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Formulas for Surface Weighted Numbers on Graph

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ABSTRACT

The boundary value problem differential operator on the graph of a specific structure is discussed in this article. The graph has degree 1 vertices and edges that are linked at one common vertex. The differential operator expression with real-valued potentials, the Dirichlet boundary conditions, and the conventional matching requirements define the boundary value issue. There are a finite number of eigenvalues in this problem.The residues of the diagonal elements of the Weyl matrix in the eigenvalues are referred to as weight numbers. The eigenvalues are monomorphic functions with simple poles.The weight numbers under consideration generalize the weight numbers of differential operators on a finite interval, which are equal to the reciprocals of the squared norms of eigenfunctions. These numbers, along with the eigenvalues, serve as spectral data for unique operator reconstruction. The contour integration is used to obtain formulas for surfacethe weight numbers, as well as formulas for the sums in the case of superficial near eigenvalues. On the graphs, the formulas can be utilized to analyze inverse spectral problems.

KEYWORDS: *boundaryproblem, Formulas for Surface, weight numbers*

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1. INTRODUCTION

Wecconsider the graph Γ which consists of m edges e_j, $m \ge 2$, $j=\overline{1,m}$, joined at a common vertex. We let the graph Γ be parameterized so that $x_j \in [0,\pi]$ where the parameter x_j corresponds to the edge e_j, the parameter $x_j = 0$ in the boundary vertex and $x_j = \pi$ in the common vertex, $j=\overline{1,m}$. We call Γ a star-shaped graph. A vector function is a graph function.

$$y(x) = \left[y_j(x_j) \right]_{j=1}^{m},$$

Where the components $y_j(x_j)$ are functions on the edges e_j correspondingly $y_j(x_j) \in L_2[0,\pi],\ j=1,2,\ldots,m$.

Differentiation of the function $g(x)$ with respect to the first parameter is $g'(x)$ denoted. Consider the variation in expression.

$$Ly := -y_j''(x_j) + p_j(x_j)\, y_j(x_j), \qquad j=1,\ldots,m \quad (1)$$

The differential operators on the graph for the boundary value problem can therefore be represented as follows:

$$Ly = \lambda y(x) \qquad (2)$$

$$\sum_{i=1}^{m} y_i(0) = 0 \qquad (3)$$

$$-y_1'(\pi) = \sum_{j=2}^{m-1} y_j'(\pi) \qquad (4)$$

$$y_1(\pi) = y_2(\pi) = \ldots = y_m(\pi) \qquad (5)$$

where λ is The spectral parameter, the equalities (3), and the conventional matching criteria (4)–(5) are all Dirichlet conditions. In (1) the functions $p_j(x_j)$ are called potentials, $p_j(x_j) \in L_2^2[0,\pi],\ p_j(x_j) \in \mathbb{R}$ .The differential operator L, given by the differential expression (1) and the conditions (3)–(5), is self-adjoin in the corresponding Hilbert space (see [1] for details). The differential operators on graphs are intensively investigated because they have applications in physics, chemistry, and nanotechnology (see [2,3]). We develop formulas for

Surface weight numbers of the problem (2)–(5) in this paper which generalize the weight numbers on a finite interval. The inverse spectral issues for differential operators on graphs can be studied using these formulas for Surface. The potentials of the differential operators on graphs have been reconstructed using weight numbers and eigenvalues, for example, in [5,6]. When the eigenvalues are superficial close but not numerous, the situation becomes more complicated. The Surface formulas are obtained by integrating over the contours in the plain of the spectral parameter that contain the superficial close eigenvalues. As with the weight matrices for the matrix differential operator in [7], the Surface formulas are obtained for the sums of the weight numbers.

Objectives of this research

The goal of this study is to provide Surface formulas for the weight numbers of the boundary problem differential operator on a Star-shaped graph.

Methodology:

On a Star-shaped graph, a descriptive research project to focus on and discover the effect of differential equations on Surface for formulae weight numbers of the boundary problem differential operator. This research was advanced and completed using books, journals, and websites.

2. Basic instructions

In this section we introduce a characteristic function of the operator L, the zeros of which coincide with the eigenvalues. We also provide auxiliary results from [8, 9],related to the eigenvalues of L.

