International Journal of Trend in Scientific Research and Development (IJTSRD)
Volume 6 Issue 6, September-October 2022 Available Online: www.ijtsrd.com e-ISSN: 2456 – 6470

A Nonlinear State Estimator Design of the Laser Dynamic System

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ABSTRACT

This paper mainly explores the existence of state estimator for the fifth-order laser dynamic systems. At the same time, we use the techniques of differential and integral inequality to design a nonlinear state estimator such that the global exponential tracking can be guaranteed. Besides, the guaranteed exponential convergence rate can be calculated correctly. Finally, several numerical simulation results are presented to illustrate the applicability and correctness of the main theorem.

KEYWORDS: *Laser dynamic system, nonlinear state estimator, exponential convergence rate, chaotic system*

How to cite this paper: Yeong-Jeu Sun | Che-Ming Chuang | Sheng-Chieh Chen "A Nonlinear State Estimator Design of the Laser Dynamic System" Published in International Journal of Trend in Scientific Research and Development (ijtsrd), ISSN: 2456-6470, Volume-6 | Issue-6, October 2022, pp.2109-2112, URL: www.ijtsrd.com/papers/ijtsrd52209.pdf

IJTSRD52209

1. INTRODUCTION

In recent years, various issues related to laser dynamic systems have been discussed and studied; see [1-5] and the references therein. Based on delay differential equation, a circuit-level model of semiconductor mode-locked laser has been present in [1]. In [2], a computational model has been provided for frequency stabilization of a laser source using the technique of extremum-seeking control. Besides, the first spectral beam combining demonstration of the solid-state lasers has been explored and proposed in [3].

On the other hand, due to the lack of measurement equipment, the estimation of the state variables of real physical systems is more difficult; see, for example, [6-15]. Meanwhile, if the state variables are chaotic signals, the estimation of the state variables is even more challenging.

This paper focuses on exploring the existence of state estimators for a fifth-order laser dynamic system. Using the techniques of differential and integral inequalities, a nonlinear state estimator will be designed to ensure that the fifth-order laser dynamic system achieves the goal of global exponential tracking. Additionally, we will derive the exponential convergence rate of such a state estimator. Finally, several computer simulation results will be shown to demonstrate the applicability and correctness of the obtained results.

2. PROBLEM FORMULATION AND MAIN RESULTS

In this paper, we consider the following nonlinear laser system [4-5], and its dynamic equation is as follows:

$$\dot{x}_1 = -c_1 x_1 - c_1 c_2 x_2 + c_1 x_3, \quad (1a)$$

$$\dot{x}_2 = c_1 c_2 x_1 - c_1 x_2 + c_1 x_4, \quad (1b)$$

$$\dot{x}_3 = c_3 x_1 - x_3 + c_2 x_4 - x_1 x_5, \quad (1c)$$

$$\dot{x}_4 = c_3 x_2 - c_2 x_3 - x_4 - x_2 x_5, \quad (1d)$$

$$\dot{x}_5 = -c_4 x_5 + x_1 x_3 + x_2 x_4, \quad (1e)$$

$$y_1(t) = c_5 x_3(t) + c_6 x_4(t), \quad (1f)$$

$$y_2(t) = c_7 x_3(t) + c_8 x_4(t), \ \forall\, t \ge 0, \quad (1g)$$

where $x(t) := [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t) \quad x_5(t)]^T \in \Re^{5\times 1}$ is the state vector, $y(t) \in [y_1(t) \quad y_2(t)] \in \Re^{2\times 1}$ is the system output, and $c_1, c_2, \cdots, c_8$ are the parameters of the system (1), with $c_1 > 0$, $c_4 > 0$, $c_1 \neq c_4$, and $c_5 c_8 \neq c_6 c_7$. In addition, we assume that the signals of $x_3(t)$ and $x_4(t)$ are bounded.

Remark 1: It is worth mentioning that when $c_1 = 2, c_2 = 0.002, c_3 = 20,$ and $c_4 = 0.25$, the above system will produce chaos phenomenon [4].

Before introducing the main theorem, we first offer the following definition.

