirical data for running a regression to show the optimal point of specified species of fish catch.

These models have helped to analysis the amount of fish to be catch in order to solve the problem of Walras Law in economics. As we have seen from the models that technology has played a significant role in the recent times of fishing. If $A = a^x$, it brought the quantity of fish available which trigger higher prices in the market. Hence, we understood from the individual model that the consumer has variety of choices depend on his/her budget line. The more species available in the market, the lesser the price of that fish.

The demonstration done on the species satisfies the $U(x) \geq U(y)$. Such function U is called the utility function of consumer. Here, there are many species of fish in the river, which could be fetch, but harvesting them can lead to the drain of resources with limited budget $w > 0$.

Individual Model

In order to maximize the growth of fish $G(x)$ and production in the fishing sector in the moment of the covid-19 and the Russia – Ukraine war, we will apply the individual model as we have different species of fish and different fishing vessel are attract in the fishing sector.

The field of consumer theory has been studied by many and thorough explanations have been done to show the rationality behind the decision making of the consumer. A consumer will always want to maximize his utility in making a choice set, (Jonathan Levin and Paul Milgrom, (2004)). From consumer perspective, consumer's choice sets are assumed to be defined by certain prices and the consumer's income or wealth. We defined the consumer problem as:

$\max_{x \in R_+^n} U(x)$

where, $p.x \leq w$.

The aim of introducing the utility function is to give a full characterization of individual demand function. More precisely, necessary and sufficient conditions are given for a map $p \to x(p)$ to be derived from a maximization problem. The proof relies on finding convex solutions to a strongly nonlinear partial differential equation.

In economics, many questions take the following form: Given an integer $n \geq 1$, a vector field $X: R^n \to R^n$,

Question 1: Can we find $2k$ functions λ and V such that

$$X(p) = \lambda(\mathrm{p})\nabla\mathrm{V}(\mathrm{p})? \ldots\ldots\ldots\ldots\ldots\ldots (1.1)$$

Question 2: Can we choose the functions λ and V in decomposition (1.1) so that λ is positive and V_i (quasi) convex?

Question 3: Can we ask λ and V to satisfy additional conditions of the following type:

$$\varphi(p, \lambda(\mathrm{p}), \nabla\mathrm{V}(\mathrm{p}))$$

Where φ is given?

Some answers:

- For λ constant (without positivity and convexity assumptions), this is a classical theorem of Poincare and the condition on X for such decompositions is given by:

$$\frac{\partial \boldsymbol{X_i}}{\partial \boldsymbol{p_j}} = \frac{\partial \boldsymbol{X_j}}{\partial \boldsymbol{p_i}} \ \forall\ \mathbf{1} \leq \boldsymbol{i}, \boldsymbol{j} \leq \boldsymbol{n}.$$

- In the case where λ non constant, that is a Frobenius condition and it can be stated as follows: $\omega \wedge d\omega = 0$

Where ω is the differential 1-form given by

$$\omega = \sum_{i=1}^{n} X_i(p) dp_i$$

- A positive answer of question 2 is given by (Ekeland and Nirenberg, (2002)).

In this research, we first show where all these questions are coming from and second we study question 3 initiated by (Ekeland and Djitte , 2006). In recent works they give necessary and sufficient conditions for a given vector field X to be decomposed in the following form:

$$X(p) = \lambda(\mathrm{p})\nabla\mathrm{V}(\mathrm{p}). \ 1.2$$

Where λ is positive, V is convex. The functions λ and V satisfy an additional condition of the form:

$$\varphi(p, \lambda(\mathrm{p}), \nabla\mathrm{V}(\mathrm{p}))$$

Where φ is given.

By the individual model, we assume in a market situation in which N goods are available. A bundle of goods is denoted by a vector x= $(x_1, x_2, x_3,\ldots, x_N)$, where x_i is the quantity of goods i. The consumer is characterized by a preference relation $\geq x$ and a budget $w > 0$. It is known at each preference relation is associated with a concave function U_so that, the consumer prefers the bundle of goods $x = (x_1, x_2, x_3,\ldots, x_N)$ to the bundle goods $y = (y_1, y_2, y_3, \ldots, y_N)$ if and only if $U(x) \geq U(y)$. Such function U is called the utility function of consumer. We also assume that the market price is given by a vector $p = (p_1, p_2, p_3, \ldots, p_N)$, where p_i is the unit price of the goods i.

