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Application of Modified Scheffe's Third Degree Polynomial Model for the Optimization of Compressive Strength of Nylon Fibre Reinforced Concrete (NFRC)

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ABSTRACT

This work is aimed at using Scheffe's Third Degree Polynomial optimization Model, Scheffe's (5, 3) for five component mixture, developed by Nwachukwu and others (2022a) to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC).The objective is to compare the results with the results of the previous work done on NFRC based on Scheffe's Second Degree Polynomial, Scheffe's (5, 2) Model by Nwachukwu and others (2022d). Through the use of Scheffe's Simplex method, the compressive strengths of NFRC with respect to Scheffe's third degree model were determined for different mix proportions. Control experiments were also carried out, and the compressive strengths determined. The adequacy of the model was evaluated using the Student's t-test and the test statistics confirmed the adequacy of the model. The maximum compressive strength of NFRC based on the Scheffe's(5, 3) model was 29.42MPa (N/mm2) which is higher than 21.96MPa,being the maximum value obtained by Nwachukwu and others (2022d) for the previous work done on NFRC based on the Scheffe's(5,2) model. However, the optimum strengths obtained from both models are higher than the minimum value specified by the American Concrete Institute (ACI), as 20MPa. Thus NFRC based on both Scheffe's models can produce the required compressive strength needed in light-weight structures and construction projects such as Bridge, Building pillars, Sidewalks, Building floors, Drainage pipes, Septic tanks etc., still satisfying all the required economic, aesthetic and safety criteria.

KEYWORDS: *NFRC, Scheffe's (5,3) Polynomial Model, Optimization, Compressive Strength, Mixture Design*

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1. INTRODUCTION

One of the ultimate aims of studying the various properties of the materials of concrete, plastic concrete and hardened concrete is to enable concrete technologist to design a concrete mix for a particular strength and durability (Shetty, 2006). The design of concrete mix is not a simple task on the account of the widely varying properties of the constituent materials, the conditions that prevail at the site of work, in particular the exposure condition, and the conditions that are demanded for a particular work for which the mix is designed. Thus, the design of concrete mix requires complete knowledge of the various properties of these constituent materials, the implication in case of change on these conditions at the site, the impact of the properties of plastic concrete on the hardened concrete and the complicated inter-relationship between the variables. All these make the task of mix design more complex and difficult. Furthermore, Concrete Mix Design according to Jackson and Dhir (1996) is the procedure which, for any given set of condition, the proportions of the constituent materials are chosen so as to produce a concrete with all the required properties for the minimum cost. In this context, the cost of any concrete includes, in addition to that of the materials themselves, the cost of the mix design, of batching, mixing and placing the concrete and of the site supervision. Based on the above criteria, the methods proposed by Hughes (1971), ACI-211(1994) and DOE (1988) can be time consuming as they involve a lot of trial mixes and deep statistical computations before the desired strength of the concrete can be attained. Therefore, optimization of the concrete mixture design remains the best option and the most efficient way of selecting concrete mix /proportion for better efficiency and performance of concrete such as workability, strength flexibility, homogeneity and durability. It is systematic and far better than the usual empirical method which involves rigorous and time consuming process. A typical example of optimization model is Scheffe's Polynomial Models. The popular Scheffe's models are the Scheffe's Second Degree model and the Scheffe's Third Degree model. Although there has been work previously has done on NFRC based on Scheffe's Second Degree model, the knowledge gathered from the works Obam (2006) and Nwachukwu and others (2022a) shows that the results from the third degree models usually have advantages over that from the second degree models. Thus in this present study, Scheffe's Third Degree Polynomial for five components mixtures (namely cement, fine aggregate, coarse aggregate, water and nylon fibre) will be presented.

Concrete, a homogeneous mixture of cement, sand, gravel and water is very strong in carrying compressive forces and hence is gaining increasing importance as building throughout the world (Syal and Goel, 2007). Concrete has no doubt remained an important material widely used in the construction industry since ancient time. Concrete, according to Neville (1990), plays an important part in all building structures owing to its numerous advantages which ranges from low built in fire resistance, high compressive strength to low maintenance. However, plain concrete, according to Shetty (2006) possesses a very low tensile strength, limited ductility and little resistance to cracking. Also internal micro cracks are inherently present in the concrete and its poor tensile strength is due to the propagation of such micro cracks, which will eventually lead to brittle fracture of the concrete. In the past, attempts have been made to impact improvement in tensile properties of concrete members by way of using conventional reinforced steel bars and also by applying restraining techniques. Although both these methods provide tensile strength to the concrete members, they however, do not increase the inherent tensile strength of concrete itself. In plain concrete and similar brittle material, micro-cracks (structural cracks) develop even before loading, particularly due to drying shrinkage or other causes of volume change. Based on several further researches, it has been recognized that the addition of small, closely spaced or uniformly dispersed fibres (eitherglass fibre, polypropylene fibre, nylon fibre, steel fibre, plastic fibre, asbestor (mineral fibre), carbon fibres, etc.) to concrete would act as crack arrester and would substantially improve its static and dynamic properties. This type of concrete is known as Fibre reinforced concrete (FRC).

FRC can be defined as a composite material consisting of mixtures of cement, mortar or concrete and discontinuous, discrete, uniformly dispersed suitable fibres as listed above. The main purposes of incorporating the fibrous materials remain to increase the concrete's durability and structural integrity and at the same time save costs. The last purpose, cost efficiency is achievable because, all fibres reduce the concrete's need for steel reinforcements. And since fibre reinforcement tends to be less expensive than steel bars (and less likely to corrode), it makes FRC more cost-effective. Nylon Fibre Reinforced Concrete (NFRC) is one form of FRC and is concrete mixture where the conventionally steel reinforcement in concrete production is replaced with nylon fibre. Nylons are high-performance semi-crystalline thermoplastics with attractive physical and mechanical properties that provide a wide range of end-use performances that are important in many industrial and construction applications. Nylon Fibres (NF) are produced when nylon are drawn, cast or extruded through spinnerets from a melt or solution. Nylon fibre has high resistance to wear, heat and chemicals and also cheaper when compared with conventional steel reinforcement. It is these basic characteristics that make NF finds extensive use as construction material in highly resistant concrete production. Concrete's compressive strength is one of the most important properties of concrete and. Compressive strength of concrete is the Strength of hardened concrete measured by the compression test. It is a measure of the concrete's ability to resist loads which tend to compress it. It is measured by crushing cylindrical concrete specimens in compression testing machine or universal testing machine. The compressive strength of the concrete cube test also provides an idea about all the characteristics of concrete.