The conditions (4)–(5) can be written as follows:

$$Y(y) := Hy'(\pi) + hy'(\pi) = 0$$

where H and h are $m\times m$ matrices :

$$H = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}, \qquad h = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{pmatrix}$$

For each fixed $j = \overline{1,m}$ let $S_j(x,\lambda)$ and $C_j(x,\lambda)$ be the solutions of the Cauchy problems

$$-S_j''(x,\lambda) + p_j(x)S_j(x,\lambda) = \lambda S_j(x,\lambda), \quad S_j(0,\lambda) = S_j'(0,\lambda) - 1 = 0,$$

$$-C_j''(x,\lambda) + p_j(x)C_j(x,\lambda) = \lambda C_j(x,\lambda), \quad C_j(0,\lambda) - 1 = C_j'(0,\lambda) = 0.$$

The functions $S_j(x,\lambda)$, $C_j(x,\lambda)$ satisfy the Volterra integral equations

$$S_j(x,\lambda) = \frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}} + \int_0^x \frac{\sin\sqrt{\lambda}(x-t)}{\sqrt{\lambda}} p_j(t)S_j(t,\lambda)dt \tag{6}$$

$$C_j(x,\lambda) = \cos\sqrt{\lambda}x + \int_0^x \frac{\sin\sqrt{\lambda}(x-t)}{\sqrt{\lambda}} p_j(t)C_j(t,\lambda)dt \tag{7}$$

Put $\tau := \operatorname{Im}\lambda$ we can obtain the following surface formulas from (6),(7) as $\sqrt{\lambda}\to\infty$:

$$S_j(x,\lambda) = \frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}} + \int_0^x \frac{\sin\sqrt{\lambda}(x-t)}{\lambda} \operatorname{Sin}\sqrt{\lambda}t\, p_j(t)dt + \int_0^x\int_0^t \frac{\sin\sqrt{\lambda}(x-t)p_j(t)}{\lambda\sqrt{\lambda}} \sin\sqrt{\lambda}(t-\xi)p_j(\xi)\sin\sqrt{\lambda}\xi d\xi\, dt + O\left(\frac{e^{|\tau|x}}{\lambda^2}\right) \tag{8}$$

$$S_j'(x,\lambda) = \cos\sqrt{\lambda}x + \int_0^x \frac{\cos\sqrt{\lambda}(x-t)}{\sqrt{\lambda}} \operatorname{Sin}\sqrt{\lambda}t\, p_j(t)dt + \int_0^x\int_0^t \frac{\cos\sqrt{\lambda}(x-t)p_j(t)}{\lambda} \sin\sqrt{\lambda}(t-\xi)p_j(\xi)\sin\sqrt{\lambda}\xi d\xi\, dt + O\left(\frac{e^{|\tau|x}}{\lambda\sqrt{\lambda}}\right) \tag{9}$$

$$C_j(x,\lambda)=\cos\sqrt{\lambda}x+\int_0^x \frac{\sin\sqrt{\lambda}(x-t)}{\sqrt{\lambda}}\cos\sqrt{\lambda}t\, p_j(t)dt+$$
$$+\int_0^x\int_0^t \frac{\sin\sqrt{\lambda}(x-t)p_j(t)}{\lambda}\sin\sqrt{\lambda}(t-\xi)p_j(\xi)\cos\sqrt{\lambda}\xi d\xi\, dt+O\left(\frac{e^{|\tau|x}}{\lambda\sqrt{\lambda}}\right) \quad (10)$$

$$C_j'(x,\lambda)=-\sqrt{\lambda}\sin\sqrt{\lambda}x+\int_0^x \cos\sqrt{\lambda}(x-t)\cos\sqrt{\lambda}t\, p_j(t)dt+$$
$$+\int_0^x\int_0^t \frac{\sin\sqrt{\lambda}(x-t)p_j(t)}{\sqrt{\lambda}}\sin\sqrt{\lambda}(t-\xi)p_j(\xi)\cos\sqrt{\lambda}\xi d\xi\, dt+O\left(\frac{e^{|\tau|x}}{\lambda\sqrt{\lambda}}\right) \quad (11)$$

