

# Design of Chaotic Secure Communication System Based on Laser Dynamic Model

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## ABSTRACT

In this paper, a novel master-slave secure communication system will be proposed and explored. Based on the state estimator of the laser dynamic system, a new type of chaotic secure communication system will be designed. This secure communication system can not only achieve global exponential tracking, but also accurately calculate the exponential convergence rate. Finally, several numerical simulation results will be provided to demonstrate the correctness and practicality of the main theorem.

**KEYWORDS:** Laser dynamic mode, secure communication system, master-slave system, secure communication system, exponential tracking

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## 1. INTRODUCTION

In recent years, various theories and designs of secure communication systems have been proposed by many researchers; see, for example, [1]-[6] and the references therein. As we know, secure communication is not only rooted in theoretical basis, but more importantly, it strengthens the security function of the actual communication system.

On the other hand, laser dynamic systems occasionally produce chaotic phenomena; at the same time, the design of their state estimators is quite challenging. The state estimator design has been proposed in [9] to ensure global exponential synchronization for the fifth-order laser dynamic model.

This paper will use the chaotic characteristics of the laser dynamic system to design a new master-slave chaotic secure communication system. Based on the control theory, it is deduced that such a secure communication system can achieve the goal of global exponential tracking. Besides, the guaranteed

exponential convergence rate of this secure communication system will be calculated simultaneously. Finally, several computer simulation results will demonstrate the effectiveness of this main theorem. In particular, throughout the paper,  $\|x\| := \sqrt{x^T \cdot x}$  represents the Euclidean norm of the column vector  $x$ , and  $|a|$  represents the absolute value of a real number  $a$ .

## 2. PROBLEM FORMULATION AND MAIN RESULTS

First, we propose a new master-slave secure communication system, and its architecture diagram is shown in Figure 1.

### Master System:

$$\dot{x}_1 = -c_1x_1 - c_1c_2x_2 + c_1x_3, \quad (1a)$$

$$\dot{x}_2 = c_1c_2x_1 - c_1x_2 + c_1x_4, \quad (1b)$$

$$\dot{x}_3 = c_3x_1 - x_3 + c_2x_4 - x_1x_5, \quad (1c)$$

$$\dot{x}_4 = c_3x_2 - c_2x_3 - x_4 - x_2x_5, \quad (1d)$$

$$\dot{x}_5 = -c_4 x_5 + x_1 x_3 + x_2 x_4, \tag{1e}$$

$$y_1(t) = c_5 x_3(t) + c_6 x_4(t), \tag{1f}$$

$$y_2(t) = c_7 x_3(t) + c_8 x_4(t), \tag{1g}$$

$$\phi(t) = Dx(t) + m_1(t), \forall t \geq 0. \tag{1h}$$

**Slave System:**

$$\dot{z}_1 = -c_1 z_1 - c_1 c_2 z_2 + \frac{c_1 \alpha_8 y_1 - c_1 \alpha_6 y_2}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7}, \tag{2a}$$

$$\dot{z}_2 = c_1 c_2 z_1 - c_1 z_2 + \frac{c_1 \alpha_5 y_2 - c_1 \alpha_7 y_1}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7}, \tag{2b}$$

$$z_3 = \frac{\alpha_8}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} y_1 - \frac{\alpha_6}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} y_2, \tag{2c}$$

$$z_4 = \frac{-\alpha_7}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} y_1 + \frac{\alpha_5}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} y_2, \tag{2d}$$

$$\dot{z}_5 = -c_4 z_5 + z_1 z_3 + z_2 z_4, \tag{2e}$$

$$m_2(t) = \phi(t) - Dz(t), \forall t \geq 0, \tag{2f}$$

where  $x(t) := [x_1(t) \ x_2(t) \ x_3(t) \ x_4(t) \ x_5(t)]^T \in \mathbb{R}^{5 \times 1}$  is state vector of the master system,  $y(t) \in [y_1(t) \ y_2(t)]^T \in \mathbb{R}^{2 \times 1}$  is output vector of the master system,  $z(t) := [z_1(t) \ z_2(t) \ z_3(t) \ z_4(t) \ z_5(t)]^T \in \mathbb{R}^{5 \times 1}$  is state vector of the slave system,  $D \in \mathbb{R}^{b \times s}$  is the two-party confidentiality matrix, and  $c_1, c_2, \dots, c_8$  are the parameters of the system, with  $c_1 > 0, c_4 > 0, c_1 \neq c_4$ , and  $c_5 c_8 \neq c_6 c_7$ . Besides,  $m_1(t)$  is the information vector,  $m_2(t)$  is the signal recovered from  $m_1(t)$ , and we assume that the signals of  $x_3(t)$  and  $x_4(t)$  are bounded.

