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# On the Thermodynamic Properties and Flow of Differentshapes of Nanoparticles in Acatheterized Stenosed Artery

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**ABSTRACT**

A modern area of fluid dynamics, called nanofluid dynamics has become increasingly popular in novel hematological treatments. Motivated by these recent developments in the field of nanotechnology, a theoretical model for catheterized stenosed artery having permeable walls, treating blood as a base fluid for various shapes of nanoparticles suspended, is presented. The disposition of blood is described to be that of a viscous nanofluid. The calculations are executed using mild stenosis conditions with heat and mass transport phenomenon. The varying shapes of nanoparticles which include bricks, platelets, cylinders and blades have been given due attention using Hamilton-Crosser equation that used to obtain expressions for thermodynamic properties like thermal conductivity and viscosity using the appropriate boundary conditions. The comparative effects for velocity and temperature have been depicted graphically. This physical model has useful application in nano-drug delivery for the cure of various cardiovascular diseases.

***KEYWORDS:*** *Composite Stenosis, Nanoparticles, Hemodynamics, Nanofluids, Hamilton-Crosser model*

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## 1. INTRODUCTION

The blood motion analysis through unhealthy arteries is a crucial domain of bioengineering research and analysis. Enhanced clinical data for mathematical models and recent developments in the computational tools has attracted ample consideration in the contemporary years. The most recurring arterial ailment is atherosclerosis. The unusual or unhealthy development in wall of artery at distinct locations in circulatory system, in medical terms is defined as stenosis[1]. Apparently, the enlargement of stenosis in an artery lessens in flow of blood. The growth of such plaques in blood vessels can cause sporadic blood flow rate, high arterial wall shear stress and boundary layer detachment[2]. Ku [3] in his review mentioned about the energy loses, turbulence and other aspects of hemodynamics.

Numerous assessments established on the principle of dynamics of fluid address the problems on blood motion in composite stenosed artery having a catheter. Catheter is designed using medical grade polyvinyl chloride and polyester constructed thermoplastic polyurethane etc. [6].The incorporation of catheter effects the hemodynamics. Srivastav [9] explored composite stenosis in presence of a catheter with permeable walls. Srivastava and Srivastava [10] analyzed a stenosed artery with catheter as well as without catheter. Srivastava et al [11] studied the problem of composite stenosed artery with catheterization considering blood modeled as non-Newtonian fluid. Ellahi et al [12] explained flow of bloodin arteries with composite stenosis while characterizing blood as a micropolar fluid.

The fast pace technological evolution over the past years has caused increasing stipulation for novel medical treatment methods, surpassing the conventional ones, to improve the therapeutic effects and lessen the side effects. Nobel laureate Richard P. Feynman introduced the term nanotechnology, and since then there have been various revolutions in this field[13]. Miscellaneous applications in energetics and bio-medical science of nanoparticles in a base fluid has been classified under nanofluid dynamics, a new branch of fluid dynamics. The term metal nanoparticles describe nano-sized metals with dimensions within 1-100 nm[14].Nanoparticles are used as new therapies for the treatment of cardiovascular diseases due to high target specificity[15]. Choi [16] explored the nanofluids. Ellahi et al [21] studied nanoparticle outcomes in arteries having composite stenosis with permeable walls. Nadeem and Ijaz [22] theoretically examined nanoparticles carrying drug in stenosed artery accompanied by slip effects on the wall. Chatterjee et al [23] used Bernstein polynomial estimation to study the effects of nanofluids in stenosed artery. Rathore and Srikanth [24] considered blood as a micro-polar nanofluid in an artery including stenosis under in the influence of catheter.

Since the chief channel of heat transmission in fluids is convection, its efficiency relies on the thermo-physical attributes of the fluids[25]. Nanoparticles possess very high thermal conductivities. There are diverse geometries associated with them. Various particle geometries either occur commonly or are schemed for distinct uses. Particle configuration is a significant nanoparticle criterion. They critically effect the nanofluid performance. In much research, spherical-shaped nanoparticles have been considered. However, practically they have limited applications. Drug delivery or clinical diagnosis have a usage of non-spherical shaped nanoparticles. Evidently, nanoparticles of geometries like brick, platelet, blade and cylinderetc have colossal relevance in drug delivery[27]. Timofeeva et al [28] studied thermal conductivity and viscosity of varying geometries of nanoparticles.

The current article is devoted to analyze the consequence of varying shapes of nanoparticlesin an artery having composite stenosis with a catheter. This is a novel comprehensive examination of mass and heat transfer of nanofluid flow in an artery having composite stenosis artery in presence of catheter with due attention given to various shapes of nanoparticles and various thermodynamic properties of nanofluids.

## 2. Formulation

Steady and laminar flow of blood in a cylindrical artery of length L has been considered. The incompressible flow in the artery is modeled using cylindrical co-ordinates. Let the velocity vector be $(u', v', w')$ here $u'$ is in $z'$ direction, $v'$ is along radial direction or $r'$ direction and $w'$ is along $\theta'$ direction. For the case of axis-symmetricity $w' = 0$, since $\theta'$ represents the circumferential direction. The configuration of composite stenosis is defined as: -

$$R'(z') = \begin{cases} R_0 - \frac{2\delta'}{R_0}(z' - d) & ;\ d \le z' \le d + \frac{R_0}{2} \\ R_0 - \frac{\delta'}{2}\left(1 + \cos\frac{2\pi}{L_0}\left(z' - d - \frac{R_0}{2}\right)\right) & ;\ d + \frac{R_0}{2} \le z' \le d + R_0 \\ R_0 & ;\ otherwise \end{cases} \quad (1)$$

where $d$ represents location of stenosis, $L_0$ is length of the stenosed part, $L$ is arterial length, $R_0$ is radius of normal artery and $R'(z')$ is arterial radius with stenosis. $R_c$ is catheter's radius (Figure 1)