- A bundle of goods x= $(x_1, x_2, x_3,\ldots, x_N)$ will cost:

$$p.x = p_1x_1 + \cdots + p_Nx_N = \sum_{i=1}^{N} p_ix_i$$

And give a satisfaction $U(x)$ to the consumer.

Therefore, the consumer chooses the best consumption (bundle of goods) by solving the following optimization problem:

$$(\mathrm{P})\begin{cases} MaxU(x) \\ p.x \leq w \\ x_i \geq o \; for\; all\; i \end{cases} \quad 1.3$$

This is the individual consumer model.

The coherence reasons let us make some assumptions on U that guarantee existence and uniqueness of the solution of problem (P).

In what follows, we assume that:

$$1.4\begin{cases} U \text{ is of class } C^{\infty} \text{ on } R^N \\ U \text{ is increasing with respect to each variable} \\ \text{The Hessian matrx } D^2U \text{ is negative definite to the orthogonal subspace of } U\nabla U \end{cases}$$

As a consequence of the fact that the utility function U is increasing, the budget constraint is saturated. That is $p.x = w$ at the optimal solution. Therefore, problem (P) takes the following form:

$$(P)\begin{cases} MaxU(x) \\ p.x = 1 \end{cases} (1.5)$$

where the budget is normalized to 1.

Under the hypothesis 1.4, for every market price, the problem (P) has a unique solution $x(P)$. from lagrangian theorem, there exist a unique scalar $\lambda(\mathrm{p})$ such that

$$\nabla \cup \left(x(p)\right) = \lambda(\mathrm{p})p, \text{ where } \lambda(\mathrm{p}) > 0 \text{ and } p.x(p) = 1. (1.6)$$

This is a standard characterization in convex analysis. The scalar $\lambda(\mathrm{p})$ is called the lagrange multiplier associated to the solution $x(p)$. Both $x(p)$ and $\lambda(\mathrm{p})$ are functions of p. The map $p \rightarrow x(p)$ is called the individual demand function. Using the implicit function theorem, we have the following properties of the demand function.

Proposition 1: Under hypothesis (1.1), the demand function $x \; is \; C^{\infty}$.

Proof.

Let $\bar{p} \gg 0$, $\bar{x}$ and $\bar{\lambda}$, the solution and the langragian multiplier associated to $\bar{p}$. Then from langrange theorem we have

$$\nabla U(\bar{x}) - \bar{\lambda}p = 0 \; (1.7)$$

$$p\bar{x} - 1 = 0. \; (1.8)$$

Let F be the map from $R^n \times R \times R^n \rightarrow R^n \times R$ defined by:

$$F(x, \lambda, \mathrm{p}) = (\nabla \mathrm{U(x)} - \lambda \mathrm{p}, \mathrm{p.x} - 1). \; (1.9)$$

Then, $F(x, \lambda, \mathrm{p}) = (0{,}0)$ and if $F_{x,\lambda}$ denote the Jacobian matrix of F with respect to the variables x and λ then:

$$F_{x,\lambda}(\bar{x}, \bar{\lambda}, \bar{p},) = \begin{bmatrix} D^2U(\bar{x}) & -\bar{p} \\ \bar{p}' & 0 \end{bmatrix}. \; (1.10)$$

Let $y \in R^n$ and $\mu \in R$ such that:

$$\begin{bmatrix} D^2U(\bar{x}) & -\bar{p} \\ \bar{p}' & 0 \end{bmatrix}\begin{pmatrix} y \\ \mu \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, (1.11)$$

Then we have:

$$D^2U(\bar{x}).y - \mu\bar{p} = 0, (1.12)$$

$\bar{p}'y = 0.$ (1.13)