Definition 1. The system (1) is exponentially state reconstructible if there exist a state estimator $E\,\dot{z}(t) = h(z(t), y(t))$ and positive numbers k and α such that, for every $t \geq 0$,

$$|x_i(t) - z_i(t)| \leq k \exp(-\alpha t), \quad \forall\, i \in \{1,2,3,4,5\},$$

where $z(t) := [z_1(t) \quad z_2(t) \quad z_3(t) \quad z_4(t) \quad z_5(t)]^T \in \Re^{5\times 1}$ expresses the reconstructed state of the system (1). In this situation, the positive number α is called the exponential convergence rate.

The main theorem of this paper is expounded as follows.

Theorem 1. The system (1) is exponentially state reconstructible. Besides, a suitable state estimator is given by

$$\dot{z}_1 = -c_1 z_1 - c_1 c_2 z_2 + \frac{c_1 \alpha_8 y_1 - c_1 \alpha_6 y_2}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7}, \tag{2a}$$

$$\dot{z}_2 = c_1 c_2 z_1 - c_1 z_2 + \frac{c_1 \alpha_5 y_2 - c_1 \alpha_7 y_1}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7}, \tag{2b}$$

$$z_3 = \frac{\alpha_8}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} y_1 - \frac{\alpha_6}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} y_2, \tag{2c}$$

$$z_4 = \frac{-\alpha_7}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} y_1 + \frac{\alpha_5}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} y_2, \tag{2d}$$

$$\dot{z}_5 = -c_4 z_5 + z_1 z_3 + z_2 z_4, \forall\, t \geq 0, \qquad 2e)$$

with the guaranteed exponential convergence rate $\alpha := \min\{c_1, c_4\}$.

Proof. Define $M \geq |x_i(t)|, \forall\, i \in \{3,4\}$, from (1), (2) with

$$e_i(t) := x_i(t) - z_i(t), \quad \forall\, i \in \{1,2,3,4,5\}, \tag{3}$$

it is easy to obtain that

$$\begin{aligned} e_3(t) &= x_3(t) - z_3(t) \\ &= \frac{\alpha_8 y_1(t) - \alpha_6 y_2(t)}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} - \left[\frac{\alpha_8}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} y_1(t) - \frac{\alpha_6}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} y_2(t)\right] \\ &= 0, \forall\, t \geq 0; \end{aligned} \tag{4}$$

$$\begin{aligned} e_4(t) &= x_4(t) - z_4(t) \\ &= \frac{\alpha_5 y_2(t) - \alpha_7 y_1(t)}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} - \left[\frac{-\alpha_7}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} y_1 + \frac{\alpha_5}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} y_2\right] \\ &= 0, \forall\, t \geq 0; \end{aligned} \tag{5}$$

$$\begin{aligned} \dot{e}_1(t) &= \dot{x}_1(t) - \dot{z}_1(t) \\ &= -c_1 x_1 - c_1 c_2 x_2 + c_1 x_3 - \left[-c_1 z_1 - c_1 c_2 z_2 + \frac{c_1 \alpha_8 y_1 - c_1 \alpha_6 y_2}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7}\right] \\ &= -c_1 x_1 - c_1 c_2 x_2 + c_1 \frac{\alpha_8 y_1 - \alpha_6 y_2}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} - \left[-c_1 z_1 - c_1 c_2 z_2 + \frac{c_1 \alpha_8 y_1 - c_1 \alpha_6 y_2}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7}\right] \\ &= -c_1 (x_1 - z_1) - c_1 c_2 (x_2 - z_2) \\ &= -c_1 e_1(t) - c_1 c_2 e_2(t), \forall\, t \geq 0; \end{aligned} \tag{6}$$

$$\begin{aligned} \dot{e}_2(t) &= \dot{x}_2(t) - \dot{z}_2(t) \\ &= c_1 c_2 x_1 - c_1 x_2 + c_1 x_4 - \left[c_1 c_2 z_1 - c_1 z_2 + \frac{c_1 \alpha_5 y_2 - c_1 \alpha_7 y_1}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7}\right] \\ &= c_1 c_2 x_1 - c_1 x_2 + c_1 \frac{\alpha_5 y_2 - \alpha_7 y_1}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7} - \left[c_1 c_2 z_1 - c_1 z_2 + \frac{c_1 \alpha_5 y_2 - c_1 \alpha_7 y_1}{\alpha_5 \alpha_8 - \alpha_6 \alpha_7}\right] \\ &= c_1 c_2 (x_1 - z_1) - c_1 (x_2 - z_2) \\ &= c_1 c_2 e_1(t) - c_1 e_2(t), \forall\, t \geq 0. \end{aligned} \tag{7}$$