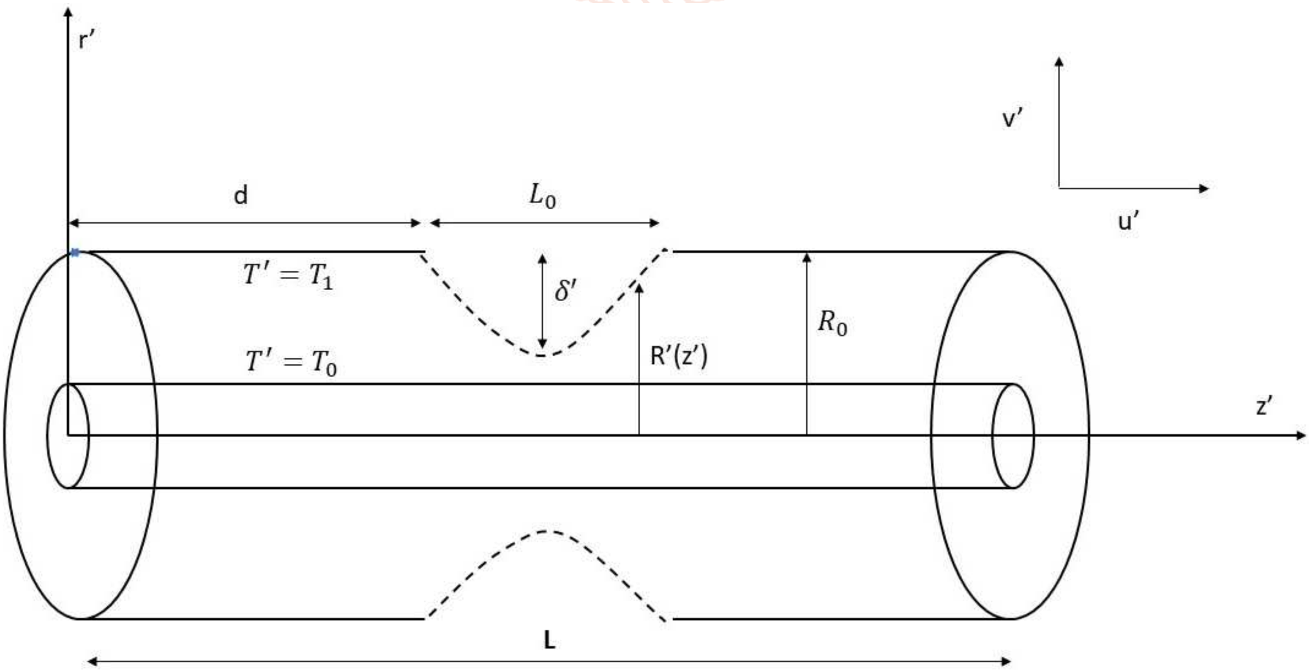


**Figure 1**

The governing equations with regard to nanoparticles are given as: -

$$\frac{\partial v'}{\partial r'}+\frac{v'}{r'}+\frac{\partial u'}{\partial z'}=0 \quad (2)$$

$$\rho_{nf}\left(v'\frac{\partial v'}{\partial r'}+u'\frac{\partial v'}{\partial z'}\right)=-\frac{\partial p'}{\partial r'}+\mu_{nf}\left(\frac{\partial^2 v'}{\partial r'^2}+\frac{1}{r'}\frac{\partial v'}{\partial r'}+\frac{\partial^2 v'}{\partial z'^2}-\frac{v'}{r'}\right) \quad (3)$$

$$\rho_{nf}\left(v'\frac{\partial u'}{\partial r'}+u'\frac{\partial u'}{\partial z'}\right)=-\frac{\partial p'}{\partial z'}+\mu_{nf}\left(\frac{\partial^2 u'}{\partial r'^2}+\frac{1}{r'}\frac{\partial u'}{\partial r'}+\frac{\partial^2 u'}{\partial z'^2}\right)+g\rho_{nf}\gamma_{nf}(T'-T_0) \quad (4)$$

$$\left(v'\frac{\partial T'}{\partial r'}+u'\frac{\partial T'}{\partial z'}\right)=\frac{k_{nf}}{\rho_{nf}c_{p_{nf}}}\left(\frac{\partial^2 T'}{\partial r'^2}+\frac{1}{r'}\frac{\partial T'}{\partial r'}+\frac{\partial^2 T'}{\partial z'^2}\right)+\frac{H}{\rho_{nf}c_{p_{nf}}} \quad (5)$$

$T_c$ is temperature of wall of catheter and $T_0$ is temperature of artery. $T'$ is nanofluid temperature. The heat generation or heat absorption parameter is represented by $H$. Now considering nanofluid model, $\mu_{nf}$ represents nanofluid viscosity, $k_{nf}$ is nanofluid thermal conductivity, $\rho_{nf}$ is nanofluid density, $\rho_{nf}\gamma_{nf}$ is nanofluid thermal expansion coefficient and $\rho_{nf}C_{p_{nf}}$ is nanofluid heat capacitance.

Boundary conditions are specified as: -

$$u'=u'_B, T'=T_0 \text{ at } r'=R'(z') \quad (6)$$

$$u'=0,\ T'=T_c \text{ at } r'=R_c \quad (7)$$

$$\frac{\partial u'}{\partial r'}=\frac{\alpha}{\sqrt{k_{nf}}}\left(u'_B-u'_p\right) \text{at } r'=R'(z') \quad (8)$$

$u'_B$ is velocity of blood at the wall of catheter and $u'_p$ is slip velocity of arterial boundary that is calculated using Darcy law. $\alpha$ is a dimensionless parameter depending upon the nanofluid and artery. Equation (8) is obtained from the Beavers and Joseph condition [35] for boundary between a free fluid and a porous medium.

$$u'_p=-\frac{k_{nf}}{\mu_{nf}}\frac{dp'}{dz'}+g\rho_{nf}\gamma_{nf}(T'-T_0) \quad (9)$$