**Remark 1:** It is noted that the master system is a fifth-order laser dynamic system, and when the parameters of the system (1) are the following values, chaos will occur in the main system [7-9]:

$$c_1 = 2, c_2 = 0.002, c_3 = 20, \text{ and } c_4 = 0.25.$$

As we all know, a high-quality master-slave secure communication system should at least make the error signal  $\tilde{e}(t) = m_2(t) - m_1(t)$  approach zero in some sense. A definition related to the master-slave secure communication system is introduced as follows.

**Definition 1 [10]:** The master-slave system (1) with (2) is called secure communication system with exponential convergence type if there are positive numbers  $k$  and  $\alpha$  such that, for every  $t \geq 0$ ,

$$|\tilde{e}(t)| = |m_2(t) - m_1(t)| \leq k \exp(-\alpha t). \tag{3}$$

In this situation, the positive number  $\alpha$  is called the exponential convergence rate.

The main theorem of this paper is presented below.

**Theorem 1:** The system (1) with (2) is a secure communication system with exponential convergence type. Meanwhile, the guaranteed exponential convergence rate is given by  $\alpha := \min\{c_1, c_4\}$ .

**Proof.** Define  $M \geq |x_i(t)|, \forall i \in \{3,4\}$  and

$$e_i(t) := x_i(t) - z_i(t), \forall i \in \{1,2,3,4,5\}. \tag{4}$$

Thus, one has, for every  $t \geq 0$ ,

$$|e_i(t)| \leq e^{-c_i t} \sqrt{e_1^2(0) + e_2^2(0)}, \forall i \in \{1,2\}; \tag{5a}$$

$$e_3(t) = e_4(t) = 0; \tag{5b}$$

$$|e_5(t)| \leq \left[ \frac{4M \sqrt{e_1^2(0) + e_2^2(0)}}{|c_4 - c_1|} + |e_5(0)| \right] e^{-\min\{c_1, c_4\}t}, \tag{5c}$$

in view of the Theorem 1 of [9]. It can be readily obtained that

$$\begin{aligned} |\tilde{e}(t)| &= |m_2(t) - m_1(t)| \\ &= |\phi(t) - Dz(t) - \phi(t) + Dx(t)| \\ &= |D[x(t) - z(t)]| \\ &\leq \|D\| \times \\ &\quad \left\| \begin{bmatrix} e_1(t) & e_2(t) & e_3(t) & e_4(t) & e_5(t) \end{bmatrix}^T \right\| \\ &\leq k \cdot e^{-\alpha t}, \forall t \geq 0, \end{aligned}$$

where

$$k = \|D\| \times \sqrt{e_1^2(0) + e_2^2(0) + \left[ \frac{4M \sqrt{e_1^2(0) + e_2^2(0)}}{|c_4 - c_1|} + |e_5(0)| \right]^2},$$

and  $\alpha = \min\{c_1, c_4\}$ , in view of (1)-(5). This completes the proof.

**3. NUMERICAL SIMULATIONS**

Consider the master-slave secure communication system of (1) and (2) with

$$c_1 = 2, c_2 = 0.002, c_3 = 20, \ c_4 = 0.25, \tag{6a}$$

$$c_5 = 2, \ c_6 = c_7 = -1, \ c_8 = 1, \tag{6b}$$

$$\text{and } D = [0.1 \ 0.1 \ -0.5 \ -0.4 \ 0]. \tag{6c}$$

According to Theorem 1, this secure communication system (1), (2) with (6) can not only make the signals  $m_1(t)$  and  $m_2(t)$  achieve global exponential synchronization targets, but also the guaranteed exponential convergence rate can be calculated as  $\alpha = 0.25$ . Furthermore, the signals  $m_1(t)$ ,  $m_2(t)$  and the error signal  $m_2(t) - m_1(t)$  are shown in Figures 2 to 4, respectively, and from Figure 4, it can be known that the signals  $m_1(t)$  and  $m_2(t)$  can be synchronized after about 5 seconds.

