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Analytical Solution of the Dynamics of Atmospheric CO_2 using the LADM-Padé Approximation Approach

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ABSTRACT

In this research paper, the Laplace transform method combined with the semi-analytical Adomian decomposition method (LADM) is proposed to solve the mathematical model of crime deterrence in society. The model is solved to obtain analytical solution to the governing parameters in the form of a rapidly convergent series to illustrate its reliability, capability, and efficiency of this hybrid method. The practical result obtained reveal, the method is accurate and an efficient tool for solving wide variety of several first and higher order models.

KEYWORDS: *Laplace Adomian Decomposition method (LADM), Padé Approximant, Analytical solution, Global warming*

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INTRODUCTION

The term global warming is defined as the unabated steady rise in the level of atmospheric carbon (iv) oxide gas beyond an allowable threshold. From pre-industrial times, the level of CO_2 was just 270ppm (Parts per million). This number has risen to a worrisome level of 395ppm (Parts per million).Misra and Verma (2013), Rasool and Schneider (1971), (Tennakone, 1990). This rising trend of atmospheric carbon (iv) oxide if not checked does not bode well for both human and the ecosystem balance. Sequel to this, group of industrialized nation under the aegis of G7 have gathered severally to brainstormed on how to mitigate the effect of climate change occasioned by the ominous rise of carbon (iv) oxide (Pluss, 1956), McMichael, Woodruff and Hales (2006). Concerted efforts have been made by these countries to reduce and stabilize the future concentrate $C0_2$ via legislation, Woodwell, Hobbie, Houghton, Melillo, Moore, Peterson, and Shaver (1983). Reports abound that shows climate change accounts for about 90% of health risks or challenges faced by human. Therefore, if this rise in carbon (iv) oxide is not checked, it will portend danger to humans. For example, extreme weather conditions such as floods, Tsunami, windstorm, and drought are all caused by climate change. The attendant effects of the above conditions are direct injuries, malnutrition, infectious diseases, and airborne disease, Marten, Jettsen, Niessen and Rothmans (1995).

Equally, increased in the presence of vectors during this period cause vector-borne disease such as malaria, diarrhoea, dengue fever and others (Alexiadis, 2007). The accompanying heat waves also evoke respiratory and cardiovascular problems to humans. Following the above adverse effects of carbon (iv) oxide in the atmosphere, a better understanding behind the factors responsible for the increased level of $C0_2$ in the atmosphere and their effects is needed. Previous studies have shown that forest biomass and human population are the primary factors responsible for the rise in the level of carbon (iv) oxide. Human activities like deforestation and burning of fossil fuel triggers the level of atmospheric $C0_2$. Upstaging the forest-biomass carbon (iv) oxide link due to human activities also lead to increase in the level of atmospheric carbon (iv) oxide, Detwiler and Hall (1988), Khasnis and Nettleman (2005), (Kurane, 2010).

Volume of studies have been devoted to understanding the interaction of human population, carbon (iv) oxide and its attendant effects on human population and climate change. Caetano, Gherardi and Yoneyama (2011) have considered the mathematical model that relates atmospheric $C0_2$, forest biomass and GDP. In this study, it was shown that clean technology and reforestation are needed to attain desired $C0_2$ level. Malhi and Grace (2000), (Ewers, 2006), Kremen, Niles, Dalton, Daily, Ehrlich, Fay, Grewal and Culley (2000), (Mahar, 1989), Olabisi, Reich, Johnson, Kapuscinski, Suh and Wilson (2009) have studied the connection between human activities and global warming using feedback control. The study reveal increased in the level of carbon (iv) oxide has destabilizing effect. The biomass-carbon (iv) oxide system have been investigated using a mathematical model. The study show that excessive deforestation destabilizes the biomass-carbon (iv) oxide equilibrium.

The Laplace transformation and Adomian decomposition method (LADM) is a hybrid semi-analytical method which is a fusion of the two methods. The method requires taking the Laplace transform of both sides of the equation using the accompanying initial condition. Thereafter, the resulting solution is then written in operator form which gives an n −fold integral proportional to the highest degree of the invertible function. The zeroth order and the recursive algorithm are then obtained which gives the solution that converges to the exact solution if it exists. The LADM originally proposed by Khuri, Songen and Mendelson (2003), (Khuri, 2004), Khuri and Alchikh (2019, 2020) is preferable over the famous Adomian decomposition method because it converges faster to the exact solution. It has been extensively applied in the following areas: linear and nonlinear PDEs, coupled systems of PDEs, numerical solution of the Duffing equation, analytical solution of an HIV model, Newell-Whitehead-Segel equation, systems of ordinary differential equations, linear and nonlinear integral equation with weak kernel, nth-order integro-differential equations, two-dimensional viscous fluid with shrinking sheet, logistics equation, numerical solution of the crime deterrence model, nonlinear function equation and convection diffusion-dissipative equations, (Fadaei, 2011), Khan, Hussain, Jafari and Khan (2010), (Yusufoglu, 2006), (Nasser, 1997), (Ongun, 2011), (Pue-on, 2013), (Cherrualt, 2002), (Dogan, 2012), (Hendi, 2011), (Wazwaz, 1999, 2010), (Waleed, 2013), (Manafianheris, 2012), Mohamed and Torky (2013), Koroma, Widattala, Kamera and Zhang (2013), Islam, Khan, Faraz and Austin (2010), Yindoula, Youssouf, Bissanga, Bassino and Some (2014), (Khuri, 2001), Al-Khaled and Allan (2005), (Doan, 2012).