We introduce matrix solutions of equation (2): $S(\lambda)=diag\left\{S_j(x_j,\lambda)\right\},\ j=1,2,...,m$ and $C(\lambda)=diag\left\{C_j(x_j,\lambda)\right\},\ j=1,2,...,m$. Every eigenvalue of problem (2)–(5) corresponds to the zero of the following characteristic function $\Delta(\lambda)$:

$$\Delta(\lambda):=\det Y(S(\lambda)) \quad (12)$$

As $S_j(\pi,\lambda)$, $S_j'(\pi,\lambda)$ are entire functions of λ, the function $\Delta(\lambda)$ is also entire. Recon-struttingthe determinant in (12), we obtain

$$\Delta(\lambda)=\sum_{k=1}^{m}\left(S_k'(\pi,\lambda)\prod_{\substack{j=1\\ j\neq k}}^{m}S_j(\pi,\lambda)\right) \quad (13)$$

Lemma 1. The number λ_0 is an eigenvalue of problem (2)–(5) of multiplicity k ifand only if λ_0 is a zero of characteristic function of multiplicity k. The statement of the Lemma 1 results from the self-adroitness of L and is proved with the same technique as in [7, Lemma 3]. From the self-adroitness of L it also follows that the eigenvalues of the boundaryproblem (2)–(5) are real.

Denote $w_j=\frac{1}{2}\int_0^\pi p_j(t)dt,\ f(z)=\prod_{j=1}^m(z-w_j)$. Let $z^{(j)},\ j=\overline{1,m-1}$ be the zeros of $f'(z),\ z^{(m)}=\sum_{j=1}^m\frac{w_j}{m}$. We will mean by $\left\{\kappa_n\right\}_{n=1}^\infty$ different sequences from l^2.

3. Results obtained

We define and investigate weight numbers based on the Weyl matrix in this paper.Let $\Phi(\lambda)=\left\{\phi_{jk}(x_j,\lambda)\right\}_{j,k=1}^m$ be the matrix solution of (2) under the conditions $\left\{\phi_{jk}(0,\lambda)\right\}_{j,k=1}^m=I,\ Y(\Phi)=0$. The matrix $M(\lambda)=\left\{-\phi_{jk}'(0,\lambda)\right\}_{j,k=1}^m$ is called the Weylmatrix and generalize the notion of the Weyl function for differential operators on intervals (see [4]). Natural spectral characteristics, such as Weyl functions and their generalizations, are frequently employed for operator reconstruction. A system of $2m$ columns of the matrix solutions $C(\lambda),\ S(\lambda)$ is fundamental, and one can show, that

$$M(\lambda)=\left(Y(S(\lambda))^{-1}Y(C(\lambda))\right) \quad (14)$$

In view of (16) the elements of the matrix $M(\lambda)=\left\{M_{k,l}(\lambda)\right\}_{k,l=1}^m$ can be calculated as

$$M_{k,l}(\lambda)=\frac{1}{\Delta(\lambda)}\left(\prod_{\substack{j=1\\ j\neq k}}^{m}S_j(x,\lambda),\ C_l(x,\lambda)\right)'\Bigg|_{x=\pi} \quad (15)$$

The elements of the matrix $M(\lambda)$ are monomorphicfunctions, and their poles may be only zeros of the characteristic function $\Delta(\lambda)$.Moreover, analogously to [7, Lemma 3], we prove the following lemma:

Lemma 2. If the number λ_0 is a pole of $M_{kl}(\lambda)$,this pole is simple.

Proof. Let λ_0 be a zero of $\Delta(\lambda)$ of multiplicity b. there are exactly b linearly independent eigenfunctions $\{y_j(x_j)\}_{j=1}^{b}$.corresponding to λ_0 Denote by K such invertible matrix that first b columns of $S(\lambda_0)$ K are equal to $\{y_j(x_j)\}_{j=1}^{b}$.

If $X(\lambda)=S(\lambda)K$, then $S(\lambda)=X(\lambda)K^{-1}$, and $M(\lambda)=K\left[Y(X(\lambda))\right]^{-1}Y(C(\lambda))$ .It is sufficient to prove that for any element of $A(\lambda)$ the number λ_0 cannot be a pole of order greater than 1, where $A(\lambda)=\left[Y(X(\lambda))\right]^{-1}Y(C(\lambda))$. If $A(\lambda)=\{A_{sl}(\lambda)\}_{s,l=1}^{m}$, then

$$A_{sl}(\lambda)=\frac{\det\left[Y(X_1(\lambda)),Y(X_2(\lambda)),\dots,Y(X_{s-1}(\lambda)),Y(C_l(\lambda)),Y(X_{s+1}(\lambda)),\dots,Y(X_m(\lambda))\right]}{\det Y(X(\lambda))}$$

The number λ_0 is zero of the numerator of multiplicity not less than $b-1$ from that the statement of the theorem follows.

