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Slightly Normal Topological Spaces of the First Kind and the Second Kind and the Third Kind

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ABSTRACT

In this paper further new generalizations of normal spaces have been made. These have been called slightly normal spaces of the first kind, the second kind and the third kind respectively. A number of important properties of these spaces have been proved.

KEYWORDS: *Slightly normal spaces, open set, closed set, finite collection, countable collection, quotient space*

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1. INTRODUCTION

This is the sixth in a series of our papers. The first, the second, the third, the fourth and the fifth such papers has appeared in 2018([9],[10],[11]), 2020[8] and in 2021[7].Two types of generalizations of normal spaces different from those considered in the last paper 2021[7] have been defined in this paper. A topological space X in which a particular pair of disjoint closed subsets can be separated by disjoint open sets will be called a slightly normal space of the first kind. Generalizing this concept, we shall call a topological space X a slightly normal space of the second kind (third kind) if there is a finite (countable) collection of mutually disjoint closed subsets in X, for which each pair can be separated by disjoint open sets. We have studied these classes closely, and established a number of important properties of these spaces which resemble those of normal spaces.

We have used the terminology and definitions of text book of S. Majumdar and N. Akhter [1], Munkres [2], Dugundji [3], Simmons [4], Kelley [5] and Hocking-Young [6].

We now define slightly normal spaces of the first kind and proceed to study them.

2. Slightly Normal Spaces of the First Kind

Definition 2.1: A topological space X will be called **slightly normal of the first kind (s. n. f. k.)** if there exist two disjoint nontrivial closed sets F_1, F_2in X such thatF_1 and F_2 can be separated by disjoint open sets. This space will be denoted by **(X; F_1, F_2).**

Example 2.1: Every normal space is **slightly normal space of the first kind**.

Example 2.2: Let $X = \mathbb{R}, \Im = \langle \mathbb{R}, \emptyset, \mathbb{Q}, \mathbb{Q}^c, (2,3) \rangle$

$\mathbb{Q}$ and $\mathbb{Q}^c$ are closed sets in X and these being open also can be separated by disjoint open sets. However, $\mathbb{Q} \cap (2,3)$and $\mathbb{Q}^c \cap (2,3)$ are two disjoint closed sets which can't be separated by disjoint open sets. **Thus $(X, \Im)$is s. n. f. k. but not normal. Here X=$(X; \mathbb{Q}, \mathbb{Q}^c)$.**

Example 2.3: Let $X = \mathbb{R}, \Im = \langle \mathbb{R}, \emptyset, (1,4), (1,4)^c, (5,7)^c \rangle$

Then the disjoint closed sets (1,4) and $(1,4)^c$ can be separated by disjoint open sets, but (1,4) and (5,7) are disjoint closed sets and these can't be separated by disjoint open sets. **Thus $(X, \mathfrak{I})$ is s. n. f. k. but not normal. Here X=$(X;(1,4),(1,4)^c)$.**

Comment 2.1: It is easy to see that there are infinitely many s. n. f. k. which are not normal.

Theorem 2.1: Every normal space is slightly normal space of the first kind but the converse is not true in general.

Proof: Let X be a normal space. Let F_1, F_2 be two disjoint nontrivial closed sets in X. Now, since X is normal, there exist disjoint open sets G_1, G_2 in X such that $F_1 \subseteq G_1$ and $F_2 \subseteq G_2$. Therefore X is slightly normal space of the first kind.

To see that the converse is always not true,

let $X = \{a,b,c,d,e\}$, $\mathfrak{I} = \{X, \Phi, \{a,b\}, \{a,b,e\}, \{e\}, \{a,b,c,d\}, \{b,c,d,e\}, \{b\}, \{b,e\}, \{b,c,d\}\}$.

Then $(X, \mathfrak{I})$ is a topological space in which the closed sets of X are X, Φ, $\{c,d,e\}$, $\{e\}$, $\{a,b,c,d\}$, $\{c,d\}$, {a}, {a, c, d, e}, {a, c, d}, {a, b} .