In this present study, we seek analytical solution to the parameters governing the problem using Laplace Adomian decomposition method. The article is composed as follows: section 1 takes the introduction. In section 2, the basics, and fundamentals of the Adomian decomposition method is extensively discussed. Section 3presents an in-depth review of the hybrid Laplace Adomian decomposition method. In sections 4 & 5, the Padé approximation and application of LADM to the model problem is presented. Numerical application via simulation is contained in section 6 and finally the section 7 gives the concluding remarks.

ADOMIAN DECOMPOSITION METHOD (ADM)

Consider a general nonlinear differential equation of the form

$$F[y(x)] = g(x) \tag{1}$$

Where F is a nonlinear operator and y, g are both functions of x

Decomposing the nonlinear operator, F into two parts comprising a linear and nonlinear operator.

$$L[y(x)] + R[y(x)] + N[y(x)] = g(x) \tag{2}$$

Where L is the highest order derivative that's assumed to be invertible, R is the linear differential operator with order less than that of L, N is a nonlinear term and g is the source term

Rewriting Eq. (2) for $L[y(x)]$, we obtain

$$L[y(x)] = g(x) - R[y(x)] - N[y(x)] \tag{3}$$

Taking the inverse operator, L^{-1} on both sides of Eq. (3), we get

$$y(x) = L^{-1}g(x) - L^{-1}R[y(x)] - L^{-1}N[y(x)] \tag{4}$$

Where ϕ is the term arising from the integration of the source term.It is obtained using the following sequence depending on the order of the given equation.

$$\phi = \begin{cases} y(0) & for\ L = \frac{d}{dx} \\ y(0) + xy'(0) & for\ L = \frac{d^2}{dx^2} \\ y(0) + xy'(0) + \frac{x^2}{2!}y''(0) & for\ L = \frac{d^3}{dx^3} \\ y(0) + xy'(0) + \frac{x^2}{2}y''(0) + \frac{x^3}{3!}y'''(0) & for\ L = \frac{d^4}{dx^4} \\ \vdots & \\ \vdots & \\ y(0) + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \cdots + \frac{x^n}{n!}y^{(n)}(0) & for\ L = \frac{d^{n+1}}{dx^{n+1}} \end{cases}$$

By the standard Adomian decomposition method, we write the unknown solution as an infinite decomposition series of the form

$$y(x) = \sum_{n=0}^{\infty} y_n(x) \tag{5}$$

Putting Eq. (5) into Eq. (4), we obtain

$$\sum_{n=0}^{\infty} y_n(x) = \phi - L^{-1}R[\sum_{n=0}^{\infty} y_n(x)] - L^{-1}N[\sum_{n=0}^{\infty} y_n(x)] \tag{6}$$

Matching both sides of Eq. (6), we obtain the zeroth order component given by

$$y_0 = \phi$$

Then the recursive relation is given by

$$y_{n+1}(x) = -L^{-1}R[y_n] - L^{-1}N[y_n], n \geq 0 \tag{7}$$

The solution of the problem in Eq. (1) is obtain as limit of the decomposing series

$$y(x) = \lim_{n\to\infty} y_n(x) \tag{8}$$

Similarly, the nonlinear term can be determined by an infinite series of the Adomian polynomials. That is,

$$N[y_0, y_1, y_2, \dots, y_n] = \sum_{n=0}^{\infty} A_n \tag{9}$$

Then the $A_n'^s$ are obtained from the relation

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[N(\sum_{k=0}^{\infty} \lambda^k y_k)]_{\lambda=0}, n = 0,1,2,3 \tag{10}$$

Using Eq. (9), the first five Adomian polynomials are given as

$$A_0 = N(y_0)$$

$$A_1 = y_1 N'(y_0)$$

$$A_2 = y_2N'(y_0) + \frac{1}{2!}y_1^2N''(y_0)$$

$$A_3 = y_3N'(y_0) + y_1y_2N''(y_0) + \frac{1}{3!}y_1^3N'''(y_0)$$

$$A_4 = y_4N'(y_0) + \frac{1}{2}N''(y_0)(2y_1y_3 + y_2^2) + \frac{1}{2}N'''(y_0)y_1^2y_2 + \frac{1}{4!}N^{(iv)}(y_0)y_1^4$$