Using $T'=T_0$ at $r'=R'(z')$

$$\therefore\ u'_p=-\frac{k_{nf}}{\mu_{nf}}\frac{dp'}{dz'} \quad (10)$$

Despite extensive research, there are not many reliant universal theoretical models to define the nanoparticle thermal conductivity. Maxwell introduced the macroscopic Effective Medium Theory(EMT) which was further developed by Hamilton and Crosser for aspheric particles. This model states the thermal conductivity of two components in a mixture which is an outcome of conductivity of unblended materials, their respective compositions and the fashion in which they are spread in the mixture. It calculates the thermal conductivity ($k_{nf}$) of nanofluid. It comprises empirical shape factor $n=3/\psi$, where $\psi$ is the sphericity which is termed as the ratio of the sphere's surface area to the real particle's surface area having equal volumes. The values of n for various shapes of nanoparticles is listed in table 2.

$$\frac{k_{nf}}{k_f}=\frac{k_p+(n-1)k_f+(n-1)(k_p-k_f)\phi}{k_p+(n-1)k_f-(k_p-k_f)\phi} \quad (11)$$

where $k_f$ is the thermal conductivity of blood, while $k_p$ is nanoparticlethermal conductivity and $\phi$ is nanoparticle volume fraction.

The nanofluid density is expressed as

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_p \quad (12)$$

where $\rho_f$ is density of blood and $\rho_p$ is nanoparticle density. Similarly, nanofluid specific heat capacity is written as

$$\rho_{nf}c_{p_{nf}}=(1-\phi)\rho_f c_{p_f}+\phi\rho_p c_{p_p} \quad (13)$$

The nanofluid thermal expansion is measured as

$$\rho_{nf}\gamma_{nf}=(1-\phi)\rho_f\gamma_f+\phi\rho_p\gamma_p \quad (14)$$

where $\rho_f c_{p_f}$ and $\rho_f \gamma_f$ is the specific heat capacity and thermal expansion of blood. $\rho_p c_{p_p}$ and $\rho_p \gamma_p$ is nanoparticle specific heat capacity and nanoparticle thermal expansion.

The nanofluid viscosity is given as

$$\mu_{nf} = (1 + A\phi)\mu_f \quad (15)$$

where $A$ is specific to the shape of nanoparticles in the fluid(Table 2). $\mu_{nf}$ is nanofluid viscosity and $\mu_f$ is blood viscosity.

The non-dimensional variables are:-

$$r = \frac{r'}{R_0}, z = \frac{z'}{R_0}, v = \frac{v'}{u_{avg}}, u = \frac{u'}{u_{avg}}, p = \frac{R_0 p'}{\mu_f u_{avg}}, \sigma = \frac{d}{R_0}, R_e = \frac{R_0 u_{avg} \rho_f}{\mu_f}, G_r = \frac{g \gamma_f \rho_f R^2{}_0 (T_c - T_0)}{u_0 \mu_f},$$
$$\delta = \frac{\delta'}{R_0}, h = \frac{H R^2{}_0}{(T_c - T_0) k_f}, \Theta = \frac{T' - T_0}{T_c - T_0}, Da = \frac{k_f}{R_0^2} \quad (16)$$

In equations (16), $G_r$ is Grashof number, $u_0$ is average velocity, $R_e$ is Reynolds number ,$h$ is heat source parameter and $Da$ isDarcy number. The blood current in arteries with small diameters is sluggish therefore intensity of inertial forces is negligible hence there remains only one component parallel to the axis, as a result $v' = 0$. Using the mild stenosis condition $\delta = \delta'/R_0 \ll 1$, the transformed equations (2) to (5)are stated as:-

$$\frac{\partial u}{\partial z} = 0 \quad (17)$$

$$\frac{\partial p}{\partial z}\frac{\mu_f}{\mu_{nf}} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + [(1-\phi) + \phi \frac{\rho_p \gamma_p}{\rho_f \gamma_f}](1 + A\phi) G_r \Theta \quad (18)$$

$$\frac{\partial^2 \Theta}{\partial r^2} + \frac{1}{r}\frac{\partial \Theta}{\partial r} + h\left[\frac{k_p + (n-1)k_f - (k_p - k_f)\phi}{k_p + (n-1)k_f + (n-1)(k_p - k_f)\phi}\right] = 0 \quad (19)$$

The geometry of the stenosis and the boundary conditions under non-dimensional scheme are:-

$$R(z) = \begin{cases} 1 - 2\delta(z - \sigma); & \sigma \le z \le \sigma + \frac{1}{2} \\ 1 - \frac{\delta}{2}(1 + \cos 2\pi(z - \sigma - \frac{1}{2})); & \sigma + \frac{1}{2} \le z \le \sigma + 1 \\ 1; & otherwise \end{cases} \quad (20)$$

$$u = u_B, \Theta = 0 \ at \ r = R(z) \quad (21)$$

$$u = 0, \Theta = 1 \ at \ r = R_c/R_0 \quad (22)$$

$$\frac{\partial u}{\partial r} = \frac{\alpha}{\sqrt{D_a}}(u_B - u_p) \ at \ r = R(z) \quad (23)$$

## 3. Solution

The solution to the equations (17) to (19) using (20) to (23) is obtained analytically as: -

The temperature is given as:-

$$\Theta = \frac{1}{4(\ln R(z) - \ln R_c/R_0)}[(4 \ln R(z) - \ln R_c/R_0) + h\,[\frac{k_p + (n-1)k_f - (k_p - k_f)\phi}{k_p + (n-1)k_f + (n-1)(k_p - k_f)\phi}](R(z)^2(\ln r - \ln R_c/R_0) + r^2(-\ln R(z) + \ln R_c/R_0) + (\ln R(z) - \ln r)(R_c/R_0)^2)] \quad (24)$$