**4. CONCLUSION**

In this paper, a new master-slave secure communication system has been proposed and investigated. Based on the state estimator of the laser dynamic system, a novel type of chaotic secure communication system has been designed. This secure communication system can not only achieve

global exponential tracking, but also accurately calculate the exponential convergence rate. Finally, several numerical simulation results have been presented to show the correctness and practicality of the main theorem.

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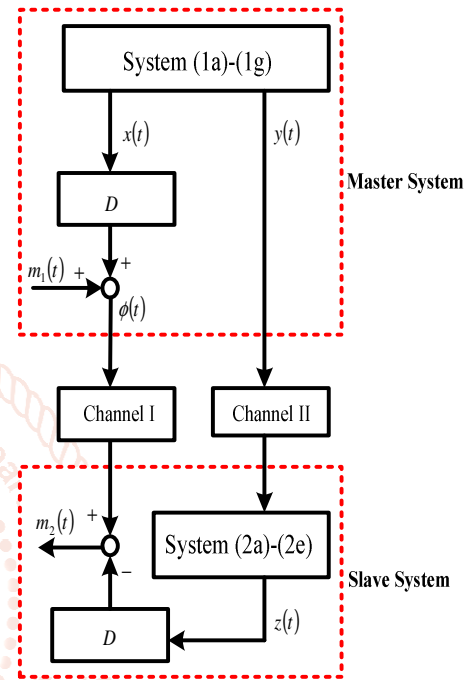
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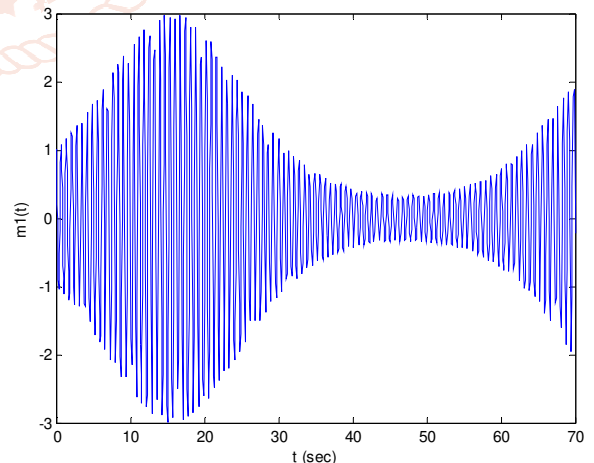
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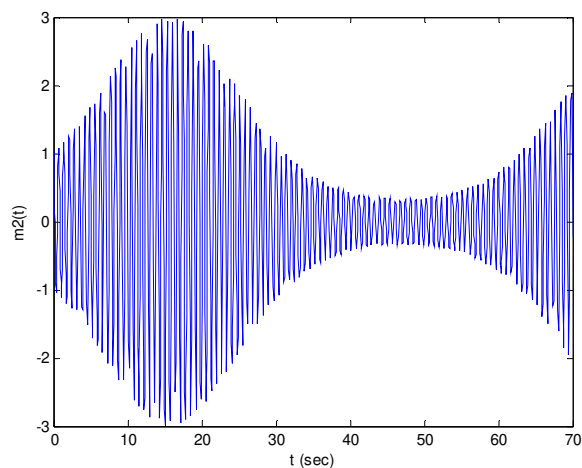
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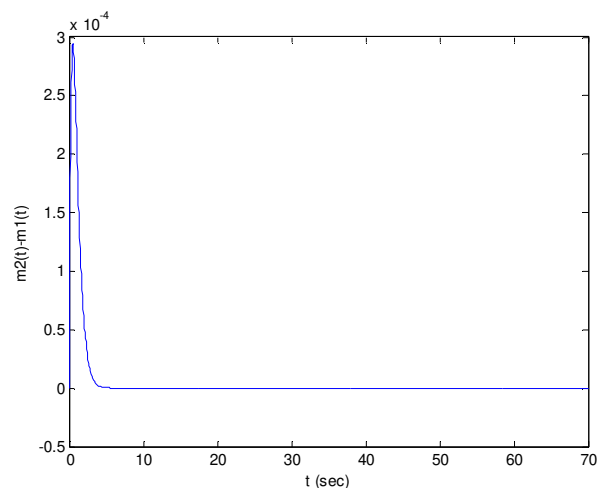
**Figure 1: Architecture diagram of secure communication system.**



**Figure 2: Real message of  $m_1(t)$  described in the master system of (1).**



**Figure 3: Recoverd message of  $m_2(t)$  described in the slave system of (2).**



**Figure 4: Error signal of  $m_2(t) - m_1(t)$ .**