$$A_5 = y_5N'(y_0) + \frac{1}{2}N''(y_0)(2y_1y_4 + 2y_2y_3) + \frac{1}{3!}N'''(y_0)(3y_1^2y_3 + 3y_1y_2^2) + \frac{4}{4!}N^{(iv)}(y_0)(y_1^3y_2) + \frac{1}{5!}N^{(v)}(y_0)y_1^5$$

$$A_6 = y_6N'(y_0) + \frac{1}{2!}N''(y_0)(2y_1y_5 + 2y_1y_4 + y_3^2) + \frac{1}{3!}N'''(y_0)(3y_1^2y_4 + y_2^3 + 6y_1y_2y_3) + \frac{1}{4!}N^{(iv)}(y_0)(4y_1^3y_3 + 6y_1^2y_2^2) + \frac{5}{5!}N^{(v)}(y_0)y_1^4y_2 + \frac{1}{6!}N^{(vi)}(y_0)y_1^6$$

$$A_7 = y_7N'(y_0) + \frac{1}{2!}N''(y_0)(2y_1y_6 + 2y_2y_5 + 2y_3y_4) + \frac{1}{3!}N'''(y_0)(3y_1^2y_5 + 3y_1y_3^2 + 3y_3y_2^2 + 6y_1y_2y_4) + \frac{1}{4!}N^{(iv)}(y_0)(4y_1^3y_4 + 12y_1^2y_2y_3 + 4y_1y_2^3) + \frac{1}{5!}N^{(v)}(y_0)(5y_1^4y_3 + 10y_1^3y_2^2) + \frac{1}{6!}N^{(vi)}(y_0)y_1^5y_2 + \frac{1}{7!}N^{(vii)}(y_0)y_1^7$$

$$A_8 = y_8N'(y_0) + \frac{1}{2!}N''(y_0)(2y_1y_7 + 2y_2y_6 + 2y_3y_5 + y_4^2) + \frac{1}{3!}N'''(y_0)(3y_1^2y_6 + 3y_4y_2^2 + 3y_2y_3^2 + 6y_1y_2y_5 + 6y_1y_3y_4) + \frac{1}{4!}N^{(iv)}(y_0)(4y_1^3y_5 + 12y_1^2y_2y_4 + 12y_1y_2^2y_3 + 6y_1^2y_3^2 + y_2^4) + \frac{1}{5!}N^{(v)}(y_0)(5y_1^4y_4 + 20y_1^3y_2y_3 + 10y_1^2y_2^3) + \frac{1}{6!}N^{(vi)}(y_0)(y_1^5y_3 + 15y_1^4y_2^2) + \frac{7}{7!}N^{(vii)}(y_0)y_1^6y_2 + \frac{1}{8!}N^{(viii)}(y_0)y_1^8$$

LAPLACE ADOMIAN DECOMPOSITION METHOD (LADM)

In this subsection, we outline the basics of the fundamentals of the fusionLaplace transformation and Adomian decomposition method (LADM)

Consider a functional differential equation of the form

$$L[u(x)] + R[u(x)] + N[u(x)] = g(x) \quad (11)$$

Subject to the initial condition

$$u(x,0) = f(x), \ \frac{\partial u(x,0)}{\partial t} = h(x) \quad (12)$$

Rearranging the above, we obtain the following relation for $L[u(x)]$

$$L[u(x)] = g(x) - R[u(x)] - N[u(x)] \quad (13)$$

Applying Laplace transform on both sides of Eq. (11), supposing the highest differential operator is of order two and using the differentiation property, we get

$$s^2\mathcal{L}\{u(x)\} - sh(x) - f(x) = \mathcal{L}\{g(x)\} - \mathcal{L}\{Ru(x)\} - \mathcal{L}\{Nu(x)\}$$

$$s^2\mathcal{L}\{u(x)\} = sh(x) + f(x) + \mathcal{L}\{g(x)\} - \mathcal{L}\{Ru(x)\} - \mathcal{L}\{Nu(x)\}$$

$$\mathcal{L}\{u(x)\} = \frac{h(x)}{s} + \frac{f(x)}{s^2} + \frac{1}{s^2}\mathcal{L}\{g(x)\} - \frac{1}{s^2}\mathcal{L}\{Ru(x)\} - \frac{1}{s^2}\mathcal{L}\{Nu(x)\} \quad (14)$$

Next, we apply the inverse transform on both sides of Eq. (14), we obtain

$$u(x) = \phi(x) - \mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\{Ru(x)\} - \frac{1}{s^2}\mathcal{L}\{Nu(x)\}\right] \quad (15)$$

Where $\phi(x)$ is the term arising from the first three terms on the right-hand side of Eq.