The velocity is given as:-

$$u=\frac{dp}{dz}\frac{(1+A\phi)}{4(\ln R(z)-\ln R_c/R_0)}(r^2(\ln R(z)-\ln R_c/R_0)+R(z)^2(-\ln r+\ln R_c/R_0)+$$
$$R_c{}^2(-\ln R(z)+\ln r))+(1+A\phi)\left[(1-\phi)+\phi\frac{\rho_p\gamma_p}{\rho_f\gamma_f}\right]G_r\{16(R(z)^2\left(\ln r-\ln\frac{R_c}{R_0}\right)+$$
$$r^2(1+\ln R(z)-\ln r)(-\ln R(z)+\ln R_c/R_0)+(\ln R(z)-\ln r)(1+\ln R(z)-$$
$$\ln R_c/R_0\Big)\left(\frac{R_c}{R_0}\right)^2\}\frac{1}{64(\ln R(z)-\ln R_c/R_0)^2}+h\left[\frac{k_p+(n-1)k_f-(k_p-k_f)\phi}{k_p+(n-1)k_f+(n-1)(k_p-k_f)\phi}\right](r^4(\ln R(z))^2+$$
$$(R(z)^2-r^2)\ln R_c/R_0)(4R(z)^2+(3R(z)^2-r^2)\ln R_c/R_0)+R(z)^2\ln r\Big(-4R(z)^2+$$
$$(-3R(z)^2+4r^2)\ln\frac{R_c}{R_0}\Big)+\ln R(z)(4R(z)^2r^2+$$
$$R(z)^2(3R(z)^2-4r^2)\ln r+(-3R(z)^4+4R(z)^2r^2-2r^4)\ln R_c/R_0)-4(r^2(\ln R(z))^2-$$
$$2R(z)^2\ln r+(R(z)^2-r^2+r^2\ln r)\ln R_c/R_0+\ln R(z)(R(z)^2+r^2-r^2(\ln r+$$
$$\ln R_c/R_0)))(R_c/R_0)^2+(\ln R(z)-\ln r)(4+3\ln R(z)-3\ln R_c/R_0)(R_c/R_0)^4))-$$
$$\frac{(-\ln r+\ln R_c/R_0)u_B}{(\ln R(z)-\ln R_c/R_0)} \quad (25)$$

The volumetric flow rate Q is defined as:-

$$Q=\int_{R_c}^{R(z)}2\pi ru\,dr \quad (26)$$

$$Q=2\pi\frac{dp}{dz}\frac{q_1(z)}{(\log R(z)-\log R_c)^2-q_2(z)} \quad (27)$$

## 4. Graphical results and discussions

Our novel analysis for the flow of various shapes of nanoparticles in a catheterized composite stenosed artery considers various parameters like shape factor $n$,volume fraction $\phi$, heat source parameter h, Grashof number $Gr$, thermal conductivity $k_{nf}$ and viscosity $\mu_{nf}$. The results are obtained analytically, and graphs are obtained for temperature and velocity. Importantly influence of various shapes of nanoparticles are obtained for cylinders, platelets, bricks and blades on temperature and velocity. Fig 2-5depicts temperature of nanofluid versus radius. Fig 6-10exhibitsvelocity of nanofluidversus radius.

Figure 2 shows the temperature θ of the nanofluid in the stenosed artery versus radial direction $r$ for distinct values of shape parameters $n$. The value of shape parameter defines the ratio of real particle's surface are to sphere's surface area with equal volumes. Greater the shape factor greater the interaction of the given shape of nanoparticle with the fluid interface, greater the viscosity. Thus, blade shaped nanoparticles show least viscosity because of greatest shape factor. And brick shaped nanoparticles have least shape factor on account of their shear thinning behavior with temperature. Consequently, brick shaped nanoparticles show maximum rise in temperature while blade shaped nanoparticles show minimum rise in temperature.

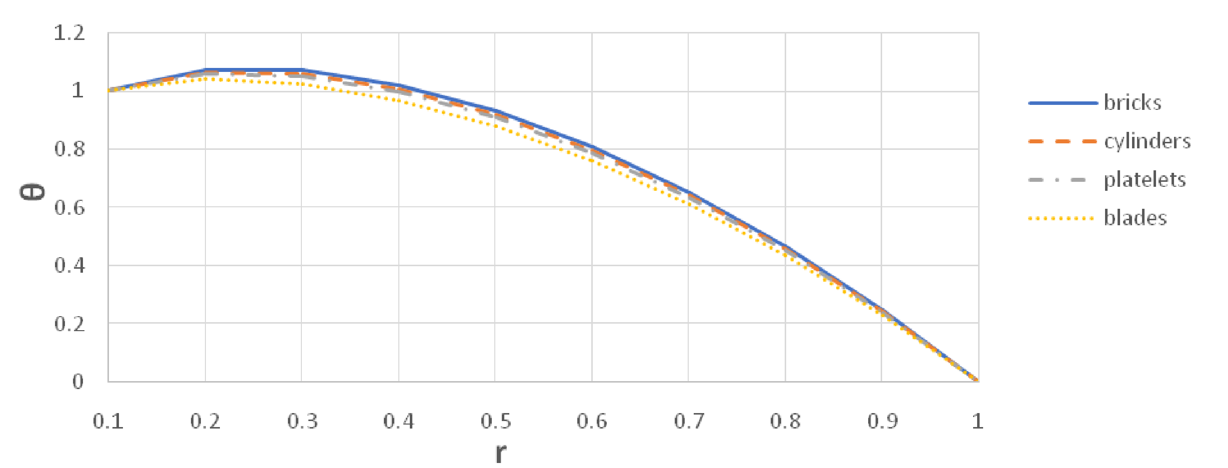


Fig 2 Temperature θ versus radial direction r various shapes of nanoparticles

Figure 3 depicts the variation of temperature θ of the nanofluid in the stenosed artery versus radial direction $r$ for distinct values of volume fraction$\phi$ for blade shaped nanoparticles in blood. With the enhancement in volume fraction for nanoparticles, the number of nanoparticles in blood increases. The increase in number of nanoparticles causes a greater increase in the transportation of nanoparticles via conduction from catheter wall to the boundary. Thus, with rise in volume fraction of nanoparticles, temperature of nanofluid rises.