From (1.3) and (1.13), y is in the orthogonal subspace to $\nabla U(x)$. Multiplying (1.12) by y', we obtain:

$$y'D^2U(\bar{x})y - \mu y'p = 0$$

Using hypothesis (1.4) and (1.13), it follows that $y = 0$ and $\mu = 0$. This proves that the matrix $F_{x,\lambda}(\bar{x}, \bar{\lambda}, \bar{p},)$ is invertible. Using the implicit function theorem, there exist an open neighbourhood v_1 of $\bar{p}$, an open neighbourhood v_2 of $(\bar{x}, \bar{\lambda})$ in $R^n \times R$ and a unique function $\varphi = (\varphi_{1,}\varphi_2), C^\infty$ from v_1 to v_2 satisfying:

$$\varphi_1\bar{p} = \bar{x}$$

$\varphi_2\bar{p} = \bar{\lambda}$ (1.14)

$F(\varphi_1(p), \varphi_2(p), p) = 0, \forall p \in v_{1.}$Since the function $p \to x(p)$ and $p \to \lambda(p)$ satisfy (1.10) on $v_{1,}$ it follows from the uniqueness that $x(p) = \varphi_1(p)$ and $\lambda(p) = \varphi_2(p)$ for all $p \in v_{1.}$ Therefore, the functions $p \to x(p)$ and $p \to \lambda(p)$ of class C^∞ in a neighbourhood of $\bar{p}$. ■

The following definition will be useful.

Definition:
The slutsky matrix associated to the demand function x, denoted by S is given by $S = (s_{ij}), i, j = 1, \dots, n$ which

$$s_{ij} := \frac{\partial x_i}{\partial p_j} - \sum_{k=1}^{n} pk\frac{\partial x_i}{\partial p_k}x_j \ (1.15)$$

The following theorem gives a characterization of a demand function from economy.

Theorem
Let $x: R^N \to R^N$ be a vector field satisfying the walras law: $p.x(p) = 1$ for all p.
Then x is a demand function if and only if the slutsky matrix S associated to the demand function x satisfies

- $Sp = 0\ p \gg 0$
- $S\ symmetric$
- $S\ negative\ semi - definite$

Model analysis
Considering the type of model we are working on, it is easy to use python to run and get solution curves with an empirical data. The population of the fish increases as the number of fishing vessels decreases and the population decreases as the fishing vessels increases. The growth rate (r), has been manipulated by the influx of the fishermen.

In recent times, covid -19 has blown the amount of fish catch as it is demonstrated in the models of the individual species of fish. The regression line in each of the model explain the correlation of the amount of fish catch and the year to achieve the optimal point and a regulator in the fishing sector.

After the collection of the several year's data from the department of fisheries, all the species available are named and the amount catch for each year. The trend has shown that, the adventure of technology (A) help sophisticated vessel can trace places where fish are in abundance, as it is understood that fish migrate and die due to high exploitation and unforeseen circumstances.

It will be obvious to maximize utility with a budget constrain as it is demonstrated in the individual model.

$$(\mathrm{P})\begin{cases} MaxU(x) \\ p.x \leq w \\ x_i \geq o\ for\ all\ i \end{cases}$$

By each of the species, individual can obtain the amount of fish that they want from the market with their budget line. The models below shows some of the individual species.

Fig1:

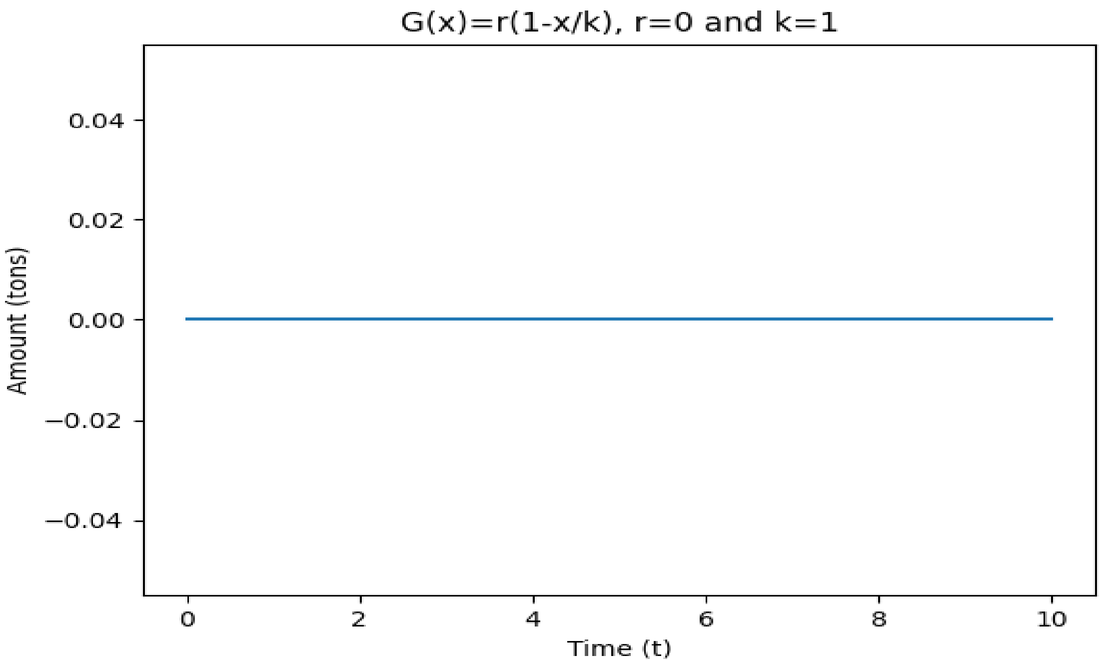


Fig. 1 show the logistic form $G(x)$ where $r = 0$ and $k = 1$ for the first time of fishing

Fig.2

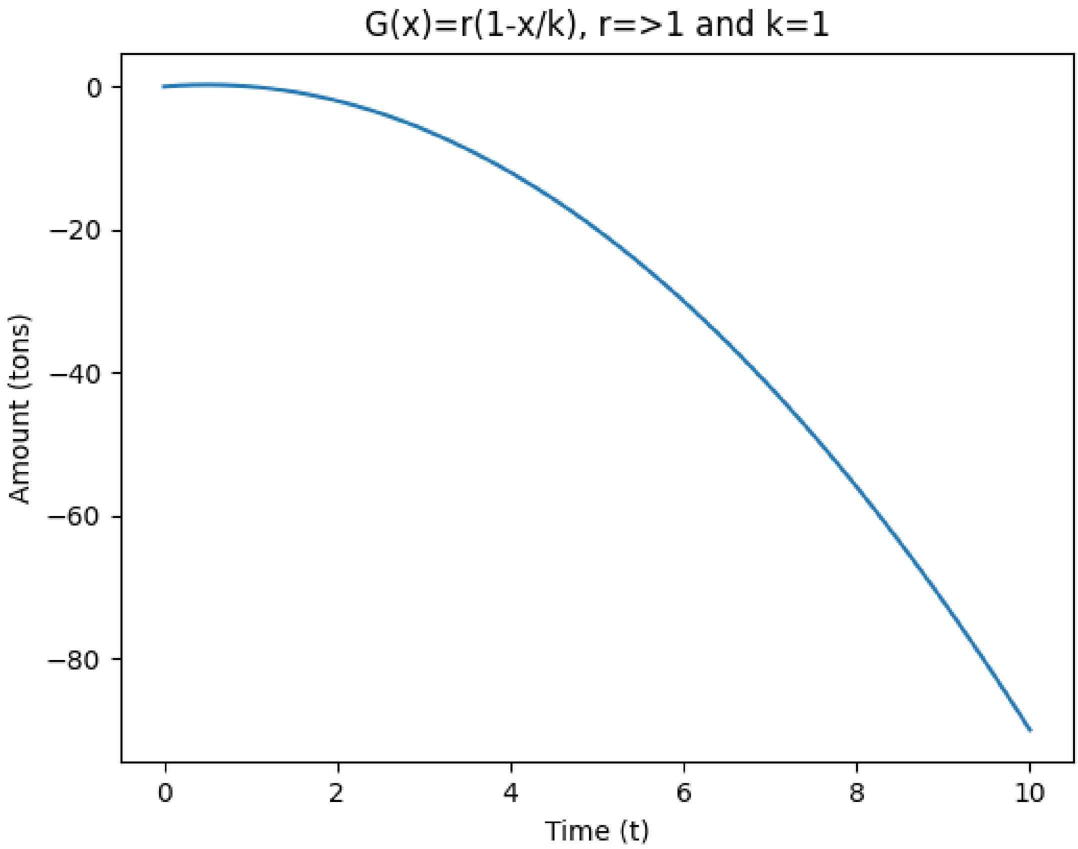


Fig.2 shows the rate $r > 0$ it always gives a negative increment.