Next, we assume the solution as decomposing series in the form

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \tag{16}$$

Similarly, the nonlinear terms are written in terms of the Adomian polynomials

$$Nu(x) = \sum_{n=0}^{\infty} A_n \tag{17}$$

Where the $A_n'^s$ represents the Adomian polynomials defined in the form

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{\infty} \lambda^i y_i\right)\right]_{i=0}, n = 0,1,2,3 \tag{18}$$

Plugging Eqs (16) and (17) into Eq. (15), we obtain

$$\sum_{n=0}^{\infty} u_n(x) = \phi(x) - \mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\{R\sum_{n=0}^{\infty} u_n(x)\} - \frac{1}{s^2}\mathcal{L}\{N\sum_{n=0}^{\infty} A_n\}\right] \tag{19}$$

Matching both sides of Eq. (19), we obtain an iterative algorithm in the form

$$u_0(x) = \phi(x)$$

$$u_1(x) = -\mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\left\{R\sum_{n=0}^{\infty} u_0(x)\right\} - \frac{1}{s^2}\mathcal{L}\left\{N\sum_{n=0}^{\infty} A_0\right\}\right]$$

$$u_2(x) = -\mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\{R\sum_{n=0}^{\infty} u_1(x)\} - \frac{1}{s^2}\mathcal{L}\{N\sum_{n=0}^{\infty} A_1\}\right] \tag{20}$$

$$u_3(x) = -\mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\left\{R\sum_{n=0}^{\infty} u_2(x)\right\} - \frac{1}{s^2}\mathcal{L}\left\{N\sum_{n=0}^{\infty} A_2\right\}\right]$$

$$\vdots$$

$$u_{n+1}(x) = -\mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\left\{R\sum_{n=0}^{\infty} u_n(x)\right\} - \frac{1}{s^2}\mathcal{L}\left\{N\sum_{n=0}^{\infty} A_n\right\}\right]$$

Then the solution of the differential equation is obtained as the sum of decomposed series in the form

$$u(x) \approx u_0(x) + u_1(x) + u_2(x) + \cdots \tag{21}$$

PADÉ APPROXIMATION

In Mathematics and other applied sciences, power series representation of a function is usually in truncated form. To approximate these functions to an appreciable degree, polynomials are used because their singularities are easily noticeable in each finite region. However, the radius of convergence may not be large enough to contain two boundaries, for this reason, power series is not always the best method to approximate a function. To overcome this inherent hurdle, a new approximation is applied to the solution obtained using power series as a quotient of two functions with varying degrees in a finite interval.

Padé approximation has been widely used to approximate several problems and has tremendous applications especially in computer calculation because it gives a better approximation without truncating its power series and still in problems where the series diverges. The different Padé approximants are obtained with the use of symbolic software Mathematica

A rational approximation to a function $f(x)$ on $[a, b]$ is the quotient of two polynomials, $P_N(x)$ and $Q_M(x)$ of degrees N and M respectively. It is denoted by $[N/M](x)$

That is, $[N/M](x) = \frac{P_N(x)}{Q_M(x)}, a \le x \le b$ (22)

Now consider the formal power series

$$f(x) = \sum_{k=0}^{\infty} c_k x^k \tag{23}$$

$$f(x) = \frac{P_N(x)}{Q_M(x)} + O(x^{N+M+1})$$

Rearranging gives

$$f(x) - \frac{P_N(x)}{Q_M(x)} = O(x^{N+M+1}) \tag{24}$$

Multiply both sides of Eq. (24) by a constant keep it unchanged, hence we impose the normalization condition.

$$Q_M(0) = 1.0 \quad (25)$$

Next, we require that $P_N(x)$ and $Q_M(x)$ have non-common factors, so we write the coefficients of the $P_N(x)$ and $Q_M(x)$ as follows

$$\left.\begin{aligned} P_N(x) &= a_0 + a_1x + a_2x^2 + \cdots + a_Nx^N \\ Q_M(x) &= b_0 + b_1x + b_2x^2 + \cdots + b_Mx^M \end{aligned}\right] \quad (26)$$

Using Eqs. (25) and (26), we multiply Eq. (23) by $Q_M(x)$. This linearizes the coefficient equation. It is given in detailed form as

$$(a_0 + a_1x + a_2x^2 + \cdots + a_Nx^N) = (b_0 + b_1x + b_2x^2 + \cdots + b_Mx^M)(c_0 + c_1x + c_2x^2 + \cdots + c_Nx^N)$$

Equating the coefficients of $x^{N+1}, x^{N+2}, \dots, x^{N+M}$ successively to zero, we obtain the system of Equations

$$\left.\begin{aligned} b_Mc_{N-M+1} + b_{M-1}c_{N-M+2} + \cdots + b_0c_{N+1} &= 0 \\ b_Mc_{N-M+2} + b_{M-1}c_{N-M+3} + \cdots + b_0c_{N+2} &= 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ b_Mc_N + b_{M-1}c_{N+1} + \cdots + b_0c_{N+M} &= 0 \end{aligned}\right] \quad (27)$$

For $j < 0$, we define $c_k = 0$ for consistency. Setting $b_0 = 1$, Eq. (22) become a set of Mlinear equations for M unknown coefficients in the denominator.