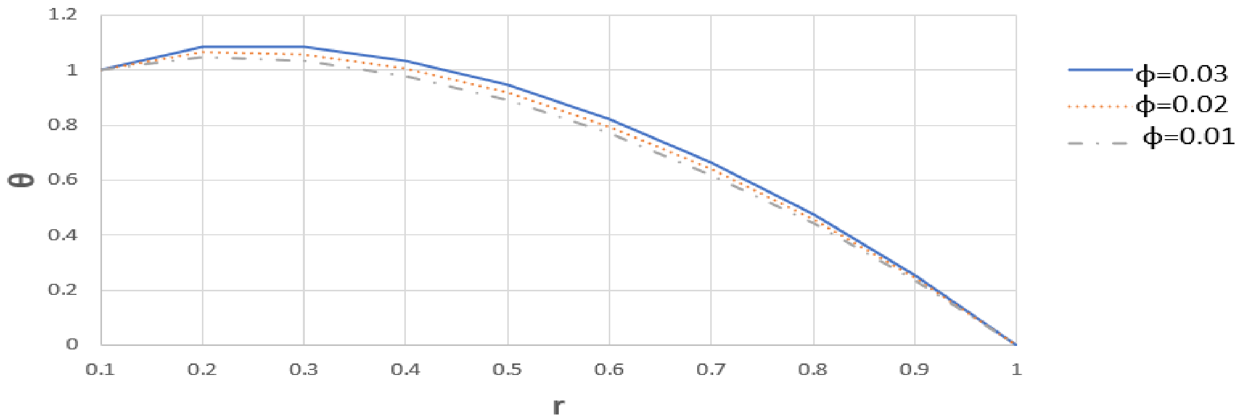


Fig 3 Temperature θ versus radial direction r for distinct values for volume fraction ϕ

Figure 4 depicts temperature θ of the nanofluid in stenosed artery versus radial direction $r$ for various values of heat source parameter $h$ for blade shaped nanoparticles in the blood. As the value of heat source parameter intensifies it causes an enhancement in heat generation in blood which enhances the value of temperature of nanofluid. Thus, temperature profile shows a rise with rise in the value of heat source parameter.

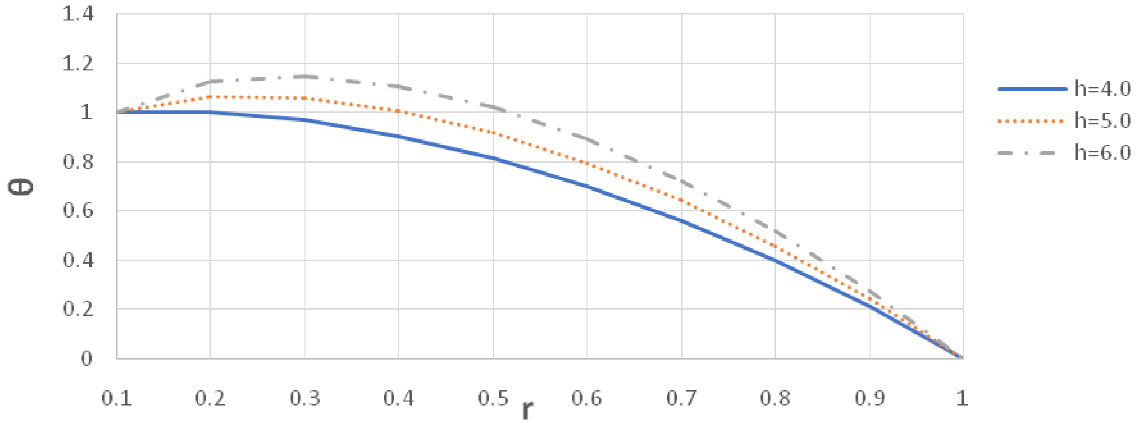


Fig 4 Temperature θ versus radial direction r for distinct values of heat source parameter h

Figure 5 shows the graph of temperature $\theta$ of the nanofluid in the stenosed artery versus thermal conductivity $k_{nf}$ for distinct shapes of nanoparticles. The graph shows that brick shaped nanoparticles have greatest conductivity while blade shaped nanoparticles have least thermal conductivity in blood. Thermal conductivity enhances with rise in temperature. Thermal conductivity in return depends on shape factor of nanoparticles. The larger shape factor causes lesser conductivity.

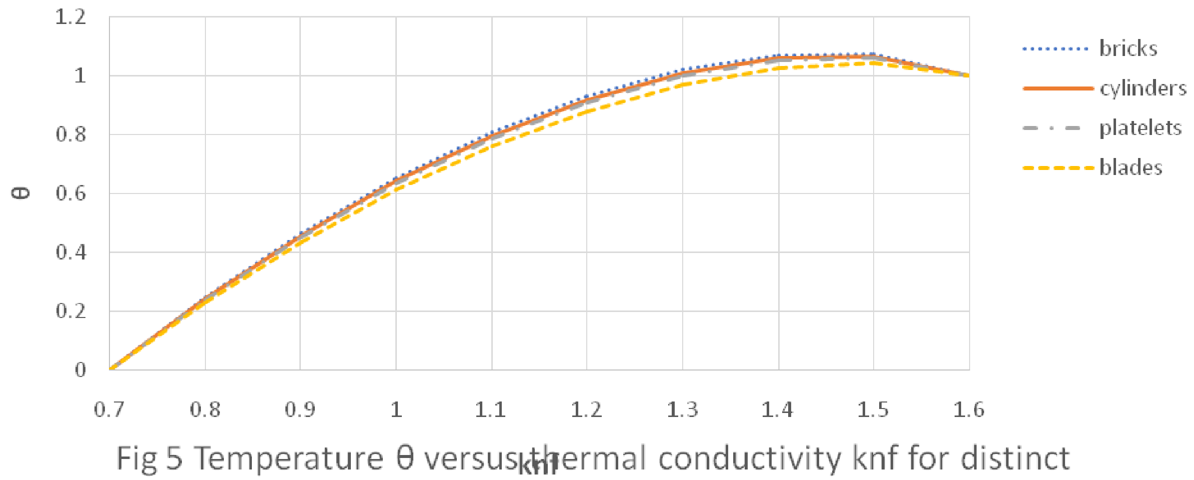


Fig 5 Temperature θ versus thermal conductivity knf for distinct values of shape parameters n