This recent t study therefore presents the application of Scheffe's Third Degree Polynomial Model for the optimization of the compressive strength of NFRC. Some related works have been done by many researchers, but none has addressed the real subject matter. For instance, Ganesh Kumar and others (2019) have carried out a study on waste nylon fibre in concrete. Samrose and Mutsuddy (2019) have investigated the durability of NFRC. Hossain and others (2012) have also investigated the effect of NF in concrete rehabilitation. Ali and others (2018) have carried out a study on NFRC through partial replacement of cement with metakaolin. Song and others (2005) also investigated the strength properties of NFRC and PFRC respectively. In recent years, many researchers have used Scheffe's method to carry out one form of optimization work or the other. Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere(2006) were also based on the use of Scheffe' mathematical model in the optimization of compressive strength of Perwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's(4,2) and Scheffe's(4,3). Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). Also, Nwachukwu and others (2022a) developed and used Scheffe's Third Degree Polynomial model, Scheffe's (5,3) to optimize the compressive strength of GFRC where they compared the results with their previous work, Nwachukwu and others (2017). Nwachukwu and others (2022c) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). Again, Nwachukwu and others (2022d) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC). And finally, Nwachukwu and others (2022b) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC). From the forgoing, it can be envisaged that no work has been done on the use of Scheffe's method to optimize the compressive strength of NFR C except, the work by Nwachukwu and others (2022d) which is based on Scheffe's Second Degree Polynomial. Henceforth, the need for this present research work.

2. SCHEFFE'S (5, 3) P0LYNOMIAL EQUATION

A simplex lattice, according to Aggarwal (2002), remains a structural representation of lines joining the atoms of a mixture, whereas these atoms are constituent components of the mixture. For NFRC mixture, the constituent elements are these five components, water, cement, fine aggregate, coarse aggregate and nylon fibre. Thus, a simplex of five-component mixture is a four-dimensional solid. An imaginary space showing a four dimensional factor space with respect to Scheffe's third degree model has been shown in the work of Nwachukwu and others (2022a). According to Obam (2009), mixture components are subject to the constraint that the sum of all the components must be equal to 1. That is:

$$X_1 + X_2 + X_3 + \dots + X_q = 1; \;\Rightarrow \sum_{i=1}^{q} X_i = 1 \qquad \textbf{(1)}$$

where $X_i \geq 0$ and i = 1, 2, 3… q, and q = the number of mixtures.

2.1. THE SIMPLEX LATTICE DESIGN

The (q, m) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well-chosen polynomial equation to represent the response surface over the entire simplex region (Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains ${}^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \dots, 1;\; i = 1, 2, \dots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe's polynomial degee, which in this present study is 3.

For example a (3, 2) lattice consists of $^{3+2-1}C_2$ i.e. $^4C_2$ = 6 points. Each X_i can take m+1 = 3 possible values; that is $x = 0, \frac{1}{2}, 1$with which the possible design points are: $(1,0,0), (0,1,0), (0,0,1), \left(\frac{1}{2},\frac{1}{2},0\right), \left(0,\frac{1}{2},\frac{1}{2}\right), \left(\frac{1}{2},0,\frac{1}{2}\right)$.The general formula for evaluating the number of coefficients/terms/points required for a given lattice is given by:

$k = \frac{(q+m-1)!}{(q-1)!.m!}$ Or $^{q+m-1}C_m$ **2(a-b)**

Where k = number of coefficients/ terms / points

q = number of components = 5 in this work

m = number of deqree of polynomial = 3 in this present work

Usingeither of Eqn. (2), $k_{(5,3)}$= 35

Thus, the possible design points for Scheffe's (5,3) lattice can be as follows:
A_1 (1,0,0,0,0); A_2 (0,1,0,0,0); A_3 (0,0,1,0,0); A_4 (0,0,0,1,0), A_5 (0,0,0,0,1); A_{112} (2/3. 1/3, 0, 0, 0,); A_{122} = (1/2, 2/3, 0,0,0); A_{113} (2/3, 0, 1/3, 0,0); A_{113} (2/3, 0, 1/3, 0,0); A_{133} (1/3, 0, 0, 2/3, 0, 0); A_{114} (2/3, 0,0,1/3,0); A_{114} (1/3, 0, 0, 2/3, 0); A_{115}, (2/3, 0, 0, 0, 1/3); A_{115} (1/3, 0,0,0, 2/3); A_{223} (0, 2/3, 1/3, 0,0); A_{223} (O, 1/3, 0,0); A_{224} (0,0 2/3, 0, 1/3, 0); A_{224} (0, 1/3, 0, 2/3,0); A_{225} (0, 2/3, 0,0, 1/3); A_{255} (0, 1/3, 0, 0, 2/3); A_{334} (0,0, 2/3, 1/3, 0); A_{344} (0,0,1/3, 2/3,0), A355 (0,0,2/3,0, 1/3); A_{355} (0,0,1/3,0, 2/3); A_{445} (0,0,0, 2/3, 1/3); A_{445} (0,0,0, 1/3, 2/3); A_{123} (1/3, 1/3, 1/3, 0,0); A_{124} (1/3, 1,3, 0, 1/3, 0); A_{125} (1/3, 1/3, 0,0, 1/3); A_{134} (134 (1/3, 0, 1/3, 1/3, 0); A_{135} (1/3, 0, 1/3, 0, 1/3); A_{145} (1/3, 0, 0,1/3,1/3); A_{234} (0,1/3, 1/3,1/3, 0); A_{235} (0,1/3, 1/3, 0, 1/3); A_{245} (0, 1/3, 0, 1/3, 1/3); A_{345} (0,0,1/3,1/3, 1/3).**(3)**

According to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable X_1, X_2, X_3,X_4 …X_q is given in the form of Eqn.(4)

$Y = b_0 + \sum bi\ xi + \sum bij xj + \sum bi\ jxjxk + + \sum bi^j_2 + \dots\ i_n x i_2 x i_n$**(4)**

where ($1 \le i \le q$, $1 \le i \le j \le k \le q$, $1 \le i_1 \le i_2 \le \dots \le i_n \le q$ respectively), b = constant coefficients and Y is the response which represents the property under investigation, which ,in this case is the compressive strength.

As this research work is based on the Scheffe's (5, 3) simplex, the actual form of Eqn. (4) for five component mixture, degree three (5, 3) has been developed by Nwachukwu and others (2022a) and will be applied subsequently.