The closed sets $\{a,b,c,d\}$ and {e} can be separated by $\{a,b,c,d\}$ and {e}, but the closed sets $\{c,d,e\}$ and {a} cannot be separated by disjoint open sets. Thus $(X, \mathfrak{I})$ is slightly normal space of the first kind but not normal.

Theorem 2.2: A topological space X is slightly normal space of the first kind if and only if there exist two disjoint nontrivial closed sets F_1, F_2 and an open set G such that $F_1 \subseteq G \subseteq \overline{G} \subseteq F_2^c$.

Proof: First, suppose that X is slightly normal space of the first kind. Then there exist disjoint nontrivial closed sets F_1, F_2 and open sets G, H in X such that $F_1 \subseteq G$ and $F_2 \subseteq H$ and $G \cap H = \phi$. It follows that $G \subseteq H^c \subseteq F_2^c$. Hence $G \subseteq \overline{G} \subseteq H^c \subseteq F_2^c$. Thus, $F_1 \subseteq G \subseteq \overline{G} \subseteq F_2^c$.

Conversely, suppose that there exist disjoint nontrivial closed sets F_1, F_2 and an open set G in X such that $F_1 \subseteq G \subseteq \overline{G} \subseteq F_2^c$. Here $F_1 \subseteq G$ and $F_2 \subseteq \overline{G}^c$. Let $\overline{G}^c = H$. Then H is open, $F_2 \subseteq H$ and $G \cap H = \phi$. Hence X is slightly normal space of the first kind.

Theorem 2.3: Let $\{X_i\}_{i \in I}$ be a non-empty family of topological spaces, and let X=$\prod_{i \in I} X_i$ be the product space. If each X_i is slightly normal of the first kind, then X is slightly normal of the first kind.

Proof: Since each X_i is slightly normal of the first kind, there exist for each i, two nontrivial closed sets F_i, H_i and two open sets U_i, V_i in X_i such that $F_i \subseteq U_i$, $H_i \subseteq V_i$, $F_i \cap H_i = \phi$, $U_i \cap V_i = \phi$.

Let F=$\prod_{i \in I} F_i$,H=$\prod_{i \in I} H_i$. Then F and H are closed sets in X. Clearly, $F \cap H = \phi$. Let $U = \prod_{i \in I} U_i$, $V = \prod_{i \in I} V_i$. Then, U and V are open sets in X, and $F \subseteq U$, $H \subseteq V$ and $U \cap V = \phi$. Therefore, X is slightly normal space of the first kind.

Theorem 2.4: Every open and one-one image of a slightly normal space of the first kind is slightly normal space of the first kind.

Proof: Let X be a slightly normal space of the first kind and Y a topological space and let $f: X \rightarrow Y$ be an open and onto mapping. Since X is slightly normal space of the first kind, there exist disjoint nontrivial closed sets F_1, F_2 and disjoint open sets G_1, G_2 in X such that $F_1 \subseteq G_1$ and $F_2 \subseteq G_2$. Since f is open, $f(F_1^c)$ and $f(F_2^c)$ are open in Y. So $(f(F_1^c))^c$ and $(f(F_2^c))^c$ are closed in Y.

Now, $F_1^c \cup F_2^c$=X and so $f(F_1^c \cup F_2^c)$ =Y, i.e., $f(F_1^c) \cup f(F_2^c)$=Y. Hence $(f(F_1^c))^c \cap (f(F_2^c))^c = \Phi$. Let $y \in (f(F_1^c))^c$. Then $y \notin f(F_1^c)$ i.e., for every $x \in F_1^c$, $f(x) \neq y$. Hence there exists $x_1 \in F_1$ such that $f(x_1) = y$, since f is onto. Thus $y \in f(F_1)$. Hence $(f(F_1^c))^c \subseteq f(F_1)$. Similarly, $(f(F_2^c))^c \subseteq f(F_2)$.

