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Analytical Iteration Method Applied to a Class of First Order Nonlinear Evolution Equations in Science

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ABSTRACT

In this research paper, we apply the novel Temimi-Ansari method to six first order nonlinear partial differential equations for exact solution namely: Burger's equation, Fisher's equation, Schrodinger equation, wave equation, advection equation and KDV equations respectively. Unlike other semi-analytical iterative methods, this method doesn't require linearization, perturbation, discretization, or the calculation of an Adomian polynomials for nonlinear terms in the Adomian decomposition method (ADM). It gives the closed form solution of the problem if it exists in finite steps of a converging series that's computationally convenient, easy to obtained and elegant. It solves the inherent problem of dealing with the nonlinear term in a straightforward way without stress. The result obtained revealed, all the chosen problems give rise to their closed form solution in simple steps which confirmed the method is powerful, reliable and has wide applicability to other nonlinear problems.

KEYWORDS: *KDV Equation, Advection equation, Schrödinger equation, Burger's equation, Fisher's equation, Temimi-Ansari method*

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INTRODUCTION

Nonlinear partial differential equations technically called evolution equations are equations that constitute the dissipative term and partial derivative of the dependent variable with respect to one or more of the independent variables. These equations feature prominently in most physical phenomena especially in the physical and applied sciences (Wazwaz, 2009). They modelled phenomena in the field of sciences, engineering, Biology, hydrodynamics, chemistry, physics, optical fibre, chemical kinetics, plasma physics, medicine and may others, Bekir, Tascan and Unsad (2015), (Feng, 2002). (Ebiwareme, 2021), Ebiwareme and Ndu (2021), (Liberty, 2021). Academics have devoted time to extensively studied these equations for analytical or approximate solution owing to their importance. All though most of the equations have closed form solutions, whereas others have solution which are difficult to obtained however due to the advent of portable computers, the solutions can be obtained with less computations. Some of the methods proposed to study these equations include: Tanh-Coth method, Asokan and Vinodh (2017), (Wazwaz, 2008), (Wazwaz, 2009), Chebyshev collocation method Gupta and Saha (2015), Vaganan and Asokan (2003), Direct similarity method, Manafian, Mahrdad and Bekir (2016), G/G' expansion method, Fan and Hongqing (1998), Homogenous balance method, (Wazwaz, 2007),extended tanh method, Fan and Zhao (2000), Gomez and Salas (2010), Variational Iteration method, Nourazr, Mohsen, Nazari-Golshan (2015), Homotopy perturbation method, Hashim, Norami and Al-Hadja (2021),Adomian decomposition method, Necdet and Konuralp (2006), Darvishi, Kheybari and Khani (2008),spectral collocation method, Khatter and Temsah (2017), linear superposition method, Ma and Fan (2011),trial equation method, Gurefe, Sonmezoglu and Misirli (2011),Fractional sub-equation method, Zhang and Zhang (2011),First Integral Method, Zhang, Zhong, Shashan, Liu, Feng and Gao (2013), (Feng, 2013), Ebiwareme and Ndu (2021), Banach contraction method, (Ebiwareme, 2021).

Most recently, Temimi and Ansari proposed a semi-analytical iteration method called (TAM) to successfully solve to linear and nonlinear functional problems. Unlike the ADM, HAM, VIM and HPM, this method solved the difficulties that often arise for nonlinear terms by writing it in the form of Adomian polynomials (ADM), construction of an Homotopy (HAM), calculation of the Lagrange multiplier (VIM) and tedious calculation of an algebraic calculation from corresponding terms as in HPM are overcome. TAM has been successfully applied to solve various problem, such as ODEs Azeez and Weli (2017), Duffing equation, Al-Jawary and Al-Razzaq (2016),Chemistry problem, Al-Jawary and Raham (2016), Nonlinear ODEs, Temimi and Ansari (2015), Thin film flow problem, (Al-Jawary, 2017),second order multi-point boundary value problems, Al-Jawary, Radhi and Ravnik (2017), Fokker-Planck's equation, Temimi and Ansari (2011), Linear and nonlinear ODEs, Azeez and Weli (2017),Newell-White-head equation, Latif, Salim, Nasreen, Alifah and Munirah (2020).