Fig.3

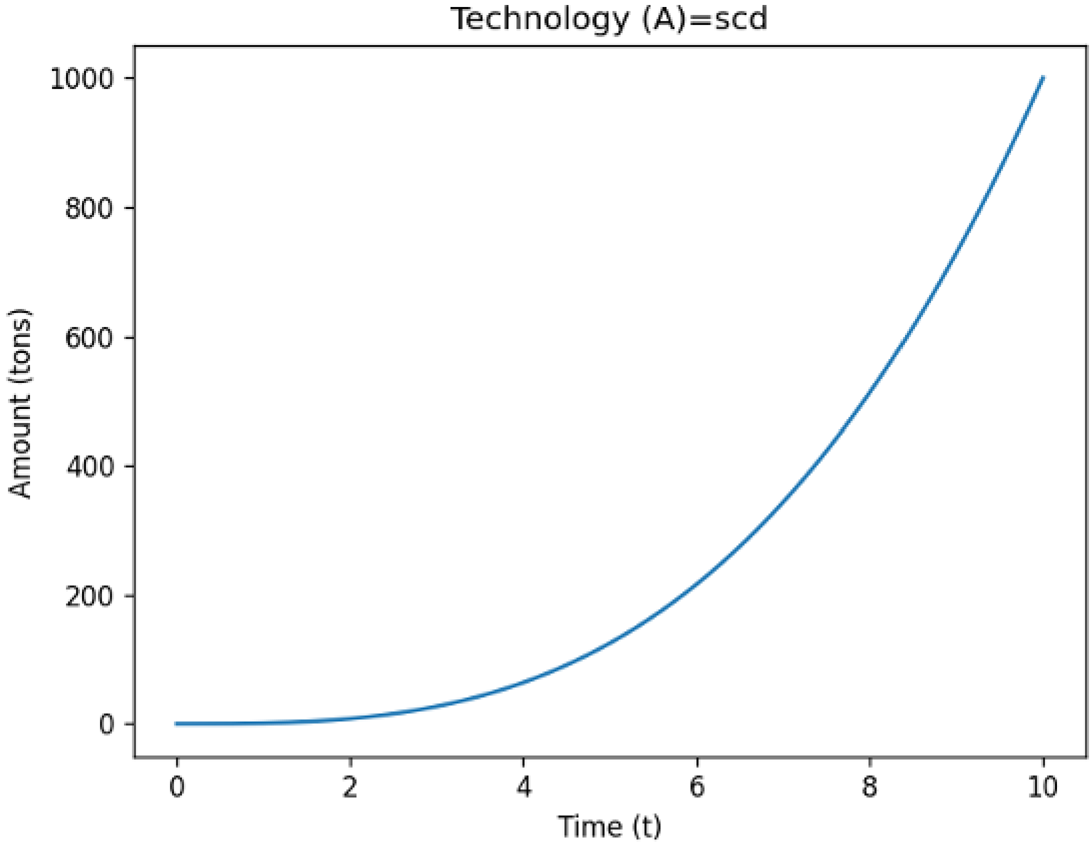
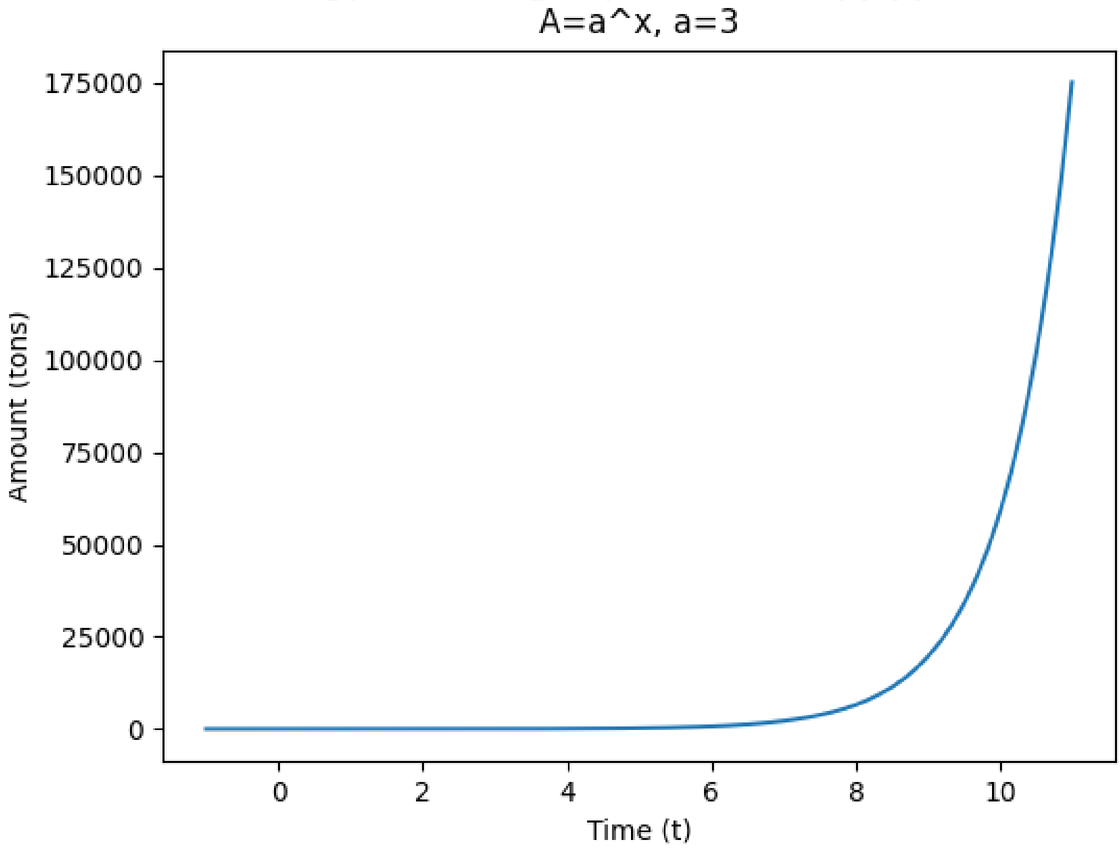