We introduce the constants $\alpha_{jn}^{k}=\operatorname*{Res}_{\lambda=\lambda_n^{(j)}} M_{kk}(\lambda)$ which are called weight numbers. We also mean by $\{\kappa_n(z)\}_{n=1}^{\infty}$ different sequences of continuous functions such as:

$$\sum_{n=1}^{\infty}\max_{|z|\le R}\left|\kappa_n(z)\right|^2<\infty$$

where

$$R=2+\max_{s=\overline{1,m}}\left|z^{(s)}\right|$$

The following two theorems summarize the paper's primary findings.

Theorem 1. Let the eigenvalues of L be enumerated as in theorem 1, $k=\overline{1,m}$ then

$$\sum_{j\in I(n)}\alpha_{jn}^{k}=\frac{2n^2}{m\pi}(m-1+\frac{\kappa_n}{n}) \quad (16)$$

$$\alpha_{ms}^{k}=\frac{(n-\frac{1}{2})^2}{m\pi}(2+\frac{\kappa_n}{n}) \quad (17)$$

Where

$$I(n)=\bigcup_{j=1}^{m-1}\left\{\min\{s:\lambda_n^s=\lambda_n^{(j)}\}\right\}.$$

Proof. To prove the theorem, consider $\lambda_n(z)=n+\frac{z}{n\pi},|z|\le R$ Substituting $\lambda=\lambda_n(z)$ into (8)–(11), we obtain

$$S_j(\pi,\lambda_n^2(z))=\frac{(-1)^n}{n\lambda_n(z)}\left(z-\tilde{\omega}_{jn}+\frac{\kappa_n(z)}{n}\right),\quad \tilde{\omega}_{jn}=\omega_j-\hat{q}_j(2n) \quad (18)$$

$$S_j'(\pi,\lambda_n^2(z))=(-1)^n\left(1+\frac{\kappa_n(z)}{n}\right) \quad (19)$$

$$C_j(\pi,\lambda_n^2(z))=(-1)^n\left(1+\frac{\kappa_n(z)}{n}\right) \quad (20)$$

$$C_j'(\pi,\lambda_n^2(z))=\frac{(-1)^n\lambda_n(z)}{n}\left(z-\tilde{\omega}_{jn}+\frac{\kappa_n(z)}{n}\right),\quad \tilde{\omega}_{jn}=\omega_j-\hat{q}_j(2n) \quad (21)$$

Where $\tilde{p}_j(l)=\frac{1}{2}\int_0^{\pi} p_j(t)\cos ltdt$ We substitute (18)–(21) into (13), (15) and get

$$\Delta(\lambda_n^2(z))=\frac{(-1)^{nm}}{n^{m-1}\lambda_n^{m-1}(z)}\left(\sum_{s=1}^{m}\prod_{\substack{j=1\\ j\neq s}}^{m}(z-\tilde{\omega}_{jn})+\frac{\kappa_n(z)}{n}\right) \tag{22}$$

$$M_{kk}(\lambda_n^2(z))\Delta(\lambda_n^2(z))=\frac{(-1)^{nm}}{n^{m-2}\lambda_n^{m-2}(z)}\left(\sum_{\substack{s=1\\ s\neq k}}^{m}\prod_{\substack{j=1\\ j\neq s, j\neq k}}^{m}(z-\tilde{\omega}_{jn})+\frac{\kappa_n(z)}{n}\right) \tag{23}$$

Let us denote $f_n(z)=\prod_{j=1}^{m}\left(z-\tilde{\omega}_{jn}\right)$, $\delta(r)$ is the circle of center 0 and radius $r>0$.

