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Soft Lattice in Approximation Space

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ABSTRACT

Rough set theory is a powerful tool to analysis the uncertain and imprecise problem in information systems. Also the soft set and lattice theory can be used as a general mathematical tool for dealing with uncertainty. In this paper, we present a new concept, soft rough lattice where the lower and upper approximations are the sub lattices and narrate some properties of soft rough lattice with some examples.

KEYWORDS: *Rough set, soft set, lattice theory, lower approximation and upper approximation of soft lattice*

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1. INTRODUCTION

Rough set theory was proposed by Prof. Z. Pawlak in 1982[7] to deal with uncertainty, vagueness and imprecise problems. The classical rough set theory is based on an equivalence relation that is, a knowledge on a universe of objects U. Knowledge is based on the ability to classify objects, and by objects we mean anything we can think of, for example real things, states, abstract concepts, process, moment of time etc. For a finite set of objects U, any subset X ⊆ U will be called a concept or a category in U and any family of concepts in U will be referred to as knowledge about U. Let us consider a classification or partition of a certain universe U, that is, a family X=$\{Y_1, Y_2, \ldots, Y_n\}$ is called a partition of U if $Y_i \cap Y_j = \emptyset$ for $Y_i, Y_j \in U$, i≠j, i,j=1,2,…,n and $\bigcup_i X_i$ = U. W know that partition(or classification) of U and an equivalence relation on U are interchangeable notion. So we consider equivalence relation R: U→ U instead of classification as it is easy to manifulate.

Let R be an equivalence relation on universe U, then U/R be the family of all equivalence classes of R referred to as categories or concepts of R, and $[x]_R$ denotes a category in R containing an element x∈ U. Here R is a knowledge on U.

In 1999 D. Molodtsov[6] introduced the concept of soft sets, which is a new mathematical tool for dealing with uncertainty. Soft set theory has potential applications in many different fields including game theory, operational research, probability theory. Maji et al[4] defined several operations on soft sets and made a theoretical study on the theory of soft sets. The combination of the theories soft set and rough set be studied by Das and Mohanty[2], Mohanty et al[5] and also by the other authors Feng et al[3].

Lattice are relatively simple structures since the basic concepts of the theory include only orders, least upper bounds, greatest lower bounds. Now the lattice plays on important role in many disciplines of computer sciences and engineering. Though the concept of lattice was introduced by Pirce and Schroder, but Boole(1930) and Birkhoff(1967)[1] gave the actual development of lattice theory.

2. PRELIMINARIES:

Definition-2.1:[8] Let U be the universal set and C be an equivalence relation(or knowledge) on U, where C is termed as indiscernibility relation. U/C be family of all equivalence class of C, known as catagories of C, for x∈U. $[x]_C$ is an equivalence class of x. The relational system K=(U,C) is called a approximation

space. The lower approximation and upper approximation of a set X⊆U under the indiscernibility relation C are defined as
$\underline{C}$X={x∈U: $[x]_C \subseteq X$} and
$\overline{C}$X={ x∈U: $[x]_C \cap X \neq \Phi$} respectively.

Example-2.2:
Let U={$h_1, h_2, \ldots, h_8$} be the various colors for a painting. Let C be the knowledge on U, we get a partition of U as

U/C={{h_1,h_2}, {h_3,h_7}, {h_5,h_6}, {h_4}, {h_8}}

Where h_1,h_2 are red color, h_3,h_7 are green color, h_5,h_6 are yellow color, h_4 *is black* color and h_8 is blue color.

Let X={h_5,h_6,h_7}⊂ U. That is X certain painting the lower and upper approximation of X are

$\underline{C}$X={h_5,h_6}. So yellow color certainly belong to the painting X.

And $\overline{C}$X={h_3, h_5,h_6, h_7}. So green and yellow color are possibly used for the painting belong to X.

Hence, $\underline{C}$X≠ $\overline{C}$X.

Therefore, the set X is rough with respect to knowledge C.