Let $w(t) := e_1^2(t) + e_2^2(t)$ with (6) and (7), it can be readily obtained that

$$\begin{aligned} \dot{w}(t) &:= 2\dot{e}_1(t) e_1(t) + 2\dot{e}_2(t) e_2(t) \\ &= 2[-c_1 e_1(t) - c_1 c_2 e_2(t)] e_1(t) + 2[c_1 c_2 e_1(t) - c_1 e_2(t)] e_2(t) \\ &= -2c_1 [e_1^2(t) + e_2^2(t)] \\ &= -2c_1 w(t), \forall\, t \geq 0. \end{aligned}$$

This implies that, for every $t \geq 0$,

$$\begin{aligned} e_i^2(t) &\leq w(t) = e^{-2c_1 t} w(0) \\ &= e^{-2c_1 t}[e_1^2(0) + e_2^2(0)], \forall\, i \in \{1,2\}. \end{aligned}$$

It follows that, for every $t \geq 0$,

$$|e_i(t)| \leq e^{-c_1 t}\sqrt{e_1^2(0) + e_2^2(0)}, \forall\, i \in \{1,2\}. \tag{8}$$

Furthermore, form (1)-(5), one has

$$\begin{aligned} \dot{e}_5(t) &= \dot{x}_5(t) - \dot{z}_5(t) \\ &= -c_4 x_5 + x_1 x_3 + x_2 x_4 - [-c_4 z_5 + z_1 z_3 + z_2 z_4] \\ &= -c_4 e_5 + e_1 x_3 + e_2 x_4 \end{aligned}$$

Owing to (8), it results that

$$\frac{d\left[e^{c_4 t}e_5(t)\right]}{dt}=c_4e^{c_4 t}e_5(t)+e^{c_4 t}\dot{e}_5(t)$$
$$=e^{c_4 t}\left[e_1(t)x_3(t)+e_2(t)x_4(t)\right]$$

$$\Rightarrow\quad e^{c_4 t}e_5(t)-e_5(0)$$
$$=\int_0^t \frac{d\left[e^{c_4 t}e_5(t)\right]}{dt}\,dt$$
$$=\int_0^t e^{c_4 t}\left[e_1(t)x_3(t)+e_2(t)x_4(t)\right]dt$$

$$\Rightarrow\quad e_5(t)=e^{-c_4 t}\int_0^t e^{c_4 t}\left[e_1(t)x_3(t)+e_2(t)x_4(t)\right]dt$$
$$+e_5(0)e^{-c_4 t}$$

$$\Rightarrow\quad |e_5(t)|$$
$$\le e^{-c_4 t}\int_0^t e^{c_4 t}\left[|e_1(t)||x_3(t)|+|e_2(t)||x_4(t)|\right]dt$$
$$+|e_5(0)|e^{-c_4 t}$$
$$\le 2Me^{-c_4 t}\sqrt{e_1^2(0)+e_2^2(0)}\int_0^t e^{c_4 t}e^{-c_1 t}\,dt$$
$$+|e_5(0)|e^{-c_4 t}$$
$$=\frac{2Me^{-c_4 t}\sqrt{e_1^2(0)+e_2^2(0)}}{c_4-c_1}\left[e^{(c_4-c_1)t}-1\right]$$
$$+|e_5(0)|e^{-c_4 t}$$
$$\le\frac{2Me^{-c_4 t}\sqrt{e_1^2(0)+e_2^2(0)}}{|c_4-c_1|}\left[e^{(c_4-c_1)t}+1\right]$$
$$+|e_5(0)|e^{-c_4 t}$$
$$\le\left[\frac{4M\sqrt{e_1^2(0)+e_2^2(0)}}{|c_4-c_1|}+|e_5(0)|\right]e^{-\min\{c_1,c_4\}t},$$
$$\forall\, t\ge 0. \qquad (9)$$

This completes the proof, in view of (4), (5), (8), and (9).