Now, $f(F_1) \subseteq f(G_1)$, $f(F_2) \subseteq f(G_2)$, f being open and one-one, $f(G_1), f(G_2)$ are open and disjoint in Y. Thus for the disjoint nontrivial closed sets $(f(F_1^c))^c, (f(F_2^c))^c$ in Y and there exist disjoint open sets $f(G_1), f(G_2)$ in Y such that $(f(F_1^c))^c \subseteq f(G_1)$ and $(f(F_2^c))^c \subseteq f(G_2)$.Hence Y is slightly normal space of the first kind.

Corollary 2.1: Every quotient space of a slightly normal space of the first kind is slightly normal space of the first kind.

Proof: Let X be a slightly normal space of the first kind and R is an equivalence relation on X. Since the projection map p:X$\rightarrow \frac{X}{R}$ is open and onto, the corollary then follows from the above Theorem 2.4.

Theorem 2.5: Let X be a slightly normal space of the first kind and Y is a subspace of X. Then Y is a slightly normal space of the first kind.

Proof: Since X is slightly normal space of the first kind, there exist disjoint nontrivial closed sets F_1, F_2 and disjoint open sets G_1, G_2 in X such that $F_1 \subseteq G_1$ and $F_2 \subseteq G_2$. Let $H_1 = Y \cap F_1$ and $H_2 = Y \cap F_2$. Then H_1, H_2 are closed in Y and $H_1 \cap H_2 = \Phi$. Also let $V_1 = Y \cap G_1$, $V_2 = Y \cap G_2$. Then $V_1 \cap V_2 = \Phi$ and $H_1 \subseteq V_1$, $H_2 \subseteq V_2$. Hence Y is slightly normal space of the first kind.

Remark 2.1: The corresponding theorem does not hold for normal spaces. The validity of the proof in Theorem 2.5 above depends on the separablity of a particular pair of disjoint closed spaces by disjoint open spaces (See Ex. of Munkres [12]).

Comment 2.1: A continuous image of a slightly normal space of the first kind need not be slightly normal space of the first kind.

For if (X,T_1) is a slightly normal space of the first kind) and (X,T_2) a space with the indiscrete topology, then the identity map $1_x : X \rightarrow X$ is continuous and onto. But (X,T_2) is not slightly normal space of the first kind.

Theorem 2.6: Each compact Hausdorff space is slightly normal space of the first kind.

Proof: Let X be a compact Hausdorff space and let A,B be two disjoint closed subsets of X. Let $x \in A$ and $y \in B$. Then $x \neq y$. Since X is Hausdorff, there exist disjoint open sets G_y and H_y such that $x \in G_y$ and $y \in H_y$. Obviously $\{H_y : y \in B\}$ is an open cover of B.

Since B is a closed subset of X, B is compact. So there exists a finite sub-cover $\{H_{y_1}, H_{y_2}, \ldots, H_{y_m}\}$ of B. Let $H_x = H_{y_1} \cup H_{y_2} \cup \ldots \cup H_{y_m}$ and $G_x = G_{y_1} \cap G_{y_2} \cap \ldots \cap G_{y_m}$. Then $B \subseteq H_x$, $x \in G_x$ and $H_x \cap G_x = \phi$. So for each $x \in A$ there exist two disjoint open sets G_x and H_x of X such that $x \in G_x$ and $B \subseteq H_x$. Hence $\{G_x : x \in A\}$ is an open cover of A. Since A is a closed subset of X, A is compact. So there exists a finite sub-cover$\{G_{x_1}, G_{x_2}, \ldots, G_{x_n}\}$of this cover A. Let $G = G_{x_1} \cup G_{x_2} \cup \ldots \cup G_{x_n}$ and $H = H_{x_1} \cap H_{x_2} \cap \ldots \cap H_{x_n}$. Then G, H are open sets of X and $A \subseteq G$, $B \subseteq H$ and $G \cap H = \phi$.

Theorem 2.7: Every metric space is slightly normal space of the first kind.