2.2. PSEUDO AND ACTUAL COMPONENTS.

In Scheffe's mix design, there exist a relationship between the pseudo components and the actual components. It has been established as Eqn.(5):

Z = A * X **(5)**

where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship

Re-arranging Eqn. (5) yields:

$X = A^{-1} * Z$ **(6)**

2.3. FORMULATION OF POLYNOMIAL EQUATION FOR SCHEFFE'S (5, 3) LATTICE

The Polynomial equation by Scheffe(1958), otherwise known as response is given in Eqn.(4). Hence, for Scheffe's (5,3) simplex lattice, the regression equation for five component mixtures has been formulated from Eqn.(4) by Nwachukwu and others (2022a) and is given as follows:

$$Y = b_0 + b_1 X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11}X_1^2 + b_{12}X_1X_2 + b_{13}X_1X_3$$
$$+ b_{14}X_1X_4 + b_{15}X_1X_5 + b_{111}X_1^3 + b_{112} X_1^2X_2 + b_{113}X_1^2X_3 + b_{114}X_1^2X_4 + b_{115}X_1^2X_5 + b_{22}X_2^2 + b_{23}X_2X_3$$
$$+ b_{24}X_2X_4 + b_{25}X_2X_5 + b_{222}X_2^3 + b_{223}X_2^2X_3 + b_{224}X_2^2X_4 + b_{225}X_2^2X_5 + b_{33}X_3^2 + b_{34}X_3X_4$$
$$+ b_{35}X_3X_5 + b_{333}X_3^3 + b_{334}X_3^2X_4 + b_{335}X_3^2X_5 + b_{44}X_4^2 + b_{45}X_4X_5 + b_{444}X_4^3 + b_{445}X_4^2X_5 + b_{55}X_5^2 + b_{555}X_5^3 \quad (7)$$

$$= b_0X_1 + b_0X_2 + b_0X_3 + b_0X_4 + b_0X_5 + b_1 X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11}X_1 - b_{11}X_1X_2 - b_{11}X_1X_3$$
$$- b_{11}X_1X_4 - b_{11}X_1X_5 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{15}X_1X_5 + b_{111}X_1^3 + b_{112} X_1X_2 - b_{112} X_1X_2^2$$
$$- b_{112} X_1X_2X_3 - b_{112} X_1X_2X_4 - b_{112} X_1X_2X_5 + b_{113}X_1 X_3 - b_{113}X_1X_2X_3 - b_{113}X_1X_3^2 - b_{113}X_1X_3X_4 - b_{113}X_1X_3X_5$$

$+ b_{114}X_1X_4 - b_{114}X_1X_2X_4 - b_{114}X_1X_3X_4 - b_{114}X_1X_4^2 - b_{114}X_1X_4X_5 + b_{115}X_1X_5 - b_{115}X_1X_2X_5$

$- b_{115}X_1X_3X_5 - b_{115}X_1X_4X_5 - b_{115}X_1X_5^2 + b_{22}X_2 - b_{22}X_1X_2 - b_{22}X_2X_3 - b_{22}X_2X_4 - b_{22}X_2X_5$

$+ b_{23}X_2X_3 + b_{24}X_2X_4 + b_{25}X_2X_5 + b_{222}X_2^3 + b_{223}X_2X_3 - b_{223}X_1X_2X_3 - b_{223}X_2X_3^2 - b_{223}X_2X_3X_4$

$- b_{223}X_2X_3X_5 + b_{224}X_2X_4 - b_{224}X_1X_2X_4 - b_{224}X_2X_3X_4 - b_{224}X_2X_4^2 - b_{224}X_2X_4X_5 + b_{225}X_2X_5$

$- b_{225}X_1X_2X_5 - b_{225}X_2X_3X_5 - b_{225}X_2X_4X_5 - b_{225}X_2X_5^2 + b_{33}X_3 - b_{33}X_1X_3 - b_{33}X_2X_3 - b_{33}X_3X_4 - b_{33}X_3X_5$

$+ b_{34}X_3X_4 + b_{35}X_3X_5 + b_{333}X_3^3 + b_{334}X_3X_4 - b_{334}X_1X_3X_4 - b_{334}X_2X_3X_4 - b_{334}X_3X_4^2 - b_{334}X_3X_4X_5$

$+ b_{335}X_3X_5 - b_{335}X_1X_3X_5 - b_{335}X_2X_3X_5 - b_{335}X_3X_4X_5 - b_{335}X_3X_5^2 + b_{44}X_4 - b_{44}X_1X_4 - b_{44}X_2X_4 - b_{44}X_3X_4$

$- b_{44}X_4X_5 + b_{45}X_4X_5 + b_{444}X_4^3 + b_{445}X_4X_5 - b_{445}X_1X_4X_5 - b_{445}X_2X_4X_5 - b_{445}X_3X_4X_5 - b_{445}X_4X_5^2$

$+ b_{55}X_5 - b_{55}X_1X_5 - b_{55}X_2X_5 - b_{55}X_3X_5 - b_{55}X_4X_5 + b_{555}X_5^3$ **(8)**

$Y = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4$

$+ \beta_{25}X_2X_5 + \beta_{34}X_3X_4 + \beta_{35}X_3X_5 + \beta_{45}X_4X_5 + \gamma_{12}X_1X_2^2 + \gamma_{13}X_1X_3^2 + \gamma_{14}X_1X_4^2 + \gamma_{15}X_1X_5^2$

$+ \gamma_{23}X_2X_3^2 + \gamma_{24}X_2X_4^2 + \gamma_{25}X_2X_5^2 + \gamma_{34}X_3X_4^2 + \gamma_{35}X_3X_5^2 + \gamma_{45}X_4X_5^2 + \beta_{123}X_1X_2X_3 + \beta_{124}X_1X_2X_4$

$+ \beta_{125}X_1X_2X_5 + \beta_{134}X_1X_3X_4 + \beta_{135}X_1X_3X_5 + \beta_{145}X_1X_4X_5 + \beta_{234}X_2X_3X_4 + \beta_{235}X_2X_3X_5$

$+ \beta_{245}X_2X_4X_5 + \beta_{345}X_3X_4X_5$ **(9)**

Where

$\beta_1 = [b_0 + b_1 + b_{11}]$; $\beta_2 = [b_0 + b_2 + b_{22}]$; $\beta_3 = [b_0 + b_3 + b_{33}]$; $\beta_4 = [b_0 + b_4 + b_{44}]$; $\beta_5 = [b_0 + b_5 + b_{55}]$;

$\beta_{12} = [b_{12} - b_{11} - b_{22} + b_{112}]$.; $\beta_{13} = [b_{13} - b_{11} - b_{33} + b_{113}]$; $\beta_{14} = [b_{14} - b_{14} - b_{11} - b_{44} + b_{114}]$;