3. NUMERICAL SIMULATIONS

Consider the laser system of (1) with

$$c_1=2,\ c_2=0.002,\ c_3=20,\ \ c_4=0.25, \qquad (10a)$$

$$c_5=2,\ \ c_6=c_7=-1,\ \text{and}\ \ c_8=1. \qquad (10b)$$

By Theorem 1, we conclude that the system (1) with (10) is exponentially state reconstructible by the state estimator

$$\dot{z}_1=-2z_1-0.004z_2+2y_1+2y_2, \qquad (11a)$$

$$\dot{z}_2=0.004z_1-2z_2+4y_2+2y_1, \qquad (11b)$$

$$z_3=y_1+y_2, \qquad (11c)$$

$$z_4=y_1+2y_2, \qquad (11d)$$

$$\dot{z}_5=-0.25z_5+z_1z_3+z_2z_4,\ \forall\, t\ge 0, \qquad (11e)$$

with the guaranteed exponential convergence rate $\alpha=0.25$. The typical state trajectories of the system (1) with (10) and those of the system (11) are depicted in Figure 1 and Figure 2, respectively. Besides, the time response of error states between the above two systems is shown in Figure 3.

4. CONCLUSION

In this paper, the existence of state estimator for the fifth-order laser dynamic system has been explored. Using the techniques of differential and integral inequalities, a nonlinear state estimator that guarantee global exponential tracking has also been provided. In addition, the guaranteed exponential convergence rate of such a state estimator has been rigorously calculated. Finally, several numerical simulation results have been offered to show the applicability and correctness of the main theorem.

ACKNOWLEDGEMENT

The authors thank the Ministry of Science and Technology of Republic of China for supporting this work under grant MOST 109-2221-E-214-014.

REFERENCES

[1] M. Shekarpour and M.H. Yavari, “An equivalent circuit model of passively mode-locked hybrid silicon laser,” IEEE Photonics Technology Letters, vol. 34, no. 11, pp. 595-598, 2022.

[2] N.M. Nieto, C.F. Rodriguez, and M.N. Portela, “Computational model of frequency stabilized laser for Extremum Seeking Controller parameter tuning,” IEEE Latin America Transactions, vol. 20, no. 3, pp. 451-457, 2022.

[3] R.J. Wang, X.J. Wang, J. Yang, L. Yuan, Y.Y. Lin, K. Liu, D.F. Cui, Y. Kou, Y. Liu, and Q.J. Peng, “Demonstration of the spectral beam combining in a Yb:YAG slab laser resonato,” IEEE Journal of Quantum Electronics, vol. 58, no. 5, pp. 1-6, 2022.

[4] R. Gilmore and C. Letellier, “The Symmetry of Chaos,” Oxford University Press, New York, 2007.

[5] H. Zeghlache and P. Mandel, “Influence of detuning on the properties of laser equations,” Journal of Optical Society of America B, vol. 2, pp. 18-22, 1985.

[6] R. Dutta, S.J. Geetha, S. Chakrabarti, and A. Sharma, “An L-1 regularized forecasting-aided state estimator for active distribution networks,” IEEE Transactions on Smart Grid, vol. 13, no. 1, pp. 191-201, 2022.

[7] L. Chen, Y. Li, M. Huang, X. Hui, and S. Gu, "Robust dynamic state estimator of integrated energy systems based on natural gas partial differential equations," IEEE Transactions on Industry Applications, vol. 58, no. 3, pp. 3303-3312, 2022.

[8] Y. Gao, K. Liu, C. Zhu, X. Zhang, and D. Zhang, "Co-estimation of state-of-charge and state-of-health for lithium-ion batteries using an enhanced electrochemical model," IEEE Transactions on Industrial Electronics, vol. 69, no. 3, pp. 2684-2696, 2022.

[9] Y. Jin, W. Kwon, and S. Lee, "Parameterized Luenberger-type H_∞ state estimator for delayed static neural networks," IEEE Transactions on Neural Networks and Learning Systems, vol. 33, no. 7, pp. 2791-2800, 2022.

[10] P. Duan, Y. Lv, Z. Duan, and G. Chen, "Distributed state estimation for uncertain linear systems with a recursive architecture," IEEE Transactions on Network Science and Engineering, vol. 9, no. 3, pp. 1163-1174, 2022.

[11] B. Jiang, H. Dai, and X. Wei, "A cell-to-pack state estimation extension method based on a multilayer difference model for series-connected battery packs," IEEE Transactions on Transportation Electrification, vol. 8, no. 2, pp. 2037-2049, 2022.