In this present article, our motivation is to apply this novel semi-analytical iteration method to solve six differentials nonlinear PDEs. To confirm the accuracy, reliability, robustness, and efficiency of this method, we seek to ascertain whether a closed form or analytical or an approximate solution will result from this method. The study is organized as follows. In section, the introduction of the study detailing PDEs and their applications in physical phenomena together with methods used hitherto to seek for their solutions. The fundamentals of the novel Temimi-Ansari method and the condition for its convergence is presented in section 2. In section 3, we apply the AIM to six different nonlinear PDEs and seek closed form solutions and finally the conclusion of the study is given in section 4.

BASICS OF THE ANALYTICAL ITERATION METHOD (AIM)

Consider the general functional differential equation in operator form as follows

$$L\big(u(x)\big) + N\big(u(x)\big) + f(x) = 0, \quad (1)$$

$$B\left(u, \frac{du}{dx}\right) = 0, \text{ or } u_1(0) = a \text{ and } u_1'(0) = b \quad (2)$$

Where x is the independent variable, $u(x)$ is an unknown function, $f(x)$ is a given known function, L is a linear operator, N is a nonlinear operator and B is a boundary operator.

To implement the TAM method, we first assume that$u_0(x)$ is an initial guess that satisfy the problem in Eq. (1) subject to Eq. (2).

$$L\big(u_0(x)\big) + f(x) = 0, B\left(u_0, \frac{du_0}{dx}\right) = 0\text{or} \quad u_0(0) = a \text{ and } u_0'(0) = b \quad (3)$$

The next approximate solution is obtained by solving the problem

$$L\big(u_1(x)\big) + N\big(u_0(x)\big) + f(x) = 0, B\left(u_1, \frac{du_1}{dx}\right) = 0, \text{ or } u_1(0) = a \text{ and } u_1'(0) = b \quad (4)$$

The next iterate of the problem become

$$L\big(u_2(x)\big) + N\big(u_1(x)\big) + f(x) = 0, B\left(u_2, \frac{du_1}{dx}\right) = 0, \text{ or } u_2(0) = a \text{ and } u_2'(0) = b \quad (5)$$

Continuing the same way to obtain the subsequent terms, the general equation of the method becomes

$$L\big(u_{n+1}(x)\big) + N\big(u_n(x)\big) + f(x) = 0, B\left(u_{n+1}, \frac{du_{n+1}}{dx}\right) = 0, \quad \text{or} \quad u_{n+1}(0) = a \quad \text{and} \quad u_{n+1}'(0) = b$$

Then the solution of the problem in Eq. (11) is given by

$$u(x) = \lim_{n\to\infty} u_n(x) \quad (6)$$

From Eq. (6), each $y(x)$ is considered alone as a solution for Eq. (1). This method easy to implement, straightforward and direct. The method gives better approximate solution which converges to the exact solution with only few members.

NUMERICAL APPLICATIONS

In this section, we apply the AIM to solve six Partial differential equations that finds usual applications in Science and Engineering. They include Fishers' equation, wave equation, advection equation, Korteweg-Devries equation (KDV), Burger's equation and Schrödinger equation respectively.

Example 1. Consider the one-dimensional Burger's equation of the form

$$u_t + uu_x - u_{xx} = 0 \quad (7)$$

Subject to the initial condition

$$u(x,0) = 2x \quad (8)$$

Applying TAM to both sides of the equation, we get

$$L(u) = u_t, N(u) = uu_x - u_{xx}, f(x,t) = 0$$

The initial problem to be solved is of the form

$$L\big(u_0(x,t)\big) + f(x,t) = 0, u_0(x,t) = 2x \quad (9)$$

Integrating both sides of the above equation subject to the initial condition, we get the initial solution as

$$u_0(x,t) = 2x$$

The second iterative solution is obtained using the equation

$$L(u_1(x,t)) + N(u_0(x,t)) + f(x,t) = 0, u_1(x,t) = 2x \quad (10)$$

Integrating both sides of the above using the initial condition yield the integral