$\beta_{15} = [b_{15} - b_{11} - b_{55} + b_{115}]$;$\gamma_{12} = [-b_{112}]$;$\gamma_{13} = [-b_{113}]$; $\gamma_{14} = [-b_{114}]$; $\gamma_{15} = [-b_{115}]$;

$\beta_{123} = [-b_{112} - b_{113} - b_{223}]$;$\beta_{124} = [-b_{112} - b_{114} - b_{224}]$;$\beta_{125} = [-b_{112} - b_{115} - b_{225}]$;$\beta_{134} = [-b_{113} - b_{114} - b_{334}]$;

$\beta_{135} = [-b_{113} - b_{115} - b_{335}]$;$\beta_{145} = [-b_{113} - b_{115} - b_{445}]$;$\beta_{23} = [b_{23} - b_{22} - b_{33} + b_{223}]$;$\beta_{24} = [b_{24} - b_{22} - b_{44} + b_{224}]$;

$\beta_{25} = [b_{25} - b_{22} - b_{55} + b_{225}]$;$\gamma_{23} = [-b_{223}]$;$\gamma_{24} = [-b_{224}]$; $\gamma_{25} = [-b_{225}]$; $\beta_{234} = [-b_{223} - b_{224} - b_{334}]$;

$\beta_{235} = [-b_{223} - b_{225} - b_{335}]$;$\beta_{245} = [-b_{224} - b_{225} - b_{445}]$;$\beta_{34} = [b_{34} - b_{33} - b_{44} + b_{334}]$;$\beta_{35} = [b_{35} - b_{33} - b_{55} + b_{335}]$;

$\gamma_{34} = [-b_{334}]$; $\gamma_{35} = [-b_{335}]$; $\beta_{345} = [-b_{334} - b_{335} - b_{445}]$;$\beta_{45}$; $= [b_{45} - b_{44} - b_{55} + b_{445}]$;$\gamma_{45} = [-b_{445}]$ **(10)**

Equation (9) is the polynomial equation for Scheffe's (5, 3) simplex

2.4. COEFFICIENTS OF THESCHEFFE'S (5, 3) POLYNOMIAL

From the work of Nwachukwu and others (2022a), the coefficients of the Scheffe's (5, 3) polynomial have been determined as under. :

$\beta_1 = Y_1$; $\beta_2 = Y_2$; $\beta_3 = Y_3$; $\beta_4 = Y_4$; and $\beta_5 = Y_5$ **11 (a-e)**

$\beta_{12} = 9/4(Y_{112} + Y_{122} - Y_1 - Y_2)$; $\beta_{13} = 9/4(Y_{113} + Y_{133} - Y_1 - Y_3)$;$\beta_{14} = 9/4(Y_{114} + Y_{144} - Y_1 - Y_4)$; **12(a-c)**

$\beta_{15} = 9/4(Y_{115} + Y_{155} - Y_1 - Y_5)$; $\beta_{23} = 9/4(Y_{223} + Y_{233} - Y_2 - Y_3)$;$\beta_{24} = 9/4(Y_{224} + Y_{244} - Y_2 - Y_4)$**13(a-c)**

$\beta_{25} = 9/4(Y_{225} + Y_{255} - Y_2 - Y_5)$; $\beta_{34} = 9/4(Y_{334} + Y_{344} - Y_3 - Y_4)$; $\beta_{35} = 9/4(Y_{335} + Y_{355} - Y_3 - Y_5)$ **14(a-c)**

$\beta_{45} = 9/4(Y_{445} + Y_{455} - Y_4 - Y_5)$; $\gamma_{12} = 9/4(3Y_{112} + 3Y_{122} - Y_1 + Y_2)$; $\gamma_{13} = 9/4(3Y_{113} + 3Y_{133} - Y_1 + Y_3)$**15(a-c)**

$\gamma_{14} = 9/4(3Y_{114} + 3Y_{144} - Y_1 + Y_4)$; $\gamma_{15} = 9/4(3Y_{115} + 3Y_{115} - Y_1 + Y_5)$;$\gamma_{23} = 9/4(3Y_{223} + 3Y_{233} - Y_2 + Y_3)$**16(a-c)**

$\gamma_{24} = 9/4(3Y_{224} + 3Y_{244} - Y_2 + Y_4)$; $\gamma_{25} = 9/4(3Y_{225} + 3Y_{255} - Y_2 + Y_5)$; $\gamma_{34} = 9/4(3Y_{334} + 3Y_{344} - Y_3 + Y_4)$**17(a-c)**

$\gamma_{35} = 9/4(3Y_{335} + 3Y_{355} - Y_3 + Y_5)$; $\gamma_{45} = 9/4(3Y_{445} + 3Y_{455} - Y_4 + Y_5)$**18(a-b)**

$\beta_{123} = 27Y_{123} - 27/4(Y_{112} + Y_{122} + Y_{113} + 133 + Y_{223} + Y_{233}) + 9/4(Y_1 + Y_2 + Y_3)$ **(19)**

$\beta_{124} = 27Y_{124} - 27/4(Y_{112} + Y_{122} + Y_{114} + Y_{144} + Y_{224} + Y_{244}) + 9/4(Y_1 + Y_2 + Y_4)$**(20)**

$\beta_{125} = 27Y_{125} - 27/4(Y_{112} + Y_{122} + Y_{115} + Y_{155} + Y_{225} + Y_{255}) + 9/4(Y_1 + Y_2 + Y_5)$ **(21)**

$\beta_{134} = 27Y_{134} - 27/4(Y_{113} + Y_{133} + Y_{114} + Y_{144} + Y_{334} + Y_{344}) + 9/4(Y_1 + Y_3 + Y_4)$**(22)**

$\beta_{135} = 27Y_{135} - 27/4(Y_{113} + Y_{133} + Y_{115} + Y_{155} + Y_{335} + Y_{355}) + 9/4(Y_1 + Y_3 + Y_5)$**(23)**

$\beta_{145} = 27Y_{145} - 27/4(Y_{114}+Y_{144}+Y_{115}+Y_{155}+Y_{445}+Y_{455}) + 9/4(Y_1+Y_4+Y_5)$ **(24)**

$\beta_{234} = 27Y_{234} - 27/4(Y_{223}+Y_{233}+Y_{224}+Y_{244}+Y_{334}+Y_{344}) + 9/4(Y_2+Y_3+Y_4)$ **(25)**

$\beta_{235} = 27Y_{235} - 27/4(Y_{223}+Y_{233}+Y_{225}+Y_{255}+Y_{335}+Y_{355}) + 9/4(Y_2+Y_3+Y_5)$ **(26)**

$\beta_{245} = 27Y_{245} - 27/4(Y_{224}+Y_{244}+Y_{225}+Y_{255}+Y_{445}+Y_{455}) + 9/4(Y_2+Y_4+Y_5)$ **(27)**

$\beta_{345} = 27Y_{345} - 27/4(Y_{334}+Y_{344}+Y_{335}+Y_{355}+Y_{445}+Y_{455}) + 9/4(Y_3+Y_4+Y_5)$ **(28)**

Where Y_i = Response Function (Compressive Strength) for the pure component, i

2.5. SCHEFFE'S (5, 3) MIXTURE DESIGN MODEL

Substituting Eqns. (11)-(28) into Eqn. (9), yields the mixture design model for the Scheffe's (5, 3) lattice.