Proof: Since every metric space is normal, therefore it is slightly normal space of the first kind.

We now define slightly normal spaces of the second kind and proceed to study them.

3. Slightly Normal Spaces of the Second Kind

Definition 3.1: A topological space X will be called **slightly normal of the second kind (s. n. s. k)** if there exists a finite collection $\mathcal{F}$ of pairwise disjoint nontrivial closed sets in X such that, for each pair F_1, F_2 in $\mathcal{F}$, F_1 and F_2 can be separated by disjoint open sets in X. This space will be denoted by **(X, $\mathcal{F}$).**

Example 3.1: Let $X = \mathbb{R}, \Im = \left(\begin{matrix}\mathbb{R}, \emptyset, \mathbb{Q}, \mathbb{Q}^c, (1,2), (1,2)^c, \\ (3,4), (3,4)^c, \cdots, (15,16), (15,16)^c\end{matrix}\right)$

Let $\mathcal{F} = \{(1,2), (3,4), (5,6), \cdots, (15,16)\}$. Then $\mathcal{F}$ is a finite collection of pair wise disjoint nontrivial closed sets in X such that, for each distinct pair F_1, F_2 in $\mathcal{F}$, F_1 and F_2 can be separated by disjoint open sets, since each of these is open as well. **Thus $(X, \Im)$ is s. n. s. k. However X is not normal** since $\mathbb{Q} \cap (1,2)$and $\mathbb{Q}^c \cap (1,2)$can't be separated by disjoint open sets.

Here X=$(X; (1,2), (3,4), (5,6), \cdots, (15,16))$.

Comment 3.1: Obviously an infinite number of such examples can be constructed.

Theorem 3.1: A topological space X is slightly normal space of the second kind if and only if there exists a finite collection $\mathcal{F}$ of pairwise disjoint nontrivial closed sets F_1, F_2 and an open set G such that $F_1 \subseteq G \subseteq \overline{G} \subseteq F_2^{\,c}$.

Proof: First, suppose that X is slightly normal space of the second kind. Then there exists a finite collection $\mathcal{F}$ of pairwise disjoint nontrivial closed sets such that for each pair F_1, F_2in $\mathcal{F}$, there exist open sets G, H in X such that $F_1 \subseteq G$ and $F_2 \subseteq H$ and $G \cap H = \phi$. It follows that $G \subseteq H^c \subseteq F_2^{\,c}$. Hence $G \subseteq \overline{G} \subseteq H^c \subseteq F_2^{\,c}$. Thus, $F_1 \subseteq G \subseteq \overline{G} \subseteq F_2^{\,c}$.

Conversely, suppose that there exists a finite collection $\mathcal{F}$ of pairwise disjoint nontrivial closed sets such that for each pair F_1, F_2 in $\mathcal{F}$, there exist an open set G in X such that $F_1 \subseteq G \subseteq \overline{G} \subseteq F_2^{\,c}$. Here $F_1 \subseteq G$ and $F_2 \subseteq \overline{G}^{\,c}$. Let $\overline{G}^{\,c} = H$. Then H is open, $F_2 \subseteq H$ and $G \cap H = \phi$. Hence X is slightly normal space of the second kind.

Theorem 3.2: Let $\{X_i\}_{i\in I}$ be a non-empty family of topological spaces, and let X=$\prod_{i\in I} X_i$ be the product space. If each X_i is slightly normal of the second kind, then X is slightly normal of the second kind.

Proof: Since each X_i is slightly normal of the second kind, there exists for each i, a finite collection $\mathcal{F}$ of pairwise disjoint nontrivial closed sets such that for each pair F_i, H_i in $\mathcal{F}$, there exist open sets U_i, V_i in X_i such that $F_i \subseteq U_i$, $H_i \subseteq V_i$, $F_i \cap H_i = \phi$, $U_i \cap V_i = \phi$.