Example-2.3:
Let U={$u_1,u_2,\ldots,u_9,u_{10}$} be the different chocolate in a jar according to their price. Let D be the knowledge on U, we get a partition of U as

U/D={{u_1,u_3, u_5 },{u_7,u_8}, {u_9,u_{10}}, {u_2,u_4}, {u_6}}

That is {u_1,u_3, u_5} are the 5rupees chocolate, {u_7,u_8} are the 10rupees chocolate, {u_9,u_{10}} are 10rupees chocolate, {u_2,u_4} are 30rupees chocolate and {u_6}is 50rupees chocolate. Let X={u_4,u_6, u_9, u_{10}}⊂ U, that is a set of chocolates are picked a random.

The lower approximation and upper approximation of X are
$\underline{D}$X={u_6,u_9,u_{10}}. So 20rupees and 50 rupees chocolate are certainly belong to X.
$\overline{D}$X={u_2, u_4,u_6,u_9, u_{10}}. So the chocolates of 20 rupees, 30 rupees and 50rupees are possibly classified belong to X.

Hence, $\underline{D}$X≠ $\overline{D}$X.

Therefore, the set X is rough with respect to knowledge D.

Definition-2.4: Let U be an initial universe, E be the set of parameters related to U. Let P(U) denotes the power set of U, A⊆ E and F be a mapping given by F:A →P(U), then the pair (F,A) is called soft set over U.

Example-2.5: Let U={$x_1,x_2,\ldots,x_8$} be the set of shop, E={$a_1, \ldots, a_5$} be the set of parameters on U that is a_1 stands for flower, a_2 stands for seed, a_3 stands for fresh, $a_4$stands for stale and a_5 stands for root vegetables. Let a mapping F:E→P(U) be given as F(a_1)={x_4,x_5, x_6}, $F(a_2)$= {x_1,x_2}, F(a_3)={x_7,x_8}, F(a_4)={x_6,x_7} and F(a_5)={ x_4,x_5} means x_4 and x_5 are root vegetables.

Let A={a_1, a_2,a_3}⊆ E then the soft set
(F,A)={(*cheap*, F(a_1)), (green, $F(a_2)$), (fresh, F(a_3))}
={(cheap, {x_4,x_5, x_6}), (green, {x_1,x_2}), (fresh, {x_7,x_8})}

Example-2.6:
Let U={$u_1,u_2,\ldots,u_{10}$} be different quality bikes available in India market, E= {$a_1, \ldots, a_5$} be the set of parameters on U that is a_1 stands for expensive, a_2 stands for good mileage, a_3 stands for sports, $a_4$stands for crusier and a_5 stands for touring. Let a mapping G:E→P(U) be given as G(a_1)={u_1,u_2, u_5} the expensive bikes are u_1, u_2 and u_5, $G(a_2)$= {u_8,u_9, u_{10}}, G(a_3)={u_3,u_7}, G(a_4)={u_4,u_6, u_7} and G(a_5)={ u_3, u_4}.

Let A={a_1, a_2,a_4}⊆ E then the soft set
(G,A)={(*expensive*, G(a_1)), (good mileage, $G(a_2)$), (cruiser, G(a_4))}
={(expensive, {u_1,u_2, u_5}), (good mileage, {u_8,u_9, u_{10}}), (cruiser, {u_4,u_6, u_7})}

Definition-2.7: Let Z=(Z,≾) be a partially ordered set. Then Z is called a lattice if for any two elements r and s of Z have a least upper bound r∨s and a greatest lower bound r∧s are in Z.

Defintion-2.8: Let (L,∨,∧) be a lattice and let S⊆ L be a subset of L. Then (S,∨,∧) is called a sublattice of (L,∨,∧) if and only if S is closed under both operations of join(∨) and meet(∧).

Example-2.9:
The power set P(S) is a lattice under the operation union and intersection. That is S={α, β,γ}
Then P(S)={Φ,{ α},{β},{ γ}, {α, β}, { β,γ}, {α, γ},S}.

Let A=(P(S),∪,∩) is a lattice.
That is { α}∪ { β }={ α, β }
{α}∩ { β }=Φ
{ β }∪{ α,β,γ }={ α,β,γ }
{ β }∩{α,β,γ }= { β }

B={T,∪,∩} is sublattice of A, where T={Φ, {α}, { β}, {α, β }}

Example-2.10: Let $A=(D_{60},|)=$ (1,2,3,4,5,6,10,12,15,20,30,60) is a lattice.
Join: 6 ∨10=lcm(6,10)=30
Meet: 6∧ 10=gcd(6,10)=2
Join: 10∨12=lcm(10,12)=60
Meet: 10∧ 12=gcd(10,12)=2
B={K, |} is a sublattice of A, where A={3,6,12}.