2.6. ACTUAL AND PSEUDO MIX RATIOS OF SCHEFFE'S (5, 3) DESIGN LATTICE

The requirement of simplex lattice design based on Eqn. (1) criteria makes it impossible to use the conventional mix ratios such as 1:2:4, 1:3:6, etc., at a given water/cement ratio for the actual mix ratio. This necessitates the transformation of the actual components proportions to meet the above criterion. Based on experience and previous knowledge from literature, the following arbitrary prescribed mix ratios are always chosen for the five vertices of Scheffe's (5, 3) lattice. See the works of Nwachukwu and others (2022a), for the figure showing the vertices of a Scheffe's (5, 3) lattice for both actual and pseudo mix ratios.

A_1 (0.67:1: 1.7: 2:0.5); A_2 (0.56:1:1.6:1.8:0.8); A_3 (0.5:1:1.2:1.7:1); A_4 (0.7:1:1:1.8:1.2) and A_5 (0.75:1:1.3:1.2:1.5), which represent water/cement ratio, cement, fine aggregate, coarse aggregate and nylon fibre respectively.

For the pseudo mix ratio, the following corresponding mix ratios at the vertices for five component mixtures are always chosen: A_1(1:0:0:0:0), A_2(0:1:0:0: 0), A_3(0:0:1:0:0), A_4(0:0:0:1:0), and A_5(0:0:0:0:1)

For the transformation of the actual component, Z to pseudo component, X, and vice versa,Eqns. (5) and (6) are used. Substituting the mix ratios from point A_1 into Eqn. (5) yields:

$$\begin{pmatrix} 0.67 \\ 1 \\ 1.7 \\ 2 \\ 0.5 \end{pmatrix} = \begin{pmatrix} A_{111} & A_{112} & A_{113} & A_{114} & A_{115} \\ A_{221} & A_{222} & A_{223} & A_{224} & A_{225} \\ A_{331} & A_{332} & A_{333} & A_{334} & A_{335} \\ A_{441} & A_{442} & A_{443} & A_{444} & A_{445} \\ A_{551} & A_{552} & A_{553} & A_{554} & A_{555} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \textbf{(29)}$$

Transforming the R.H.S matrix and solving, we obtain

A_{111}= 0.67; A_{221}= 1; A_{331}= 1.7; A_{441}= 2; A_{551}= 0.5

The same approach is used to obtain the remaining values as shown in Eqn. (30)

$$\begin{matrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{matrix} \begin{pmatrix} \\ \\ =1.7 \\ \\ \\ \end{pmatrix} \begin{pmatrix} 0.67\ 0.56\ 0.5\ 0.7\ 0.75 \\ 1.0\ 1.0\ 1.0\ 1.0\ 1.0 \\ 1.6\ 1.2\ 1.0\ 1.3 \\ 2.0\ 1.8\ 1.7\ 1.8\ 1.2 \\ 0.5\ 0.8\ 1.0\ 1.2\ 1.5 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix} \quad \textbf{(30)}$$

Considering mix ratios at the mid points from Eqn.(3) and substituting these pseudo mix ratios in turn into Eqn.(30) will yield the corresponding actual mix ratios.

For instance, considering point A_{112}we have:

$$\begin{matrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{matrix} \begin{pmatrix} \\ \\ =1.7 \\ \\ \\ \end{pmatrix} \begin{pmatrix} 0.67\ 0.56\ 0.5\ 0.7\ 0.75 \\ 1.0\ 1.0\ 1.0\ 1.0\ 1.0 \\ 1.6\ 1.2\ 1.0\ 1.3\ 0 \\ 2.0\ 1.8\ 1.7\ 1.8\ 1.2 \\ 0.5\ 0.8\ 1.0\ 1.2\ 1.5 \end{pmatrix} \begin{pmatrix} 0.63 \\ 0.331 \\ = 1.67 \\ 01.90 \\ 0\ 1.60 \end{pmatrix} \left\{ \quad \right\} \quad \textbf{(31)}$$

Solving, $Z_1 = 0.63$; $Z_2 = 1.00$; $Z_3 = 1.67$' $Z_4 = 1.90$; $Z_5 = 1.60$

The same approach goes for the remaining mid-point mix ratios.

Hence, to generate the polynomial coefficients, thirty-five (35) experimental tests will be carried out and the corresponding mix ratios are depicted in Table 1.

Table 1: Actual and Pseudo Mix Ratio for NFRC based on Scheffe's (5.3) Lattice.