Figure 6 exhibits velocity profile $u$ versus radial direction r for various shapes of nanoparticles. Brick shaped nanoparticles in blood show highest velocity profile while blade shaped nanoparticles show a least velocity profile. The consequence of the shape of nanoparticles on the velocity is because of the viscosity dependence-relation of the shape of the respective nanoparticle at a given temperature. Platelets and cylinders have almost same viscosity in nanofluid due to elongated structures, thus they show an overlapping profile for velocity. Blade shaped nanoparticles show maximum viscosity while brick shaped nanoparticles have least viscosity.

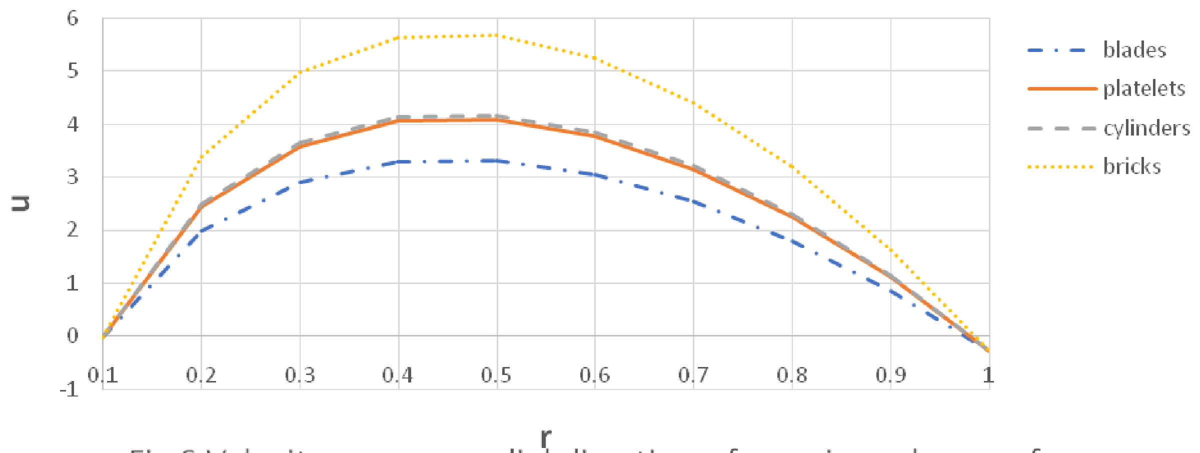


Fig 6 Velocity u versus radial direction r for various shapes of nanoparticles

Figure7 depicts the variation of velocity $u$ of the nanofluid versus radial direction $r$using distinct values of volume fraction $\phi$ of blade shaped nanoparticles in the blood. It is seen that the velocity decreases with rise in the value of volume fraction of nanoparticles in the nanofluid. This is so because as the volume fraction increases, the number of nanoparticles in the blood increases, which makes the nanofluid more viscous. The enhancement in viscosity causes an enhancement in the friction force which reduction in velocity.

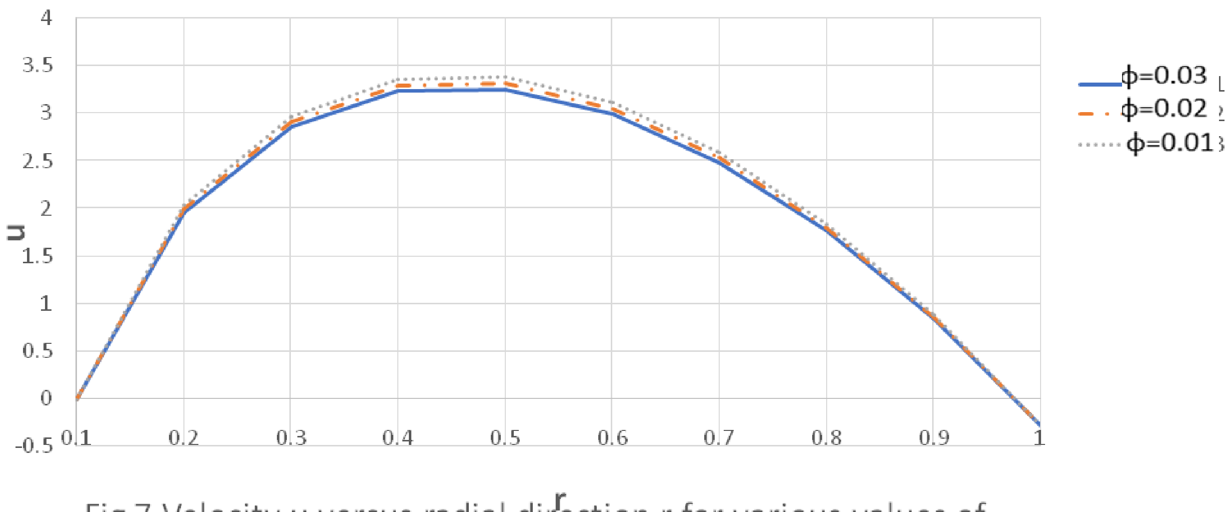


Fig 7 Velocity u versus radial direction r for various values of volume fraction ϕ

Figure8 shows the variation of velocity $u$ of the nanofluid versus radial direction $r$ for distinct values of Grashof number ($Gr$) for blade shaped nanoparticles in the blood. The results show that velocity rises with the rise in the value of Grashof number. Grashof number is the ratio of upthrust by fluid density because of temperature difference torestraining forces because of viscosity of nanofluid. The increase in value of Grashof number reduces the viscosity of the nanofluid which thus increases the velocity.