[12] Y.J. Sun, "Simple exponential observer design for the generalized Liu chaotic system," International Journal of Trend in Scientific Research and Development, Vol. 2, No. 1, pp. 953-956, 2017.

[13] Y.J. Sun, "Exponential state observer design for a class of uncertain chaotic and non-chaotic systems," International Journal of Trend in Scientific Research and Development, Vol. 3, No. 1, pp. 1158-1161, 2018.

[14] Y.J. Sun, "Design of state estimator for a class of generalized chaotic systems," International Journal of Trend in Scientific Research and Development, Vol. 3, No. 6, pp. 1029-1032, 2019.

[15] Y.J. Sun, "An exponential observer design for a class of chaotic systems with exponential nonlinearity," International Journal of Trend in Scientific Research and Development, Vol. 5, No. 1, pp. 1550-1552, 2020.

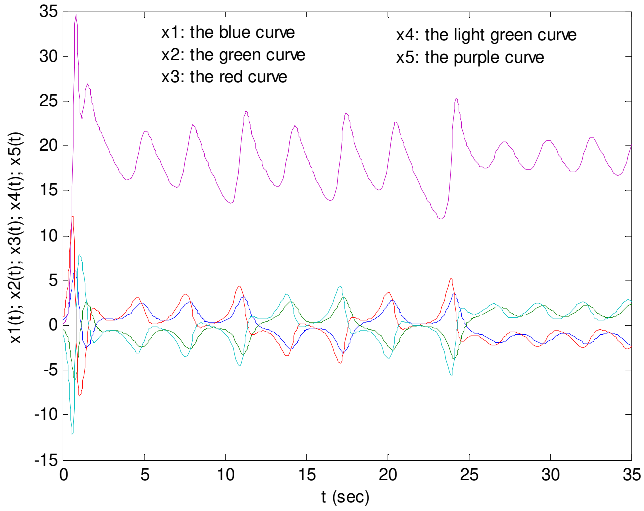

Figure 1: Typical state trajectories of the system (1) with (10).

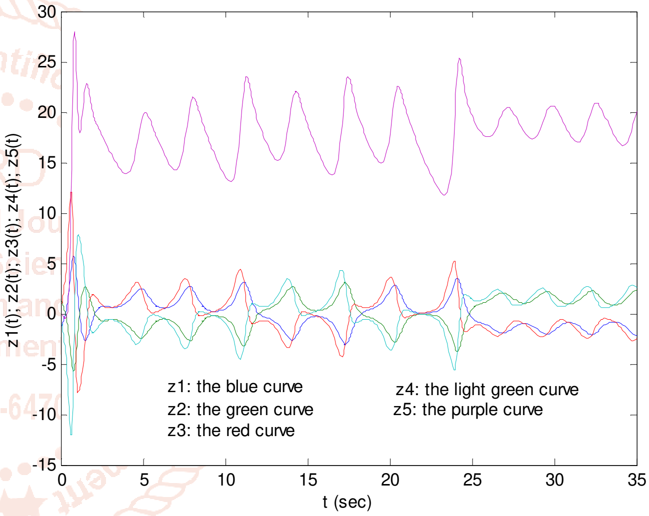

Figure 2: Typical state trajectories of the system (11).

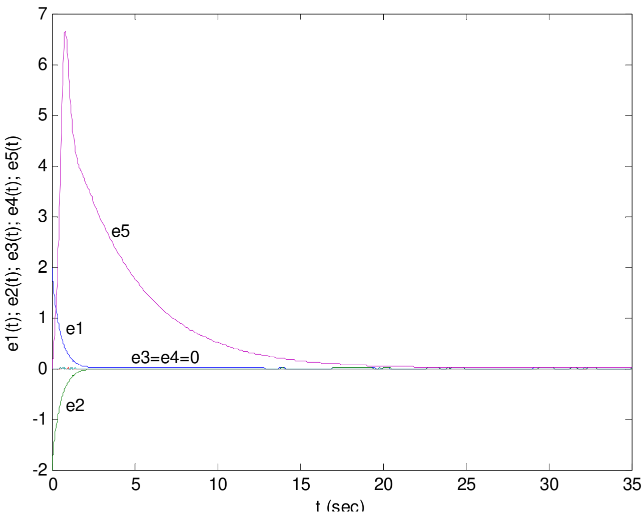

Figure 3: The time response of error states.