Points	Pseudo Component					Response Symbol	Actual Component				
	X_1	X_2	X_3	X_4	X_5		Z_1	Z_2	Z_3	Z_4	Z_5
1	1	0	0	0	0	Y_1	0.67	1.00	1.70	2.0	0.5
2	0	1	0	0	0	Y_2	0.56	1.00	1.60	1.8	0.8
3	0	0	1	0	0	Y_3	0.50	1.00	1.20	1.7	1.0
4	0	0	0	1	0	Y_4	0.70	1.00	1.00	1.8	1.2
5	0	0	0	0	1	Y_5	0.75	1.00	1.30	1.2	1.5
112	0.67	0.33	0	0	0	Y_{112}	0.63	1.00	1.67	1.9	1.6
122	0.33	0.67	0	0	0	Y_{122}	0.60	1.00	1.63	1.8	0.7
113	0.67	0	0.33	0	0	Y_{113}	0.61	1.00	1.54	1.9	0.6
133	0.33	0	0.67	0	0	Y_{133}	0.56	1.00	1.37	1.8	0.8
114	0.67	0	0	0.33	0	Y_{114}	0.68	1.00	1.47	1.9	0.7
144	0.33	0	0	0.67	0	Y_{144}	0.69	1.00	1.23	1.8	0.9
115	0.67	0	0	0	0.33	Y_{115}	0.70	1.00	1.57	1.7	0.8
155	0.33	0	0	0	0.67	Y_{115}	0.72	1.00	1.43	1.4	1.1
223	0	0.67	0.33	0	0	Y_{223}	0.55	1.00	1.40	1.7	0.8
233	0	0.33	0.67	0	0	Y_{233}	0.52	1.00	1.20	1.7	0.9
224	0	0.67	0	0.33	0	Y_{224}	0.61	1.00	1.67	1.8	0.9
244	0	0.33	0	0.67	0	Y_{244}	0.66	1.00	1.73	1.8	1.0
225	0	0.67	0	0	0.33	Y_{225}	0.63	1.00	1.50	1.6	0.7
255	0	0.33	0	0	0.67	Y_{255}	0.69	1.00	1.40	1.4	0.6
234	0	0	0.67	0.33	0	Y_{334}	0.57	1.00	1.13	1.7	1.0
344	0	0	0.33	0.67	0	Y_{344}	0.64	1.00	1.07	1.7	1.1
335	0	0	0.67	0	0.33	Y_{355}	0.58	1.00	1.23	1.5	1.1
355	0	0	0.33	0	0.67	Y_{335}	0.67	1.00	1.27	1.3	1.3
445	0	0.33	0	0	0.67	Y_{445}	0.72	1.00	1.10	1.6	1.3
455	0	0	0	0.67	0.33	Y_{445}	0.73	1.00	1.20	1.4	1.4
123	0.33	0.33	0.33	0	0	Y_{123}	0.57	1.00	1.49	1.8	0.7
124	0.33	0.33	0	0.33	0	Y_{124}	0.64	1.00	1.09	1.8	0.8
125	0.33	0.33	0	0	0.33	Y_{125}	0.66	1.00	1.52	1.6	0.9
134	0.33	0.33	0	0.33	0	Y_{134}	0.62	1.00	1.29	1.8	0.8
135	0.33	0	0.33	0	0.33	Y_{135}	0.63	1.00	1.39	1.6	0.9
145	0.33	0	0	0.33	0.33	Y_{145}	0.70	1.00	1.32	1.6	1.0
234	0	0.33	0.33	0.33	0	Y_{234}	0.58	1.00	1.25	1.7	0.9
235	0	0.33	0.33	0	0.33	Y_{235}	0.60	1.00	1.32	1.5	1.0
245	0	0.33	0	0.33	0.33	Y_{245}	0.67	1.00	1.29	1.5	1.1
345	0	0	0.33	0.33	0.33	Y_{345}	0.64	1.00	1.6	1.5	1.2

2.7. THE CONTROL POINTS

Thirty five(35) different controls were predicted which according to Scheffe's (1958) ,their summation should not be greater than one. The same approach for component transformation adopted for the initial experimental points are also adopted for the control points and the results are shown in Table 2.

Table 2: Actual and Pseudo Component of NFRC Based onScheffe (5,3) Lattice for Control Points

Points	Pseudo Component					Control Points	Actual Component				
	X_1	X_2	X_3	X_4	X_5		Z_1	Z_2	Z_3	Z_4	Z_5
1	0.25	0.25	0.25	0.25	0	C_1	0.61	1	1.38	1.83	0.5
2	0.25	0.25	0.25	0	0.25	C_2	0.62	1	1.45	1.68	0.8
3	0.25	0.25	0	0.25	0.25	C_3	0.67	1	1.40	1.70	1
4	0.25	0	0.25	0.25	0.25	C_4	0.66	1	1.30	1.68	1.2
5	0	0.25	0.25	0.25	0.25	C_5	0.63	1	1.28	1.63	1.5
112	0.20	0.20	0.2	0.20	0.20	C_{112}	0.64	1	1.36	1.70	0.65
122	0.30	0.30	0.30	0.10	0	C_{122}	0.59	1	1.45	1.83	0.75
113	0.30	0.30	0.30	0	0.10	C_{113}	0.59	1	1.48	1.77	0.85
133	0.30	0.30	0	0.30	0.10	C_{133}	0.65	1	1.42	1.80	1
114	0.30	0	0.30	0.30	0.10	C_{114}	0.64	1	1.30	1.77	0.9
144	0	0.30	0.30	0.30	0.10	C_{144}	0.60	1	1.27	1.71	1
115	0.10	0.30	0.30	0.30	0	C_{115}	0.60	1	1.31	1.79	1.55
155	0.30	0.10	0.30	0.30	0	C_{155}	0.62	1	1.33	1.83	1.1
223	0.30	0.10	0.30	0.30	0	C_{223}	0.63	1	1.41	1.85	1.25
233	0.10	0.20	0.30	0.40	0	C_{233}	0.61	1	1.25	1.79	1.35
224	0.30	0.20	0.10	0.40	0	C_{224}	0.64	1	1.35	1.85	0.89
244	0.20	0.20	0.10	0.10	0.40	C_{244}	1.40	1	1.04	1.59	1.08
225	0.30	0.10	0.30	0.20	0.10	C_{225}	0.62	1	1.36	1.77	0.92
255	0.25	0.25	0.15	0.15	0.20	C_{255}	0.61	1	1.51	3.16	0.91
334	0.30	0.30	0.20	0.10	0.10	C_{334}	0.68	1	1.56	1.96	0.98
344	0.10	0.30	0.30	0.30	0	C_{344}	1.30	1	1.31	1.79	0.95
335	0.25	0.15	0.20	0.20	0.20	C_{335}	0.65	1	0.96	1.05	0.97
355	0.15	0.25	0.20	0.20	0.20	C_{355}	0.64	1	1.37	1.71	0.79
445	0.10	0.20	0.30	0.40	0	C_{445}	0.61	1	1.25	1.79	0.99
455	0.30	0.10	0.20	0.30	0.10	C_{455}	0.61	1	1.31	1.72	1.03
123	0.25	0.10	0.40	0	0.25	C_{123}	0.61	1	1.39	1.66	0.98
124	0.30	0.20	0.40	0.10	0	C_{124}	0.58	1	1.41	1.82	0.83
125	0.15	0.15	0.20	0.10	0.40	C_{125}	0.65	1	1.36	1.57	1.11
134	0.10	0.30	0	0.30	0.30	C_{134}	0.67	1	1.34	1.65	1.10
135	0.25	0.20	0.20	0.20	0.15	C_{135}	0.74	1	1.38	2.08	0.88
145	0.10	0.10	0.10	0.30	0.40	C_{145}	0.68	1	1.27	1.57	1.19
234	0.40	0.20	0.10	0.10	0.20	C_{234}	0.73	1	1.61	1.87	1.03
235	0.25	0.25	0.15	0.25	0.10	C_{235}	0.63	1	1.39	1.78	0.93
245	0.15	0.20	0.10	0.25	0.30	C_{245}	0.66	1	1.34	1.64	1.09
345	0.30	0.10	0.20	0.25	0.15	C_{345}	0.64	1	1.34	1.75	0.96

The actual component as transformed from Eqn. (30), Table (1) and (2) were used to measure out the quantities of water (Z_1), cement (Z_2), fine aggregate (Z_3), coarse aggregate (Z_4) and nylonfibre (Z_5) in their respective ratios for the concrete cube strength test.