It can be proved that $\tilde{z}_n^{(j)}=z^{(j)}+O(1), n\to\infty$, where $\tilde{z}_n^{(j)}$, $j=1,2,3,...,m-1$ are the zeros of $f_n'(z)$ if $z\in\delta(R)$, then for sufficiently large n, $\lambda_n^2(z)$ runs across the simple closed contour, which surrounds $\lambda_n^{(j)}$, $j=1,2,3,...,m-1$ Integrating $M_{kk}(\lambda)$, after the substitution $\lambda=\lambda_n^2(z)$ we have

$$\sum_{l\in I(n)}\alpha_{jn}^k=\frac{1}{2\pi i}\int_{z\in\delta(R)}\frac{2\lambda_n(z)}{n\pi}M_{kk}(\lambda_n^2(z))dz.$$

The following formula is obtained from the previous one and (22), (23):

$$\sum_{l\in I(n)}\alpha_{jn}^k=\frac{1}{2\pi i}\int_{z\in\delta(R)}\frac{2\lambda_n^2(z)}{m\pi}\frac{\sum_{\substack{s=1\\ s\neq k}}^{m}\prod_{\substack{j=1\\ j\neq s, j\neq k}}^{m}\left(z-\tilde{\omega}_{jn}\right)+\frac{\kappa_n(z)}{n}}{\prod_{j=1}^{m-1}\left(z-\tilde{z}_n^{(j)}\right)+\frac{\kappa_n(z)}{n}}dz \tag{24}$$

The remainder $\frac{\kappa_n(z)}{n}$ can be excluded from the denominator of (24) with Taylor expansion as $\min_{|z|=R}\prod_{j=1}^{m-1}\left|z-\tilde{z}_n^{(j)}\right|>1$ if n is large enough. Besides, $\lambda_n^2(z)=n^2\left(1+\frac{\kappa_n(z)}{n}\right), |z|\leq R$ after the designation

$$g_{kn}(z)=\frac{\sum_{\substack{s=1\\ s\neq k}}^{m}\prod_{\substack{j=1\\ j\neq s, j\neq k}}^{m}\left(z-\tilde{\omega}_{jn}\right)}{\prod_{j=1}^{m-1}\left(z-\tilde{z}_n^{(j)}\right)}$$

we get

$$\sum_{l\in I(n)}\alpha_{jn}^k=\frac{2n^2}{2m\pi^2 i}\left(\int_{z\in\delta(R)}g_{kn}(z)dz+\frac{\kappa_n}{n}\right) \tag{25}$$

We note that $\delta(r)$ contains all $\tilde{z}_n^{(j)}$, $j=1,2,3,...,m-1$ for $r\geq R$ and large n. Thus,

$$\int_{z\in\delta(R)}g_{kn}(z)dz=\int_{z\in\delta(r)}g_{kn}(z)dz$$

the numerator of the fraction $g_{kn}(z)$ is a polynomial of degree $m-2$ with leading coefficient $m-1$, and its denominator is a polynomial of degree $m-1$ with leading coefficient 1. For $z\in\delta(r)$ there is the equality

$g_{kn}(z)=\frac{m-1}{z}+O(r^{-2})$, and

$$\frac{1}{2\pi i}\int_{z\in\delta(r)}g_{kn}(z)=m-1+O(r^{-1}).$$

As $r\to\infty$ we obtain (16). Formula (17) is proved analogously.

Theorem 2. Let $z^{(s)}$ be a zero of $f'(z)$ of multiplicity $b(s)>0,\ 1\le t\le m$ Denote $N(s)=\{1\le j<m: z^{(s)}\ne z^{(j)}\}$, $N'(s)=\{1\le j<m: z^{(s)}= z^{(j)}\}$, and $W(s)=\{1\le j<m: z^{(s)}\ne \omega_j\}$ if $t\in W(s)$, then

$$\sum_{l\in N'(s)} \alpha_{\ln}^t = \frac{2n^2}{m\pi}(\Omega_{ts}+\kappa_n) \tag{26}$$

Else

$$\sum_{l\in N'(s)} \alpha_{\ln}^t = \frac{2n^2}{m\pi}(\theta_s+\kappa_n) \tag{27}$$

Where

$$\Omega_{ts} = -\frac{\prod_{j=1}^{m}(z^{(s)}-\omega_j)}{(z^{(s)}-\omega_j)^2\prod_{j\in N(s)}(z^{(s)}-z^{(j)})}, \qquad \theta_s = b(s)\frac{\prod_{j\in N(s)}(z^{(s)}-\omega_j)}{\prod_{j\in N(s)}(z^{(s)}-z^{(j)})}$$

and the product over empty set is understood as 1.