$$\begin{bmatrix} c_{N-M+1} & c_{N-M+2} \cdots . c_{N+1} \\ c_{N-M+2} & c_{N-M+3} \dots\dots c_{N+2} \\ \dots\dots\dots & \dots\dots\dots \\ c_N & c_{N+1} \dots\dots\dots c_{N+M+1} \end{bmatrix} \begin{bmatrix} b_M \\ b_{M-1} \\ \vdots \\ b_1 \end{bmatrix} = \begin{bmatrix} C_{N+1} \\ C_{N+2} \\ \vdots \\ C_{N+M} \end{bmatrix} \quad (28)$$

Solving the above system in Eq. (28), the coefficients b_i for $i = 1,2, \dots .M$ may be found. Since the coefficients of the numerator, $c_0, c_1, c_2, \dots .., c_k$ is known. We equate the coefficients of $1, x, x^2, \dots, x^N, x^{N+M}$ to obtain the remaining coefficients $a_0, a_1, a_2, \dots, a_N$

$$\begin{aligned} &x^0: c_0 - a_0 = 0 \\ &x^1: c_0b_1 + c_1 - a_1 = 0 \\ &x^2: c_0b_2 + c_1b_1 + c_2 - a_2 = 0 \\ &\dots\dots\dots\dots\dots\dots\dots\dots \\ &x^M: c_0b_M + c_1b_{M-1} + c_M - a_M = 0 \\ &x^N: c_0b_N + c_1b_{N-1} + c_N - a_N = 0 \end{aligned}$$

Writing the above in explicit form, we obtain

$$\begin{aligned} a_0 &= c_0 \\ a_1 &= c_1 + b_1c_0 \\ a_2 &= c_2 + b_1c_1 + b_2c_0 \\ &\dots\dots\dots\dots\dots\dots \\ a_N &= c_N + \textstyle\sum_{k=1}^{N} b_kc_{N-k}, N = min(N, M) \end{aligned} \quad (29)$$

In view of Eqs. (28) and (29), the numerator and denominator of the Pade approximant are all determined which agrees with the original series to the order of $x^{N+M.}$

Now, to solve the system (29) for the set of unknowns, we assume that the Eqs. (28) and (29) are non-singular, so we can obtain the solution via the determinant. See [42-45]

$$[N/M] = \frac{\begin{bmatrix} a_{N-M+1} & a_{N-M+2} & \dots\dots & a_{N+1} \\ a_N & a_{N+1} & \dots\dots & a_{N+M} \\ \vdots & \vdots & & \vdots \\ \sum_{j=M}^{N} a_{j-M}x^j & \sum_{j=M-1}^{N} a_{j-M+1}x^j & & \sum_{j=0}^{N} a_jx^j \end{bmatrix}}{\begin{bmatrix} a_{N-M+1} & a_{N-M+2} \dots\dots & a_{N+1} \\ a_N & a_{N+1} \dots\dots\dots & a_{N+M} \\ \vdots & \vdots & \vdots \\ x^M & x^{M-1} & 1 \end{bmatrix}} \quad (30)$$

To obtain the diagonal Pade approximants of different orders such as $[2/2], [4/4], [6/6]$, we use symbolic software Mathematica

APPLICATION OF LADM TO THE MODEL

Given the mathematical model for global warming as follows

$$\frac{dX}{dt} = Q_0 + \lambda N - \alpha X - \lambda_1 XF \quad (31)$$

$$\frac{dN}{dt} = sN\left(1 - \frac{N}{L}\right) - \theta XN + \pi\phi NF \quad (32)$$

$$\frac{dF}{dt} = \mu F\left(1 - \frac{F}{M}\right) - \phi NF + \pi_1\lambda_1 XF \quad (33)$$

Rearranging the above Eqs$(31) - (33)$, we obtain

$$\frac{dX}{dt} = Q_0 + \lambda N - \alpha X - \lambda_1 XF \quad (34)$$

$$\frac{dN}{dt} = \frac{s}{L}N(L - N) - \theta XN + \pi\phi NF \quad (35)$$

$$\frac{dF}{dt} = \frac{\mu}{L}F(M - F) - \phi NF + \pi_1\lambda_1 XF \quad (36)$$

$$X(0) > 0, N(0) \geq 0, F(0) \geq 0 \quad (37)$$

Where the parameters X, N and F have their usual meanings

$X =$ amount of CO_2 in the atmosphere

$N =$ Available human population

$F =$ Forest Biomass

Taking the Laplace Transform of both sides, we get

$$\mathcal{L}\left\{\frac{dX}{dt}\right\} = \mathcal{L}\{Q_0\} + \mathcal{L}\{\lambda N\} - \mathcal{L}\{\alpha X\} - \mathcal{L}\{\lambda_1 XF\} \quad (38)$$

$$\mathcal{L}\left\{\frac{dN}{dt}\right\} = \frac{s}{L}\mathcal{L}\{N(L - N)\} - \mathcal{L}\{\theta XN\} + \mathcal{L}\{\pi\phi NF\} \quad (39)$$

$$\mathcal{L}\left\{\frac{dF}{dt}\right\} = \frac{\mu}{M}\mathcal{L}\{F(M - F)\} - \mathcal{L}\{\phi NF\} + \mathcal{L}\{\pi_1\lambda_1 XF\} \quad (40)$$