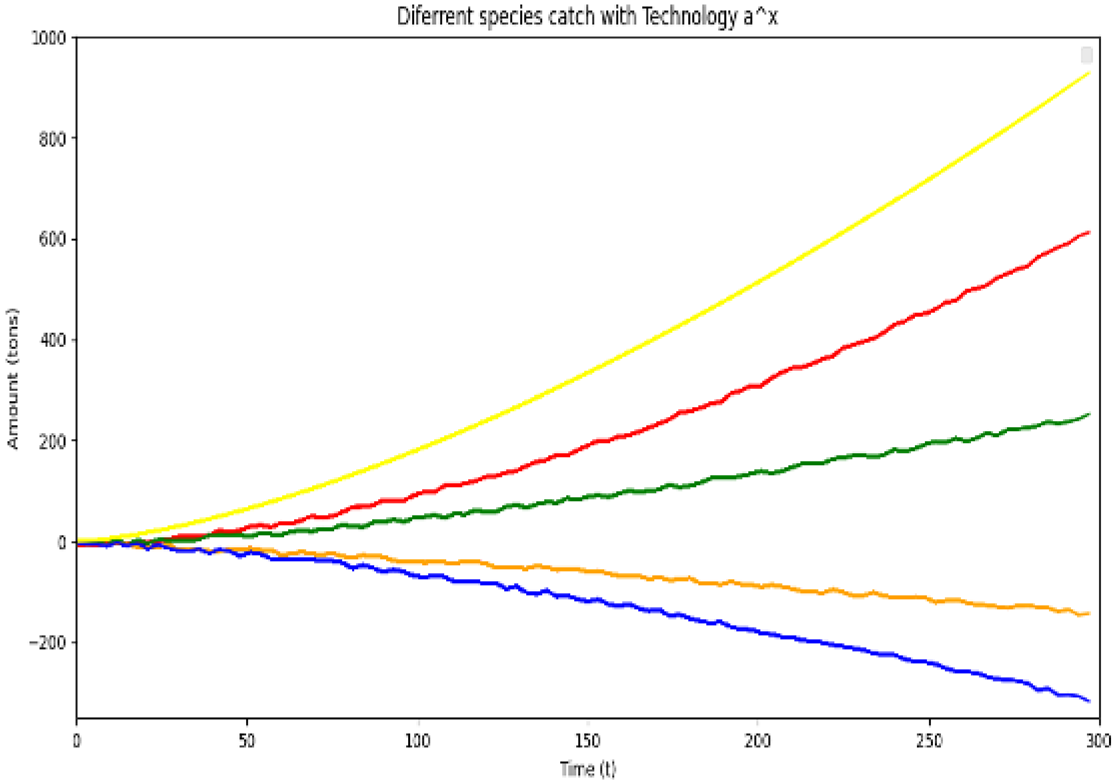
Fig.3 this is where technology is taking in a linear form. That is technology $(A) = scd$, thus $s = 1, c = 1$ *and* $d = 1$, Which shows that the amount of fish catch moves exponentially.