$$\int_0^t u_{1t}(x,t)dt = \int_0^t (u_{0xx} - u_0 u_{0x})dt$$

Solving the above integral yield, the second iterative solution as

$$u_1(x,t) = 2x - 4xt \quad (11)$$

The next iterate of the problem is given by

$$L(u_2(x,t)) + N(u_1(x,t)) + f(x,t) = 0, u_2(x,t) = 2x$$

Taking the inverse operator of both sides of the above yield

$$\int_0^t u_{2t}(x,t)dt = \int_0^t (u_{1xx} - u_1 u_{1x})dt \quad (12)$$

Plugging in the derivatives and evaluating, we obtain the third iterate as

$$u_2(x,t) = 2x - 4xt + 8t^2 x - \frac{16}{3} t^3 x \quad (13)$$

Similarly, the next iterate is obtained with the problem

$$L(u_3(x,t)) + N(u_2(x,t)) + f(x,t) = 0, u_3(x,t) = 2x \quad (14)$$

Integrating both sides using the initial conditions, we get the fourth iterate as follows

$$\int_0^t u_{3t}(x,t)dt = \int_0^t (u_{2xx} - u_2 u_{2x})dt \quad (15)$$

Substituting the derivatives, we obtain the solution as

$$u_3(x,t) = 2x - 4xt + 8xt^2 - 16xt^3 + \frac{64}{3}xt^4 - \frac{64}{3}xt^5 + \frac{128}{9}xt^6 - \cdots \quad (16)$$

Using the relation, $u(x,t) = \lim_{n\to\infty} u_n(x,t)$, the closed form solution of the problem is obtained

$$u(x,t) = 2x(1 - 2t + 4t^2 - 8t^3 + \cdots)$$

$$u(x,t) = \frac{2x}{1+2t} \quad (17)$$

Example 2. Consider the Fisher's equation as follows

$$u_t = u_{xx} + u(1-u) \quad (18)$$

Subject to the initial condition

$$u(x,0) = \alpha \quad (19)$$

To implement TAM, we have the following

$$L(u) = u_t, N(u) = -(u_{xx} + u(1-u)), f(x,t) = 0$$

The first problem to be solved is given by the equation

$$L(u_0(x,t)) + f(x,t) = 0, u_0(x,t) = \alpha \quad (20)$$

Taking the inverse operator of both sides yield the first iterative solution as

$$u_0(x,t) = \alpha$$

The second iterative solution is obtained using the relation

$$L(u_1(x,t)) + N(u_0(x,t)) + f(x,t) = 0, u_1(x,t) = \alpha \quad (21)$$

Integrating both sides of the above using the initial condition yield the integral

$$\int_0^t u_{1t}(x,t)dt = \int_0^t (u_{0xx} - u_0 u_{0x})dt$$

Solving the above integral yield, the second iterative solution as

$$u_1(x,t) = \alpha + \alpha(1-\alpha)t \quad (22)$$

The third iterate is solved using the relation

$$L(u_2(x,t)) + N(u_1(x,t)) + f(x,t) = 0, u_2(x,t) = \alpha$$

Taking the inverse operator of both sides using the initial condition, we get

$$\int_0^t u_{2t}(x,t)dt = \int_0^t (u_{1xx} - u_1 u_{1x})dt \quad (23)$$

Solving the above after plugging in the derivatives, we get the solution as

$$u_2(x,t) = \alpha + \alpha(1-\alpha)t + \alpha(1-\alpha)\alpha(1-2\alpha)\frac{t^2}{2!} + \left[-\alpha^2(1-\alpha)^2\frac{t^3}{3!}\right] \quad (24)$$

The fourth iterative solution is obtained using the problem

$$L(u_3(x,t)) + N(u_2(x,t)) + f(x,t) = 0, u_3(x,t) = \alpha \quad (25)$$

Integrating both sides using the given initial conditions, we obtained the integral as follows

$$\int_0^t u_{3t}(x,t)dt = \int_0^t (u_{2xx} - u_2 u_{2x})dt \quad (26)$$

Solving the integral above yield the iterative solution below.

$$u_3(x,t) = \alpha + \alpha(1-\alpha)t + \alpha(1-\alpha)\alpha(1-2\alpha)\frac{t^2}{2!} + \alpha(1-\alpha)(1-6\alpha+6\alpha^2)\frac{t^3}{3!} + \cdots \quad (27)$$