3. MATERIALS AND METHODS

3.1. MATERIALS

The materials under investigation in this research work are cement, water, fine and coarse aggregates and nylon fibre. The cement is Dangotecement, a brand of Ordinary Portland Cement, conforming to British Standard Institution BS 12 (1978). The fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite of 20mm size was obtained from a local stone market and was downgraded to 4.75mm. The same size and nature of nylonfibre used previously by Nwachukwu and others (2022d) is the same as the one being used in this present work. Also, potable water from the clean water source was used in this experimental investigation.

3.2. METHOD

3.2.1. SPECIMEN PREPARATION / BATCHING/ CURING

The specimens for the compressive strength were concrete cubes. They were cast in steel mould measuring 15cm*15cm*15cm. The mould and its base were damped together during concrete casting to prevent leakage of mortar. Thin engine oil was applied to the inner surface of the moulds to make for easy removal of the cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 70 mix ratios were to be used to produce 140 prototype concrete cubes. Thirty five (35) out of the 70 mix ratios were as control mix ratios to produce 70 cubes for the conformation of the adequacy of the mixture design given by Eqn. (9), whose coefficients are given in Eqns. (11) – (28). Curing commenced 24hours after moulding. The specimens were removed from the moulds and were placed in clean water for curing. After 28days of curing the specimens were taken out of the curing tank.

3.2.2. COMPRESSIVE STRENGTH TEST

Compressive strength testing was done in accordance with BS 1881 – part 116 (1983) - Method of determination of compressive strength of concrete cube and ACI (1989) guideline. Two samples were crushed for each mix ratio and in each case, the compressive strength was then calculated using Eqn.(32)

$$\text{Compressive Strength} = \frac{\text{Average failure Load (N) } P}{\text{Cross- sectional Area (mm}^2\text{) } A} \quad \textbf{(32)}$$

4. RESULTS AND DISCUSSION

4.1. COMPRESSIVE STRENGTHRESULTSFOR THE INITIAL EXPERIMENTAL TESTS.

The results of the compressive strength ($R_{esponse,}$ Y_i) based on a 28-days strength is presented in Table 3. These are calculated from Eqn.(32)

Table 3: 28th Day Compressive Strength Test Results for NFRC Based on Scheffe's (5, 3) Model for the Initial Experimental Tests.

Points	Experimental Number	Response, Y_i, MPa	Response Symbol	$\sum Y_i$	Average Response Y, MPa
1	1A 1B	18.89 19.44	Y_1	38.33	19.17
2	2A 2B	20.76 21.00	Y_2	41.76	20.88
3	3A 3B	17.32 16.98	Y_3	34.30	17.15
4	4A 4B	15.40 15.38	Y_4	30.78	15.39
5	5A 5B	18.25 17.78	Y_5	36.03	18.02
112	6A 6B	22.65 23.21	Y_{112}	45.86	22.93
122	7A 7B	21.23 21.33	Y_{122}	42.56	21.28
113	8A 8B	24.54 25.11	Y_{113}	49.65	24.83
133	9A 9B	29.43 29.41	Y_{133}	58.84	29.42
114	10A 10B	18.17 18.19	Y_{114}	36.36	18.18
144	11A 11B	26.45 26.49	Y_{144}	52.94	26.47
115	12A 12B	22.77 22.86	Y_{115}	45.63	22.82

155	13A 13B	18.23 18.43	Y_{155}	36.66	18.33
223	14A 14B	23.33 24.02	Y_{223}	47.35	23.68
233	15A 15B	17.05 17.10	Y_{233}	34.15	17.08
224	16A 16B	20.86 20.97	Y_{224}	41.83	20.92
244	17A 17B	22.54 22.64	Y_{244}	45.18	22.59
225	18A 18B	19.67 18.99	Y_{225}	38.66	19.33
255	19A 19B	19.82 19.79	Y_{255}	39.61	19.81
334	20A 20B	20.22 20.41	Y_{334}	40.63	20.32
344	21A 21B	27.23 27.35	Y_{344}	54.58	27.29
335	22A 22B	19.43 19.25	Y_{335}	38.68	19.34
355	23A 23B	21.32 21.34	Y_{355}	42.66	21.33
445	24A 24B	19.75 19.79	Y_{445}	39.54	19.77
455	25A 25B	21.38 21.41	Y_{455}	42.79	21.40
123	26A 26B	23.23 23.42	Y_{123}	46.65	23.33
124	27A 27B	24.54 25.12	Y_{124}	49.66	24.83
125	28A 28B	20.48 20.64	Y_{125}	41.12	20.56
134	29A 29B	18.34 17.98	Y_{134}	36.32	18.16
135	30A 30B	23.21 23.35	Y_{135}	46.56	23.28
145	31A 31B	22.74 22.77	Y_{145}	45.51	22.76
234	32A 32B	20.76 20.59	Y_{234}	41.35	20.68
235	33A 33B	19.54 19.64	Y_{235}	39.18	19.59
245	34A 34B	17.87 18.03	Y_{245}	35.90	17.95
345	35A 35B	21.44 22.02	Y_{345}	43.46	21.73

4.2. COMPRESSIVE STRENGTHRESULTSFOR THE EXPERIMENTAL (CONTROL) TEST.

Table 4 shows the 28th day Compressive strength results for the Experimental (Control) Test

Table 4: 28TH Day Compressive Strength Values for NFRC Based on Scheffe's (5, 3) Modelfor the Experimental (Control) Tests.