Applying the formula for Laplace transform in the first derivative, we obtain

$$w\mathcal{L}\{X\} - X(0) = \mathcal{L}\{Q_0\} + \lambda\mathcal{L}\{N\} - \alpha\mathcal{L}\{X\} - \lambda_1\mathcal{L}\{XF\} \quad (41)$$

$$w\mathcal{L}\{N\} - N(0) = \frac{s}{L}\mathcal{L}\{N(L - N)\} - \theta\mathcal{L}\{XN\} + \pi\phi\mathcal{L}\{NF\} \quad (42)$$

$$w\mathcal{L}\{F\} - F(0) = \frac{\mu}{M}\mathcal{L}\{F(M - F)\} - \phi\mathcal{L}\{NF\} + \pi_1\lambda_1\mathcal{L}\{XF\} \quad (43)$$

Using the initial condition in Eq. (37), Eqs (41) - (43) reduced to

$$w\mathcal{L}\{X\} = \frac{Q_0}{w} + \lambda\mathcal{L}\{N\} - \alpha\mathcal{L}\{X\} - \lambda_1\mathcal{L}\{XF\} \quad (44)$$

$$w\mathcal{L}\{N\} = \frac{s}{L}\mathcal{L}\{N(L - N)\} - \theta\mathcal{L}\{XN\} + \pi\phi\mathcal{L}\{NF\} \quad (45)$$

$$w\mathcal{L}\{F\} = \frac{\mu}{M}\mathcal{L}\{F(M - F)\} - \phi\mathcal{L}\{NF\} + \pi_1\lambda_1\mathcal{L}\{XF\} \quad (46)$$

Further simplification by dividing both sides by w, we get

$$\mathcal{L}\{X\} = \frac{Q_0}{w^2} + \frac{\lambda}{w}\mathcal{L}\{N\} - \frac{\alpha}{w}\mathcal{L}\{X\} - \frac{\lambda_1}{w}\mathcal{L}\{A\} \quad (47)$$

$$\mathcal{L}\{N\} = \frac{s}{wL}\mathcal{L}\{N(L - N)\} - \frac{\theta}{w}\mathcal{L}\{B\} + \frac{\pi\phi}{w}\mathcal{L}\{C\} \quad (48)$$

$$\mathcal{L}\{F\} = \frac{\mu}{wM}\mathcal{L}\{F(M - F)\} - \frac{\phi}{w}\mathcal{L}\{C\} + \frac{\pi_1\lambda_1}{w}\mathcal{L}\{A\} \quad (49)$$

Where $A = XF, B = XN, C = NF$ (50)

By the Laplace transform decomposition method, we represent the solution as infinite series of the form

$$X = \sum_{N=0}^{\infty} X_n, N = \sum_{n=0}^{\infty} N_n, F = \sum_{n=0}^{\infty} F_n \quad (51)$$

Where the terms $X_n,$ N_n and F_n are to be obtained via the recursive relation. Similarly, the nonlinear operators, A, B and C are decomposed as follows

$$A = \sum_{N=0}^{\infty} A_n, B = \sum_{n=0}^{\infty} B_n, C = \sum_{n=0}^{\infty} C_n \quad (52)$$

Where A_n, B_n and C_n are the Adomian polynomials. The first eight of these polynomials are given by

$A_0 = X_0F_0$

$A_1 = X_0F_1 + X_1F_0$

$A_2 = X_0F_2 + X_1F_1 + X_2F_0$

$A_3 = X_0F_3 + X_1F_2 + X_2F_1 + X_3F_0$

$A_4 = X_0F_4 + X_1F_3 + X_2F_2 + X_3F_1 + X_4F_0$ (53)
$A_5 = X_0F_5 + X_1F_4 + X_2F_3 + X_3F_2 + X_4F_1 + X_5F_0$
$A_6 = X_0F_6 + X_1F_5 + X_2F_4 + X_3F_3 + X_4F_2 + X_5F_1 + X_6F_0$
$A_7 = X_0F_7 + X_1F_6 + X_2F_5 + X_3F_4 + X_4F_3 + X_5F_2 + X_6F_1 + X_7F_0$
$A_8 = X_0F_8 + X_1F_7 + X_2F_6 + X_3F_5 + X_4F_4 + X_5F_3 + X_6F_2 + X_7F_1 + X_8F_0$