Continuing in similar fashion the subsequent terms will be determined, hence the solution of the problem is obtained using the relation

$$u(x,t) = \lim_{n\to\infty} u_n(x,t)$$

$$u(x,t) = \alpha + \alpha(1-\alpha)t + \alpha(1-\alpha)\alpha(1-2\alpha)\frac{t^2}{2!} + \alpha(1-\alpha)(1-6\alpha+6\alpha^2)\frac{t^3}{3!} + \cdots$$

$$u(x,t) = \frac{\alpha e^t}{1-\alpha+\alpha e^t} \quad (28)$$

Example 3. Let's consider the first-order wave equation in one-dimension as follows

$$u_t + ku_{xx} = 0, k > 0 \quad (29)$$

Subject to the initial condition

$$u(x,0) = \sin\left(\frac{\pi x}{l}\right), u_x(0,t) = \frac{x}{l}\cos\left(\frac{-k\pi t}{l}\right)$$
$$u(0,t) = \sin\left(\frac{-k\pi t}{l}\right), u_t(x,0) = \frac{-k\pi}{l}\cos\left(\frac{\pi x}{l}\right) \quad (30)$$

Applying TAM to both sides, we get the following terms

$$L(u) = u_t, N(u) = ku_{xx}, u_0(x,0) = \sin\left(\frac{\pi x}{l}\right)$$

The first iterate is obtained by solving the problem of the form

$$L\big(u_0(x,t)\big) + f(x,t) = 0, u_0(x,t) = \sin\left(\frac{\pi x}{l}\right) \quad (31)$$

Integrating both sides subject to the initial condition, we get the initial solution as

$$u_0(x,t) = \sin\left(\frac{\pi x}{l}\right)$$

The second iterative solution is obtained by solving the problem

$$L\big(u_1(x,t)\big) + N\big(u_0(x,t)\big) + f(x,t) = 0, u_1(x,t) = \sin\left(\frac{\pi x}{l}\right) \quad (32)$$

Integrating both sides of the above using the initial condition yield the integral below

$$\int_0^t u_{1t}(x,t)dt = \int_0^t (u_{0xx} - u_0u_{0x})dt \quad (33)$$

Solving the above integral yield, the second iterative solution as

$$u_1(x,t) = \left[\sin\left(\frac{\pi x}{l}\right) - \cos\left(\frac{\pi x}{l}\right)\right]\left(\frac{k\pi t}{l}\right) \quad (34)$$

The next iterate of the problem is given by

$$L\big(u_2(x,t)\big) + N\big(u_1(x,t)\big) + f(x,t) = 0, u_2(x,t) = \sin\left(\frac{\pi x}{l}\right)$$

Taking the inverse operator of both sides of the above yield

$$\int_0^t u_{2t}(x,t)dt = \int_0^t (u_{1xx} - u_1u_{1x})dt \quad (35)$$

Plugging in the derivatives and evaluating, we obtain the third iterate as

$$u_2(x,t) = \sin\left(\frac{\pi x}{l}\right)\left[1 - \frac{1}{2!}\left(\frac{k\pi t}{l}\right)^2\right] - \cos\left(\frac{\pi x}{l}\right)\left[\frac{k\pi t}{l} - \frac{1}{3!}\left(\frac{k\pi t}{l}\right)^3\right] \quad (36)$$

Continuing in the same way, the succeeding terms are obtained, and the closed form solution of the problem is given by

$$u(x,t) = \sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{k\pi t}{l}\right) - \cos\left(\frac{\pi x}{l}\right)\sin\left(\frac{k\pi t}{l}\right)$$

Using trigonometric identity, the above reduced to the form

$$u(x,t) = \sin\left[\pi\left(\frac{x-kt}{l}\right)\right] \quad (37)$$

Example 4. Let's consider the homogenous advection equation of the form

$$u_t + uu_x = 0 \quad (38)$$

Subject to the initial condition

$$u(x,0) = -x \quad (39)$$

Implementing TAM on both sides of the equation, we have the following terms

$$L(u) = u_t, N(u) = uu_x, f(x,t) = 0$$

The first problem to be solved for the first iterate is given by

$$L\big(u_0(x,t)\big) + f(x,t) = 0, u_0(x,t) = -x \quad (40)$$

Integrating both sides of the above equation subject to the initial condition, we get the initial solution as

$$u_0(x,t) = -x \quad (41)$$

The second iterative solution is obtained using the equation

$$L\big(u_1(x,t)\big) + N\big(u_0(x,t)\big) + f(x,t) = 0, u_1(x,t) = -x \quad (42)$$