Control Points	Experimental Number	Response, MPa	Average Response, MPa
C_1	1A 1B	19.32 19.47	19.40
C_2	2A 2B	19.65 19.59	19.62
C_3	3A 3B	18.89 19.03	18.96
C_4	4A 4B	16.18 15.78	15.98
C_5	5A 5B	20.32 19.55	19.94
C_{112}	6A 6B	22.00 22.22	22.11
C_{122}	7A 7B	22.30 22.34	22.32
C_{113}	8A 8B	25.28 25.43	25.36
C_{133}	9A 9B	27.39 27.46	27.43
C_{114}	10A 10B	20.12 20.23	20.18
C_{144}	11A 11B	24.87 25.49	25.18
C_{115}	12A 12B	22.08 22.24	22.16
C_{155}	13A 13B	17.88 18.11	18.00
C_{223}	14A 14B	24.23 24.52	24.38
C_{233}	15A 15B	18.56 18.65	18.61
C_{224}	16A 16B	21.44 21.54	21.49
C_{244}	17A 17B	23.44 23.75	23.60
C_{225}	18A 18B	19.02 18.88	18.95
C_{255}	19A 19B	21.21 20.79	21.00
C_{334}	20A 20B	18.65 19.22	18.94
C_{344}	21A 21B	26.32 26.46	26.39
C_{335}	22A 22B	19.33 19.31	19.32
C_{355}	23A 23B	22.47 22.46	22.47
C_{445}	24A 24B	20.21 20.24	20.23
C_{455}	25A 25B	23.08 22.76	22.92

C_{123}	26A 26B	23.11 23.35	23.23
C_{124}	27A 27B	23.75 23.43	23.59
C_{125}	28A 28B	22.48 22.62	22.55
C_{134}	29A 29B	19.42 19.53	19.48
C_{135}	30A 30B	24.37 24.35	21.36
C_{145}	31A 31B	21.86 21.84	21.85
C_{234}	32A 32B	22.94 22.76	22.85
C_{235}	33A 33B	20.56 20.62	20.59
C_{245}	34A 34B	16.89 17.36	17.13
C_{345}	35A 35B	19.86 19.83	19.85

4.3. SCHEFFE'S (5,3) REGRESSION MODEL FOR COMPRESSIVE STRENGTH OF PFRC

By substituting the values of the responses from Table 3 into Eqns. (11) through (28), the coefficients (in MPa) of the Scheffe's third degree polynomial were determined as follows:

$\beta_1 = 19.19; \beta_2 = 20.88; \beta_3 = 17.15; \beta_4 = 15.39; \beta_5 = 18.02; \beta_{12} = 9.32; \beta_{13} = 1.71; \beta_{14} = 22.66; \beta_{15} = 8.87; \beta_{23} = 6.14; \beta_{24} = 16.29; \beta_{25} = 0.54; \beta_{34} = 33.91; \beta_{35} = 12.38; \beta_{45} = 17.46; \gamma_{12} = 302.56; \gamma_{13} = 361.94; \gamma_{14} = 292.57; \gamma_{15} = 275.13; \gamma_{23} = 266.74; \gamma_{24} = 281.34; \gamma_{25} = 257.76; \gamma_{34} = 317.14; \gamma_{35} = 276.48; \gamma_{45} = 283.82; \beta_{123} = -181.09; \beta_{124} = -98.31; \beta_{125} = -153.61; \beta_{134} = -308.98; \beta_{135} = -99.58; \beta_{145} = -125.53; \beta_{234} = -211.64; \beta_{235} = -158.81; \beta_{245} = -167.07; \beta_{345} = -173.32$ **(33)**

Substituting the values of these coefficients in Eqn.(33) into Eqn. (9), we obtain the polynomial model for the optimization of the compressive strength of the concrete cubes made using nylon fibre (NFRC) based on Scheffe's (5,3) polynomial given in Eqn.(34).

$$Y = 19.19X_1 + 20.88X_2 + 17.18X_3 + 15.39X_4 + 18.02X_5 + 9.32X_1X_2 + 1.71X_1X_3 + 22.66X_1X_4 + 8.87X_1X_5 + 6.14X_2X_3 + 16.29X_2X_4 + 0.54X_2X_5 + 33.91X_3X_4 + 12.38X_3X_5 + 17.46X_4X_5 + 302.56X_1X_2^2 + 361.94X_1X_3^2 + 292.17X_1X_4^2 + 275.13X_1X_5^2 + 266.74X_2X_3^2 + 281.34X_2X_4^2 + 257.76X_2X_5^2 + 317.14X_3X_4^2 + 276.48X_3X_5^2 + 283.82X_4X_5^2 - 181.09X_1X_2X_3 - 98.31X_1X_2X_4 - 153.61X_1X_2X_5 - 308.98X_1X_3X_4 - 99.58X_1X_3X_5 - 125.53X_1X_4X_5 - 211.64X_2X_3X_4 - 158.81X_2X_3X_5 - 167.07X_2X_4X_5 - 173.32X_3X_4X_5 \quad \textbf{(34)}$$

Then, by substituting the pseudo mix ratio of points c_1, c_2, c_3, c_4, c_5, ……..c_{345} of Table 2 into Eqn.(34), we obtain the third degree model responses for the control points.

4.4. VALIDATION AND TEST OF ADEQUACY OF THE MODEL

Here, the Student's – T - test is adopted to check if there is any significant difference between the lab responses (compressive strength results) given in Table 4 and model responses from the control points based on Eqn.(34). The procedures for using the Student's – T - test have been explained by Nwachukwu and others (2022 c). The outcom of the test shows that there is no significant difference between the experimental results and model results. Thus, the model isvery adequate for predicting the compressive strength of NFRC based on Scheffe's(5,3) polynomial.

4.5. DISCUSSION OF RESULTS

The maximum NFRC compressive strength of 29.42MPa.corresponding to mix ratio of 0.56:1.00:1.37:1.80:0.80 for water, cement, fine aggregate, coarse aggregate and nylon fibre respectively was obtained through the Scheffe's third degree lattice. The lowest strength was found to be 15.39MPa corresponding to mix ratio of 0.70:1.00:1.00:1.80:1.20. The maximum strength value from the model was found to be greater than the minimum value specified by the American Concrete Institute for the compressive strength of good concrete. Using the model, compressive strength of NFRC of all points in the simplex can be determined based on third degree model.

5. CONCLUSION

In this work so far, Scheffe's Third Degree Polynomial Model, Scheffe's (5, 3) was used to predict the mix proportions as well as a model for predicting the compressive strength of NFRC cubes. Using Scheffe's (5, 3) simplex model, the values of the compressive strength were obtained for NFRC. As confirmed through the student's t-test, there is good correlation between the strengths predicted by the models and the corresponding experimentally observed results. The maximum attainable compressive strength of NFRC predicted by the Scheffe's (5, 3) model at the 28th day was 29.42MPa. This value is higher than the maximum value (21.96 MPa) obtained by Nwachukwu and others (2022d) for NFRC based on Scheffe's (5,2) model. However, both values meet the minimum standard requirement stipulated by American Concrete Institute (ACI) of 20MPa for the compressive strength of good concrete. With the model, any desired strength of Nylon Fibre Reinforced Concrete, given any mix proportions can be easily predicted and evaluated. Thus the problem of having to go through vigorous and laborious mix- design procedures to obtain a desiring strength of NFRC has been reduced by the utilization of this Scheffe's optimization model.

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