$B_0 = X_0N_0$
$B_1 = X_0N_1 + X_1N_0$
$B_2 = X_0N_2 + X_1N_1 + X_2N_0$
$B_3 = X_0N_3 + X_1N_2 + X_2N_1 + X_3N_0$
$B_4 = X_0N_4 + X_1N_3 + X_2N_2 + X_3N_1 + X_4N_0$ (54)
$B_5 = X_0N_5 + X_1N_4 + X_2N_3 + X_3N_2 + X_4N_1 + X_5N_0$
$B_6 = X_0N_6 + X_1N_5 + X_2N_4 + X_3N_3 + X_4N_2 + X_5N_1 + X_6N_0$
$B_7 = X_0N_7 + X_1N_6 + X_2N_5 + X_3N_4 + X_4N_3 + X_5N_2 + X_6N_1 + X_7N_0$
$B_8 = X_0N_8 + X_1N_7 + X_2N_6 + X_3N_5 + X_4N_4 + X_5N_3 + X_6N_2 + X_7N_1 + X_8N_0$

$C_0 = N_0F_0$
$C_1 = N_0F_1 + N_1F_0$
$C_2 = N_0F_2 + N_1F_1 + N_2F_0$
$C_3 = N_0F_3 + N_1F_2 + N_2F_1 + N_3F_0$
$C_4 = N_0F_4 + N_1F_3 + N_2F_2 + N_3F_1 + N_4F_0$ (55)
$C_5 = N_0F_5 + N_1F_4 + N_2F_3 + N_3F_2 + N_4F_1 + N_5F_0$
$C_6 = N_0F_6 + N_1F_5 + N_2F_4 + N_3F_3 + N_4F_2 + N_5F_1 + N_6F_0$
$C_7 = N_0F_7 + N_1F_6 + N_2F_5 + N_3F_4 + N_4F_3 + N_5F_2 + N_6F_1 + N_7F_0$
$C_8 = N_0F_8 + N_1F_7 + N_2F_6 + N_3F_5 + N_4F_4 + N_5F_3 + N_6F_2 + N_7F_1 + N_8F_0$

Putting Eqs. (51) and (52) into Eqs (47) - (49)

$\mathcal{L}\{\sum_{n=0}^{\infty} X_n\} = \frac{Q_0}{w^2} + \frac{\lambda}{w}\mathcal{L}\{\sum_{n=0}^{\infty} N_n\} - \frac{\alpha}{w}\mathcal{L}\{\sum_{n=0}^{\infty} X_n\} - \frac{\lambda_1}{w}\mathcal{L}\{\sum_{n=0}^{\infty} A_n\}$ (56)

$\mathcal{L}\{\sum_{n=0}^{\infty} N_n\} = \frac{s}{wL}\mathcal{L}\{\sum_{n=0}^{\infty} N_n\,(L - \sum_{n=0}^{\infty} N_n)\} - \frac{\theta}{w}\mathcal{L}\{\sum_{n=0}^{\infty} B_n\} + \frac{\pi\phi}{w}\mathcal{L}\{\sum_{n=0}^{\infty} C_n\}$ (57)

$\mathcal{L}\{\sum_{n=0}^{\infty} F_n\} = \frac{\mu}{wM}\mathcal{L}\{\sum_{n=0}^{\infty} F_n\,(M - \sum_{n=0}^{\infty} F_n)\} - \frac{\phi}{w}\mathcal{L}\{\sum_{n=0}^{\infty} C_n\} + \frac{\pi_1\lambda_1}{w}\mathcal{L}\{\sum_{n=0}^{\infty} A_n\}$ (58)

Matching both sides of Eqs (56) – (58) yield the following iterative algorithms

$\mathcal{L}\{X_0\} = \frac{Q_0}{w^2}$
$\mathcal{L}\{X_1\} = \frac{\lambda}{w}\mathcal{L}\{N_0\} - \frac{\alpha}{w}\mathcal{L}\{X_0\} - \frac{\lambda_1}{w}\mathcal{L}\{A_0\}$
$\mathcal{L}\{X_2\} = \frac{\lambda}{w}\mathcal{L}\{N_1\} - \frac{\alpha}{w}\mathcal{L}\{X_1\} - \frac{\lambda_1}{w}\mathcal{L}\{A_1\}$ (59)
$\mathcal{L}\{X_3\} = \frac{\lambda}{w}\mathcal{L}\{N_2\} - \frac{\alpha}{w}\mathcal{L}\{X_2\} - \frac{\lambda_1}{w}\mathcal{L}\{A_2\}$
$\vdots$
$\mathcal{L}\{X_{n+1}\} = \frac{\lambda}{w}\mathcal{L}\{N_n\} - \frac{\alpha}{w}\mathcal{L}\{X_n\} - \frac{\lambda_1}{w}\mathcal{L}\{A_n\}$