Integrating both sides of the above using the initial condition yield the integral equation

$$\int_0^t u_{1t}(x,t)dt = -\int_0^t (u_0u_{0x})dt$$

Solving the above integral yield, the second iterative solution as

$$u_1(x,t) = -x - xt \quad (43)$$

The next iterate of the problem is given by

$$L\big(u_2(x,t)\big) + N\big(u_1(x,t)\big) + f(x,t) = 0, u_2(x,t) = -x$$

Taking the inverse operator of both sides of the above yield

$$\int_0^t u_{2t}(x,t)dt = -\int_0^t (u_1u_{1x})dt \quad (44)$$

Plugging in the derivatives and evaluating, we obtain the third iterate as

$$u_2(x,t) = -x - xt - xt^2 - \frac{1}{3}xt^3 \quad (45)$$

Similarly, the next iterate is obtained by solving the problem below

$$L\big(u_3(x,t)\big) + N\big(u_2(x,t)\big) + f(x,t) = 0, u_3(x,t) = -x$$

Integrating both sides using the initial conditions, we get the fourth iterate as follows

$$\int_0^t u_{3t}(x,t)dt = -\int_0^t (u_2 u_{2x})dt \quad (46)$$

Substituting the derivatives, we obtain the solution as

$$u_3(x,t) == -x - xt - xt^2 - \frac{1}{3}xt^3 - \frac{2}{3}xt^4 - \cdots \quad (47)$$

Using the relation, $u(x,t) = \lim_{n\to\infty} u_n(x,t)$, the closed form solution of the problem is obtained

$$u(x,t) = -x(1 + t + t^2 + t^3 + t^4 + \cdots)$$

$$u(x,t) = \frac{x}{t-1} \quad (48)$$

Example 5. Consider the Korteweg-de Vries (KDV) equation which takes the form

$$u_t - 6uu_x + u_{xxx} = 0 \quad (49)$$

Subject to the initial condition

$$u(x,0) = -\frac{k^2}{2}\sec h^2\left(\frac{kx}{2}\right) \quad (50)$$

Applying TAM to both sides of the equation yield the expressions

$$L(u) = u_t, N(u) = -6uu_x + u_{xxx}, f(x,t) = 0$$

The initial problem to be solved is of the form

$$L(u_0(x,t)) + f(x,t) = 0, u_0(x,t) = -\frac{k^2}{2}\sec h^2\left(\frac{kx}{2}\right) \quad (51)$$

Integrating both sides of the above equation subject to the initial condition, we get the initial solution as

$$u_0(x,t) = -\frac{k^2}{2}\sec h^2\left(\frac{kx}{2}\right) \quad (52)$$

The second iterative solution is obtained using the relation

$$L(u_1(x,t)) + N(u_0(x,t)) + f(x,t) = 0, u_1(x,t) = -\frac{k^2}{2}\sec h^2\left(\frac{kx}{2}\right) \quad (53)$$

Integrating both sides of the above using the initial condition yield the integral

$$\int_0^t u_{1t}(x,t)dt = \int_0^t (6u_0 u_{0x} - u_{0xxx})dt \quad (54)$$

Solving the above integral yield, the second iterative solution as

$$u_1(x,t) = -\frac{k^2}{2}\sec h^2\left(\frac{kx}{2}\right) - \frac{k^5 \sec h^2\left(\frac{kx}{2}\right)}{2}\mathrm{Tan}\, h\left(\frac{kx}{2}\right) \quad (55)$$

The next iterate of the problem is given by

$$L(u_2(x,t)) + N(u_1(x,t)) + f(x,t) = 0, u_2(x,t) = 2x \quad (56)$$

Taking the inverse operator of both sides of the above yield

$$\int_0^t u_{2t}(x,t)dt = \int_0^t (6u_1 u_{1x} - u_{1xxx})dt \quad (57)$$

Plugging in the derivatives and evaluating, we obtain the third iterate as

$$u_2(x,t) = -\frac{k^2}{2}\sec h^2\left(\frac{kx}{2}\right) - \frac{k^5 \sec h^2\left(\frac{kx}{2}\right)}{2}\mathrm{Tan}\, h\left(\frac{kx}{2}\right) - \frac{k^8}{8}sech^4\left[\frac{kx}{2}\right](2 - \cosh[kx])t^2 + \cdots \quad (58)$$