Let F=$\prod_{i\in I} F_i$, H=$\prod_{i\in I} H_i$. Then F and H are closed sets in X. Clearly, $F \cap H = \phi$. Let $U = \prod_{i\in I} U_i$, $V = \prod_{i\in I} V_i$. Then, U and V are open sets in X, and $F \subseteq U$, $H \subseteq V$ and $U \cap V = \phi$. Therefore, X is slightly normal space of the second kind.

Theorem 3.3: Every open and one-one image of a slightly normal space of the second kind is slightly normal space of the second kind.

Proof: The proof of the Theorem 3.3 of the above is almost similar to the proof of the Theorem 2.4.

Corollary 3.1: Every quotient space of a slightly normal space of the second kind is slightly normal space of the second kind.

Proof: The proof of the Corollary 3.1 follows from the proof of the Corollary 2.1.

Theorem 3.4: Let X be a slightly normal space of the second kind and Y is a subspace of X. Then Y is a slightly normal space of the second kind.

Proof: The proof of the Theorem 3.4 is most similar to the proof of the Theorem 2.5.

Remark 3.1: The corresponding theorem does not hold for normal spaces. The validity of the proof in Theorem 3.4 above depends on the separablity of a particular pair of disjoint closed spaces by disjoint open spaces (See Ex. of Munkres [12]).

Comment 3.1: A continuous image of a slightly normal space of the second kind need not be slightly normal space of the second kind.

For if (X, T_1) is a slightly normal space of the second kind and (X, T_2) a space with the indiscrete topology, then the identity map $1_x : X \to X$ is continuous and onto. But (X, T_2) is not slightly normal space of the second kind.

Theorem 3.5: Each compact Hausdorff space is slightly normal space of the second kind.

Proof: The proof of the Theorem 3.5 of the above is almost similar to the proof of the Theorem 2.6.

We now define slightly normal spaces of the third kind and proceed to study them.

4. Slightly Normal Spaces of the Third Kind

Definition 4.1: A topological space X will be called **slightly normal of the third kind (s. n. t. k)** if there exists a countable collection $\mathcal{C}$ of pairwise disjoint nontrivial closed sets in X such that, for each pair F_1, F_2 in $\mathcal{C}$, F_1 and F_2 can be separated by disjoint open sets in X. This space will be denoted by **(X, $\mathcal{C}$).**

Example 4.1: Let $X = \mathbb{R}, \mathfrak{I} = \left\langle \begin{matrix} \{\mathbb{R}, \emptyset, \mathbb{Q}, \mathbb{Q}^c\} \cup \\ \{(n, n+1), (n, n+1)^c \mid n \in \mathbb{N}\} \end{matrix} \right\rangle$

Let $\mathcal{C} = \{(n, n+1) \mid n \in \mathbb{N}\}$. **Then (X, $\mathcal{C}$) is clearly s. n. t. k. But X is not normal** since $\mathbb{Q} \cap (1,2)$ and $\mathbb{Q}^c \cap (1,2)$ are disjoint closed sets which can't be separated by disjoint open sets.

Example 4.2: Let $X = \mathbb{C}, \mathfrak{I} = \left\langle \begin{matrix} \{\mathbb{C}, \emptyset, \mathbb{Q}, \mathbb{Q}^c\} \cup \\ \left\{\left\{D_n = \left\{z \in \mathbb{C} \,\middle|\, |z-n| < \tfrac{1}{3}\right\}, D_n^c\right\} \middle| n \in \mathbb{N}\right\} \end{matrix} \right\rangle$

Let $\mathcal{C} = \{D_n \mid n \in \mathbb{N}\}$. Then $\mathcal{C}$ is a countable collection of pair wise disjoint closed sets such that, for each pair D_{n_1} and D_{n_2} $(n_1 \neq n_2)$ can be separated by disjoint open sets since each D_n is both open and closed. **Hence X is s. n. t. k. However X is not normal** since $\mathbb{Q} \cap D_1$ and $\mathbb{Q}^c \cap D_1$ are disjoint closed sets which can't be separated by disjoint open sets.