Fig.4



With technology $A = a^x$ has move exponentially to increase the amount of fish catch.

Fig.5



As we understood that, there are different, species catch at the same time. Technology has made it easily to catch fish but it resulted to the downing of the amount of fish.

Fig.5

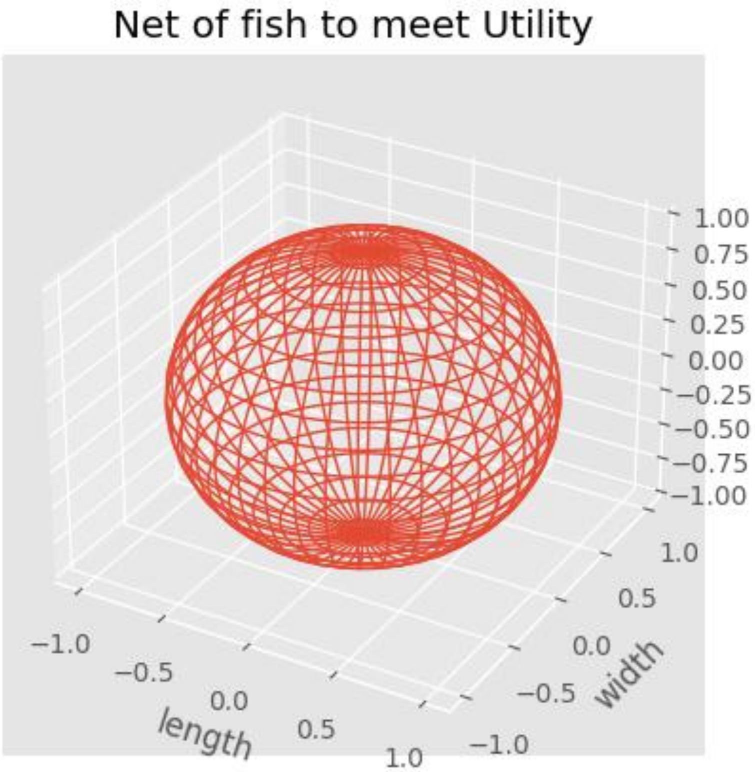
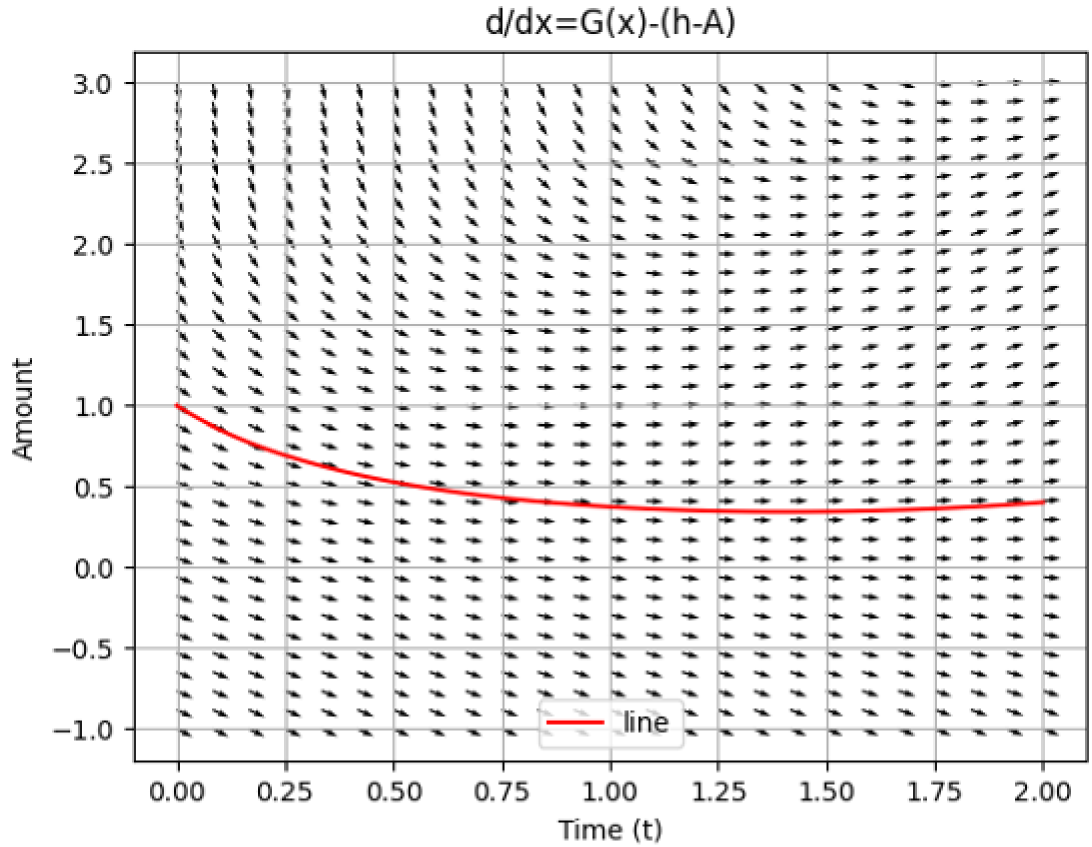


Fig.6



A solution curve, effect of Technology on Fishing

The directional field has shown that the effect of technology reduces the amount of fish available which has a great impact on the individual utility.

Discussion and Conclusion

The Food and Agriculture Organization (FAO) assessment of fisheries reveals that the share of fish biomass within biologically sustainable levels has exhibited a downward trend, declining from 90% in 1974, 68.8% in 2013 to 64.4% in 2019; see Status of FAO (2022). In order to ensure sustainability, the resource must be used efficiently and effectively. This necessitates the utilization of the resource using appropriate and efficient technology.

The discrete optimal control concept has only been stressed to show how easily technology contributes to the depletion of our natural resources. In order to demonstrate that there is an optimal control in the fishing industry, a nonlinear technique is used to introduce the utility function into the individual model.

A social planner may use a variety of methods to accomplish the goal of altering the pace at which the resource is consumed. To teach the consumer population to change their consumption behaviors as necessary would be an indirect and relatively long-term strategy. A different strategy would be to limit the price, which would impact the percentage of the population that has access to the resource. However, this might have significant economic repercussions, particularly if the resource in issue is crucial to the consumption society's economy.

The analysis of the species of the fishes shows a trend of fish catch in the Gambia for ten years. It generate the statistical analysis of the sample species taken. There are many fishes in the river of the Gambia. But each can show that an optimal point by using the best-fit line. If we are to consider “for others” as we demonstrated to mitigate the gap of scarcity of fish in our various market, we have to implement policies that would guide to the optimal point.

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