3. SOFT ROUGH LATTICE:

Let L be an lattice, E be the set of parameter and P(L) denotes the set of all sublattice of L. The collection W=(F,A) is soft lattice over L, where F is a mapping given by F:A → P(L). Then S=(L,F,A) is called soft lattice approximation space.

Definition-3.1: For X ⊆L, we define lower and upper approximation as

$W_*(X) = \bigcup\{F(a) \in P(L): F(a) \subseteq X\}$ and

$W^*(X) = L - W_*(X^c)$

If $W_*(X) \neq W^*(X)$, then X is called soft rough lattice. Otherwise X is soft definable lattice. Here, $W_*(X)$ and $W^*(X)$ are sublattices.

Example-3.2: Let S={α, β,γ} be a set and L=(S,⊆) = {Φ,{ α},{β},{ γ}, {α, β}, { β,γ}, {α, γ},S} be a lattice. Let E={$a_1, \ldots a_5$} be the set of parameters and D={a_1, a_2, a_3, a_5}. Let F: D→ P(L) be a mapping given by F(a_1)={{α},{α, β }}, F(a_2)={ Φ,{α},{ γ}, {α, γ }}, F(a_3)={{α},{α, γ}}, F(a_4)= {Φ,{α}, S}, F(a_5)={ {β}, { β,γ } } be sublattices of L. Let X={{α},{α, γ}}∈P(L). Then $W_*(X)$ ={{α},{α, γ}} and $W^*(X)$= {Φ,{α},{ γ},{α, γ},{α, β},S}

Theorem-3.3:
Let W=(F,A) be soft lattice over L, S=(L,F,A) be a soft lattice approximation space and X, Y⊆ L, we have

A. $W_*(\Phi) = W^*(\Phi) = \Phi$
B. $W_*(L) \neq L$, $W^*(L) = L$
C. X⊆ Y⇒$W_*(X) \subseteq W_*(Y)$
D. X⊆ Y⇒ $W^*(X) \subseteq W^*(Y)$

Proof:
A. According to definition of soft lattice lower and upper approximation, we have

$W_*(\Phi) = \bigcup\{F(a) \in P(L): F(a) \subseteq \Phi\} = \Phi$

and $W^*(\Phi) = L - W_*(\Phi^c) = L - L = \Phi$

B. $W_*(L) = \bigcup\{F(a) \in P(L): F(a) \subseteq L\} \neq L$ and $W^*(L) = L - W_*(L^c) = L$

C. Assume that X⊆ Y
By definition, F(a)⊆ X⇒ u∈$W_*(X)$
So, u∈ F(a)⊆ Y
Therefore, $W_*(X) \subseteq W_*(Y)$

D. Suppose that X⊆ Y
$\Rightarrow X^c \subseteq Y^c$
$\Rightarrow W_*(X^c) \supseteq W_*(Y^c)$
$\Rightarrow$ L-$W_*(X^c) \subseteq L - W_*(Y^c)$
$\Rightarrow W^*(X) \subseteq W^*(Y)$

Theorem-3.4:
Let W=(F,A) be soft lattice over L, S=(L,F,A) be a soft lattice approximation space and X, Y⊆ L, we have

(a) $W^*(X \cup Y) = W^*(X) \cup W^*(Y)$
(b) $W_*(X \cap Y) = W_*(X) \cap W_*(Y)$
(c) $W_*(X \cup Y) \supseteq W_*(X) \cup W_*(Y)$
(d) $W^*(X \cap Y) \subseteq W^*(X) \cap W^*(Y)$
(e) $W^*(-X) = -W_*(X)$
(f) $W_*(-X) = -W^*(X)$

Proof:
These can be prove directly.

4. CONCLUSION:

This paper aims to define soft rough lattice which is a new mathematical model to deal with uncertain and vague concept. Some properties are proved. Frther research can be made for the new model soft rough lattice.

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