Theorem 4.1: A topological space X is slightly normal space of the third kind if and only if there exists a countable collection $\mathcal{C}$ of pairwise disjoint nontrivial closed sets F_1, F_2 and an open set G such that $F_1 \subseteq G \subseteq \overline{G} \subseteq F_2^{\,c}$.

Proof: First, suppose that X is slightly normal space of the third kind. Then there exists a countable collection $\mathcal{C}$ of pairwise disjoint nontrivial closed sets such that for each pair F_1, F_2 in $\mathcal{C}$, there exist open sets G, H in X such that $F_1 \subseteq G$ and $F_2 \subseteq H$ and $G \cap H = \phi$. It follows that $G \subseteq H^c \subseteq F_2{}^c$. Hence $G \subseteq \overline{G} \subseteq H^c \subseteq F_2{}^c$. Thus, $F_1 \subseteq G \subseteq \overline{G} \subseteq F_2{}^c$.

Conversely, suppose that there exists a countable collection $\mathcal{C}$ of pairwise disjoint nontrivial closed sets such that for each pair F_1, F_2 in $\mathcal{C}$, there exist an open set G in X such that $F_1 \subseteq G \subseteq \overline{G} \subseteq F_2{}^c$. Here $F_1 \subseteq G$ and $F_2 \subseteq \overline{G}^c$. Let $\overline{G}^c = H$. Then H is open, $F_2 \subseteq H$ and $G \cap H = \phi$. Hence X is slightly normal space of the third kind.

Theorem 4.2: Let $\{X_i\}_{i\in I}$ be a non-empty family of topological spaces, and let X= $\prod_{i\in I} X_i$ be the product space. If each X_i is slightly normal of the third kind, then X is slightly normal of the third kind.

Proof: Since each X_i is slightly normal of the third kind, there exists for each i, a countable collection $\mathcal{C}$ of pairwise disjoint nontrivial closed sets such that for each pair F_i, H_i in $\mathcal{C}$, there exist open sets U_i, V_i in X_i such that $F_i \subseteq U_i$, $H_i \subseteq V_i$, $F_i \cap H_i = \phi$, $U_i \cap V_i = \phi$.

Let F= $\prod_{i\in I} F_i$,H= $\prod_{i\in I} H_i$. Then F and H are closed sets in X. Clearly, $F \cap H = \phi$. Let $U = \prod_{i\in I} U_i$, $V = \prod_{i\in I} V_i$. Then, U and V are open sets in X, and $F \subseteq U$, $H \subseteq V$ and $U \cap V = \phi$. Therefore, X is slightly normal space of the third kind.

Theorem 4.3: Every open and one-one image of a slightly normal space of the third kind is slightly normal space of the third kind.

Proof: The proof of the Theorem 4.3 is most similar to the proof of the Theorem 2.4.

Corollary 4.1: Every quotient space of a slightly normal space of the third kind is slightly normal space of the third kind.

Proof: The proof of the Corollary 4.1 of the above is almost similar to the proof of the Corollary 2.1.

Theorem 4.4: Let X be a slightly normal space of the third kind and Y is a subspace of X. Then Y is a slightly normal space of the third kind.

Proof: The proof of the Theorem 4.4 follows from the proof of the Theorem 2.5.

Remark 4.1: The corresponding theorem does not hold for normal spaces. The validity of the proof in Theorem 4.4 above depends on the separablity of a particular pair of disjoint closed spaces by disjoint open spaces (See Ex. of Munkres [12]).

Comment 4.1: A continuous image of a slightly normal space of the third kind need not be slightly normal space of the third kind.

For if (X, T_1) is a slightly normal space of the third kind and (X, T_2) a space with the indiscrete topology, then the identity map $1_x : X \to X$ is continuous and onto. But (X, T_2) is not slightly normal space of the third kind.

Theorem 4.5: Each compact Hausdorff space is slightly normal space of the third kind.

Proof: The proof of the Theorem 4.5 is most similar to the proof of the Theorem 2.6.

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