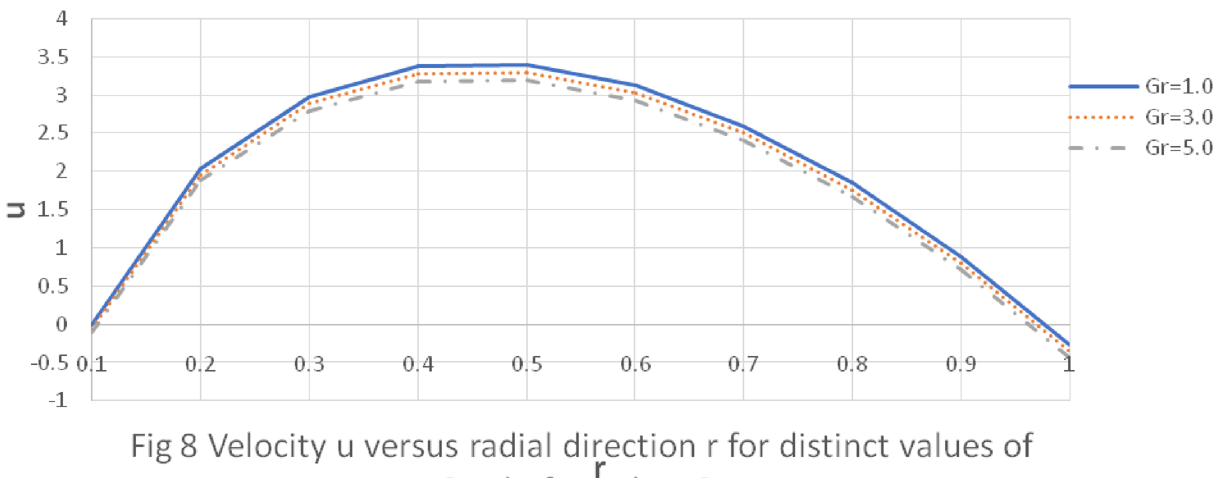


Fig 8 Velocity u versus radial direction r for distinct values of Grashof number Gr

Figure9 exhibits the variation of velocity $u$ of the nanofluid versus radial direction $r$using distinguished values of heat source parameter $h$ for blade shaped nanoparticles. The enhancement in heat source parameter causes an increase in the temperature which decreases the viscosity of nanofluids hence increasing its velocity.

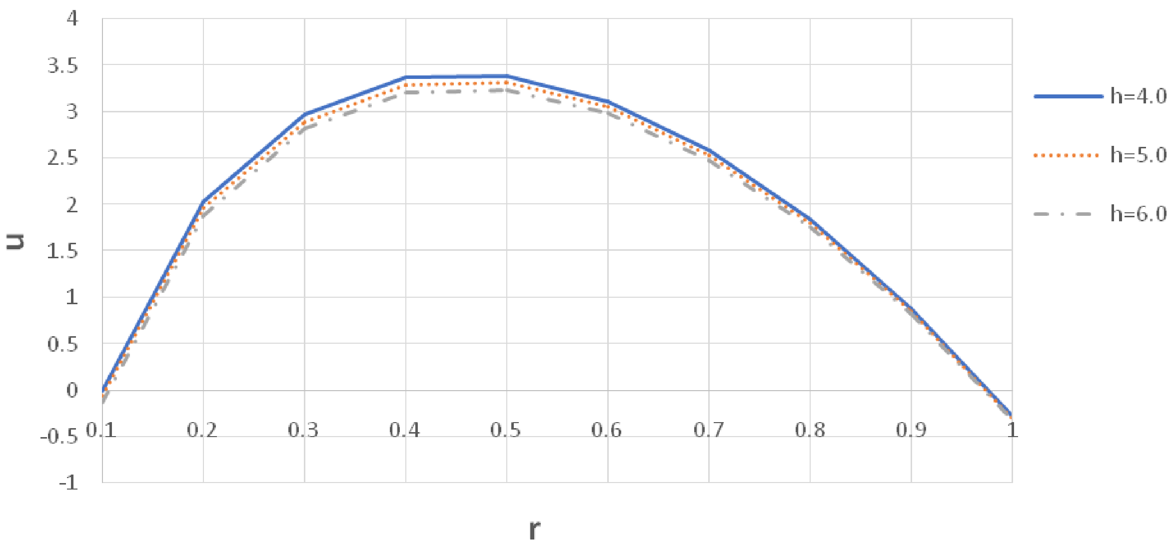


Fig 9 Velocity u against radial direction r for distinct values of heat source parameter h