Continuing in the same, the succeeding term are obtained in a similar manner

Using the relation, $u(x,t) = \lim_{n\to\infty} u_n(x,t)$, the closed form solution of the problem is obtained

$$u(x,t) = -\frac{k^2}{2}sech^2\left[\frac{k}{2}(x - k^2 t)\right] \quad (59)$$

Example 6. Consider the Schrödinger equation in the form

$$u_t + iu_{xx} = 0, u(x,0) = 1 + \cos h(2x) \quad (60)$$

Applying TAM to both sides of the equation, we get the constants

$$L(u) = u_t, N(u) = iu_{xx}, f(x,t) = 0$$

The first problem to be solved is given by

$$L(u_0(x,t)) + f(x,t) = 0, u_0(x,t) = 1 + \cos h(2x) \quad (61)$$

Taking the inverse operator of both sides subject to the initial condition, we get the initial solution as

$$u_0(x,t) = 1 + \cos h(2x) \quad (62)$$

The next iterative solution is obtained using the equation

$$L(u_1(x,t)) + N(u_0(x,t)) + f(x,t) = 0, u_1(x,t) = 1 + \cos h(2x)$$

Integrating both sides of the above using the initial condition yield the integral

$$\int_0^t u_{1t}(x,t)dt = \int_0^t (iu_{0xx})dt \quad (63)$$

Solving the above integral yield, the second iterative solution as

$$u_1(x,t) = 1 + \cos h(2x) + 4i\cos h\,(2x) \quad (64)$$

The third iterative solution of the problem is given by

$$L(u_2(x,t)) + N(u_1(x,t)) + f(x,t) = 0, u_2(x,t) = 1 + \cos h(2x) \quad (65)$$

Taking the inverse operator of both sides of the above yield

$$\int_0^t u_{2t}(x,t)dt = \int_0^t (iu_{1xx})dt$$

Plugging in the derivatives and evaluating, we obtain the third iterate as

$$u_2(x,t) = 1 + \cos h(2x) + 4i\cos h\,(2x) + \frac{(4it)^2}{2!}\cos h(2x) \quad (66)$$

Similarly, the next iterate is obtained with the problem

$$L(u_3(x,t)) + N(u_2(x,t)) + f(x,t) = 0, u_3(x,t) = 1 + \cos h(2x) \quad (67)$$

Integrating both sides using the initial conditions, we get the fourth iterate as follows

$$\int_0^t u_{3t}(x,t)dt = \int_0^t (iu_{2xx})dt \quad (68)$$

Substituting the derivatives, we obtain the solution as

$$u_3(x,t) = 1 + \cos h(2x) + 4i \cos h\,(2x) + \frac{(4it)^2}{2!}\cos h(2x) + \frac{(4it)^3}{3!}\cos h(2x) \quad (69)$$

Using the relation, $u(x,t) = \lim_{n\to\infty} u_n(x,t)$, the closed form solution of the problem is obtainedas

$$u(x,t) = 1 + \cos h(2x)\left(1 + (4it) + \frac{(4it)^2}{2!} + \frac{(4it)^3}{3!} + \frac{(4it)^4}{4!} + \cdots\right)$$

$$u(x,t) = [1 + \cos h(2x)]e^{4it} \quad (70)$$

CONCLUDING REMARKS

In this research article, the closed form, or exact solutions for six different nonlinear partial differential equation is investigated using the novel Analytical Iteration Method (AIM). The efficiency of the method is confirmed by solving the Burgers, Fisher's, Advection, Schrödinger, wave and KDV equations respectively. The method gives an analytical solution which converges rapidly to the exact solution with more terms considered. This solution is easily verifiable in few steps of iteration subject to the initial condition and is computationally convenient since it doesn't require perturbation, discretization, and linearization. It is observed the method is reliable, efficient, and applicable to all class of nonlinear problems in the fields of Science and Engineering.

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