Proof. Denote by r such positive number that the circle $|z-z^{(s)}|\le r$ does notcontain $z^{(j)},\ j\in N(s)$ and $|z^{(s)}|+r<R,\ r\ge C>0$ we call the circumference of thatcircle $\gamma(s)$ the following analogue of the formulae (25) can be proved:

$$\sum_{l\in N'(n)} \alpha_{ln}^t = \frac{n^2}{m^2\pi^2 i}\left(\int_{\gamma(s)} \frac{\sum_{\substack{k=1\\k\ne t}}^{m}\prod_{\substack{j=1\\ j\ne k,\, j\ne t}}^{m}\left(z-\tilde{\omega}_{jn}\right)}{\prod_{j=1}^{m-1}\left(z-\tilde{z}_n^{(j)}\right)}dz+\frac{\kappa_n}{n}\right) \tag{28}$$

We designate

$$F_t(z)=\frac{\sum_{\substack{k=1\\k\ne t}}^{m}\prod_{\substack{j=1,\\ j\ne k,\, j\ne t}}^{m}(z-\omega_n)}{\prod_{j=1}^{m}(z-z^{(j)})}$$

As $\omega_j-\tilde{\omega}_{jn}=\kappa_n$ and the coefficients of $f'(z), f_n'(z)$ depend on $\{\omega_j\}_{j=1}^{m}, \{\tilde{\omega}_{jn}\}_{j=1}^{m}$ polynomially, we have

$$\frac{\sum_{\substack{k=1\\k\ne t}}^{m}\prod_{\substack{j=1,\\ j\ne k,\, j\ne t}}^{m}(z-\tilde{\omega}_{jn})}{\prod_{j=1}^{m}(z-\tilde{z}_n^{(j)})}-F_t(z)=\kappa_n(z) \tag{29}$$

Where $z\in\gamma(s)$ We integrate the fraction $F_t(z)$.

First we consider $b(s)>1$. Then $z^{(s)}$ is a zero of $f(z)$ of multiplicity $b(s)+1$ and cardinality of $W(s)$ is $m-b(s)-1$ in the case when $p\in W(s)$ the function $F_p(z)$ has no pole inside $\gamma(s)$, and $\alpha_{sn}^p=\frac{2n^2}{m\pi}\kappa_n$, what is the same as (26). If $p\notin W(s)$ then

$$F_t(z) = \frac{b(s)\left(z-z^{(s)}\right)^{b(s)-1}\prod_{j\in W(s)}\left(z-\omega_j\right)+\left(z-z^{(s)}\right)^{b(s)}\sum_{k\in W(s)}\prod_{\substack{j\in W(s)\\ j\neq k}}\left(z-\omega_j\right)}{\left(z-z^{(s)}\right)^{b(s)}\prod_{j\in N(s)}\left(z-z^{(j)}\right)}$$

and

$$\int_{\gamma(s)} F_t(z)dz = \frac{b(s)\prod_{j\in W(s)}\left(z^{(s)}-\omega_j\right)}{\prod_{j\in N(s)}\left(z^{(s)}-z^{(j)}\right)} \qquad (30)$$

formula (29) follows from (28)–(30).

Further, let $b(s)=1$ When $z^{(s)}$ is a zero of $f(z)$, computations are the same as in the case $b(s)>1$ so we assume $f(z^{(s)})\neq 0$, and consequently $p\in W(s)$. Rewriting $F_p(z)$ as

$$F_t(z) = \left(\frac{f(z)}{z-\omega_t}\right)'\left(f'(z)\right)^{-1} = \frac{1}{z-\omega_t} - \frac{f(z)}{(z-\omega_t)^2 f'(z)}$$

and integrating over $\gamma(s)$, we obtain (27).

4. Conclusion

This article is divided into three parts as a consequence of the research. The first section comprises an introduction, the second section covers preliminaries, and the third section contains the proofs of the second and third theorems, as well as the justification of the approach for extracting two-point boundary value issues from finite text. On a Star-shaped graph, a set of eigenvalues of the asymptotic formula for boundary condition coefficients and formulas for Surface weight numbers of the boundary problem differential operator. Furthermore, the word weight numbers can be stated to be taken into account. In some years, these are the leftovers of the Weyl matrix's oblique elements.These are well-known functions with simple poles that can only have a limited set of attributes. The assumed weight numbers were generalized to the weight numbers of differential operators over a finite time period, equivalent to the reciprocal of the particular squared norms. For the unique reconstruction of operators, these values, coupled with particular properties, serve as spectral data. We find the unbalanced duct for the weight numbers using contour integration, and fours for the values in the case of closely spaced free. Finally, in graphs, formulas can be employed to assess inverse spectral.

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