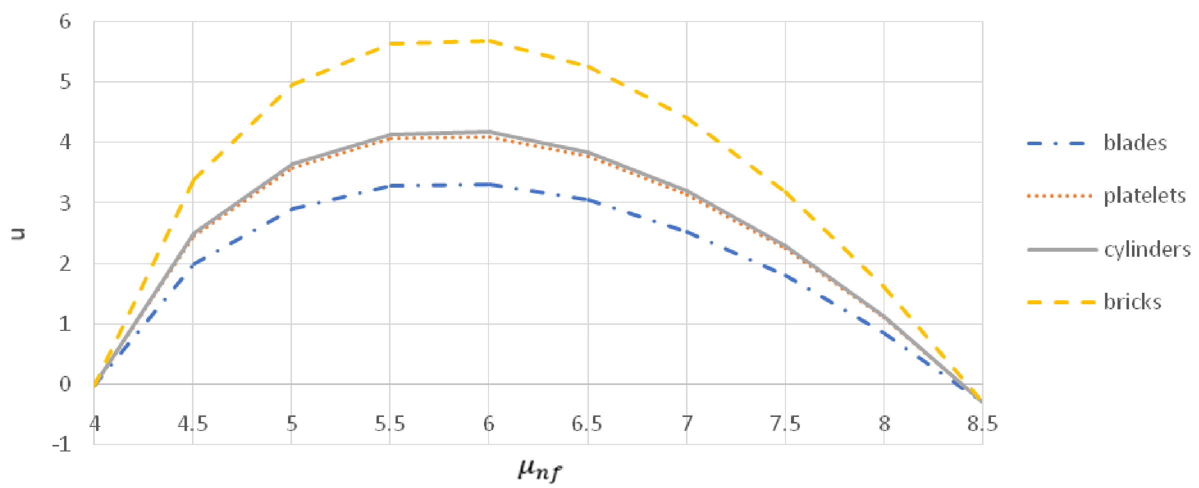


Fig 10 Velocity u versus viscosity $\mu_{nf}$ of nanofluid for various shapes of nanoparticles

Figure 10shows the graph of velocity $u$ against viscosity $\mu_{nf}$ of nanofluid for various shapes of nanoparticles. Brick shaped nanoparticles show highest velocity profile and blade shaped nanoparticles show least velocity profile on account of their varying shape factors. The blade shaped nanoparticles show highest viscositywhile brick shaped nanoparticles show least viscosity. The increase inviscosity of nanofluid causes a decrease in velocity of nanofluid.

## 5. Conclusions

The flow of a nanofluid is modeled in acomposite stenosed artery having a catheter with prime focus on various shapes of nanoparticles in blood and various thermodynamic properties of nanofluids. The results obtained can be summarized as:-

1. The nanoparticles having the shape of blades show least rise in temperature while brick shaped nanoparticles show maximum rise in temperature.
2. The rise is the number of nanoparticles in the blood causes a rise in temperature of nanofluid.
3. The increase in heat source parameter causes an elevation in temperature of nanofluid.
4. The increase in thermal conductivity raises temperature of nanofluid.
5. The greatest increase in thermal conductivity is observed for bricks and least is observed for blades.

6. Blade shaped nanoparticles in blood show least velocity while brick shaped nanoparticles show highest velocity.
7. The rise in the number of nanoparticles in the blood causes a decrease in velocity of nanofluid.
8. The enhancement in the value of Grashof number brings a rise velocity of nanofluid.
9. The enhancement in heat source parameter brings a rise in velocity of nanofluid.
10. The enhancement in viscosity of nanofluid raises its velocity.
11. The greatest increase in viscosity is observed for bricks and least is observed for blades.

Thus, the shape of the nanoparticle can be selected based on the purpose of use and the respective thermodynamic properties can be controlled. The developed mathematical model has an avant-grade approach in the cure of cardiovascular diseases.

## 6. Appendix

The thermo physical properties of the blood are:-

| | |
|---|---|
| $C_p$ | 3594J/KgK |
| $\rho$ | 1063Kg/m$^3$ |
| $k$ | 0.492W/mK |
| $\gamma$ | 0.18x10$^{-5}$1/K |

Table 1

| Shapes of nanoparticles | $n$ | $A$ |
|---|---|---|
| Platelets | 5.7 | 37.1 |
| Blades | 8.6 | 14.6 |
| Cylinders | 4.9 | 13.5 |
| Bricks | 3.7 | 1.9 |

Table 2

$$q_1(z) = (logR(z) - logR_c)^2(4R(z)\alpha Da(R(z)^2(1 - 2R(z) + 2logR_c) - R_c^2) + \sqrt{Da}(R(z)^4(-3 + 4logR(z) - 4logR_c) + 4R(z)^2R_c^2 - R_c^4) + R(z)\alpha(R(z)^4(1 - logR(z) + logR_c) - 2R(z)^2R_c^2 + (1 + logR(z) - logR_c)R_c^4))/(16/(1 + A\phi)(R(z)\alpha(logR(z) - logR_c) - \sqrt{Da}))$$

$$q_2(z) = (logR(z) - logR_c)^2\Bigg(2R(z)\alpha\Bigg(6\left(R(z)^4(-4 + 3logR(z) - 3logR_c) + 4R(z)^2(2 + logR(z) - logR_c)R_c^2 - \left(4 + 7logR(z) + 4(logR(z))^2 - (7 + 8logR(z))logR_c + 4(logR_c)^2\right)R_c^4\right) + 2h\left[\frac{k_p + (n-1)k_f - (k_p - k_f)\phi}{k_p + (n-1)k_f + (n-1)(k_p - k_f)\phi}\right](R(z)^2 - R_c^2)\left(R(z)^4(6 - 9logR(z) + 4logR(z))^2 + (9 - 8logR(z))logR_c + 4(logR_c)^2\right) + 4R(z)^2(-3 + (logR(z) - logR_c)^2)R_c^2 + \left(6 + 9logR(z) + 4logR(z)\right)^2 - (9 + 8logR(z))logR_c + 4(logR_c)^2\Bigg)R_c^4\Bigg) + \sqrt{Da}(12(R(z)^4(9 - 8logR(z) + 8logR_c) - 4R(z)^2)(3 + 2logR(z) - 2logR_c)(logR_c)^2(3 + 4logR(z) - 4logR_c)(logR_c)^4) + h\left[\frac{k_p + (n-1)k_f - (k_p - k_f)\phi}{k_p + (n-1)k_f + (n-1)(k_p - k_f)\phi}\right](-R(z)^6\left(27 - 46logR(z) + 24(logR(z))^2 + 2(23 - 24logR(z)logR_c + 24logR_c)^2\right) + 9R(z)^4(7 - 4logR(z) + 4logR_c)R_c^2 - 9R(z)^2(5 + 2logR(z) - 2logR_c)R_c^4 + (9 + 8logR(z) - 8logR_c)R_c^6)))/(768(logR(z) - logR_c))(R(z)\alpha(logR(z) - logR_c) - \sqrt{Da}))$$

## 7. Conflicts of interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

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