$\mathcal{L}\{N_0\} = 0$
$\mathcal{L}\{N_1\} = \frac{s}{wL}\mathcal{L}\{N_0(L - N_0)\} - \frac{\theta}{w}\mathcal{L}\{B_0\} + \frac{\pi\phi}{w}\mathcal{L}\{C_0\}$
$\mathcal{L}\{N_2\} = \frac{s}{wL}\mathcal{L}\{N_1(L - N_1)\} - \frac{\theta}{w}\mathcal{L}\{B_1\} + \frac{\pi\phi}{w}\mathcal{L}\{C_1\}$ (60)
$\mathcal{L}\{N_3\} = \frac{s}{wL}\mathcal{L}\{N_2(L - N_2)\} - \frac{\theta}{w}\mathcal{L}\{B_2\} + \frac{\pi\phi}{w}\mathcal{L}\{C_2\}$
$\vdots$

$$\mathcal{L}\{N_{n+1}\} = \frac{s}{wL}\mathcal{L}\{N_n(L - N_n)\} - \frac{\theta}{w}\mathcal{L}\{B_n\} + \frac{\pi\phi}{w}\mathcal{L}\{C_n\}$$

$$\mathcal{L}\{F_0\} = 0$$
$$\mathcal{L}\{F_1\} = \frac{\mu}{wM}\mathcal{L}\{F_0(M - F_0)\} - \frac{\phi}{w}\mathcal{L}\{C_0\} + \frac{\pi_1\lambda_1}{w}\mathcal{L}\{A_0\}$$
$$\mathcal{L}\{F_2\} = \frac{\mu}{wM}\mathcal{L}\{F_1(M - F_1)\} - \frac{\phi}{w}\mathcal{L}\{C_1\} + \frac{\pi_1\lambda_1}{w}\mathcal{L}\{A_1\} \qquad (61)$$
$$\mathcal{L}\{F_3\} = \frac{\mu}{wM}\mathcal{L}\{F_2(M - F_2)\} - \frac{\phi}{w}\mathcal{L}\{C_2\} + \frac{\pi_1\lambda_1}{w}\mathcal{L}\{A_2\}$$
$$\vdots$$
$$\mathcal{L}\{F_{n+1}\} = \frac{\mu}{wM}\mathcal{L}\{F_n(M - F_n)\} - \frac{\phi}{w}\mathcal{L}\{C_n\} + \frac{\pi_1\lambda_1}{w}\mathcal{L}\{A_n\}$$

NUMERICAL APPLICATION

In this section, we apply the LADM to the numerical solution of the model using simulation.

Applying the inverse Laplace transform to both sides of Eqs (59) – (61) gives

$$\mathcal{L}\{X_0\} = \frac{Q_0}{w^2}, \mathcal{L}\{N_0\} = \frac{1}{w}, \mathcal{L}\{F_0\} = \frac{1}{w} \qquad (62)$$

Substitution of Eq (62) into the second equations in Eqs (59) – (61), we get

$$\mathcal{L}\{X_1\} = \frac{\lambda}{w^2} - \frac{\alpha Q_0}{w^2} + \frac{\lambda_1 Q_0}{w^3}$$
$$\mathcal{L}\{N_1\} = \frac{s(WL-\ )}{w^3 L} - \frac{QQ_0}{w^3} + \frac{\pi\phi}{w^2} \qquad (63)$$
$$\mathcal{L}\{F_1\} = \frac{\mu}{wM}\left(\frac{wM - 1}{w^2}\right) - \frac{\phi}{w^3} + \frac{\pi_1\lambda_1 Q_0}{w^3}$$

Putting the values of $\mathcal{L}\{X_1\}, \mathcal{L}\{N_1\}$ and $\mathcal{L}\{F_1\}$ into the second Eqs. (59) – (61), we obtain

$$\mathcal{L}\{X_2\} = \frac{\lambda}{w^2} - \frac{\alpha Q_0}{w^2} + \frac{\lambda_1 Q_0}{w^3}$$
$$\mathcal{L}\{N_2\} = \frac{s(WL-1)}{w^3 L} - \frac{QQ_0}{w^3} + \frac{\pi\phi}{w^2} \qquad (64)$$
$$\mathcal{L}\{F_2\} = \frac{\mu}{wM}\left(\frac{wM - 1}{w^2}\right) - \frac{\phi}{w^3} + \frac{\pi_1\lambda_1 Q_0}{w^3}$$

Evaluating the Laplace transform of the quantities on the RHS of Eqs. (62) – (64), and applying the inverse Laplace transform, we obtain the values, $X_1(t), N_1(t), F_1(t)$and $X_2(t), N_2(t), F_2(t)$. Similarly, the other higher order solutions $X_3(t), X_4(t), \ldots, X_n(t)$, $N_3(t), N_4(t), \ldots, N_n(t)$, $F_3(t), F_4(t), \ldots, F_n(t)$ are obtained recursively in a similar fashion using Eqs. (59) – (61)

Now to obtain the solution of the parameters of interest in explicit form, we apply LADM to the model by taking the following values via simulation. We take $X(0) = 1, N(0) = 1, F(0) = 1,$ for the three components of the model. Next, we take $Q_0 = 1, \lambda = 0.05, \alpha = 0.03, \lambda_1 = 0.0001, s == 0.01, L = 1000, \theta = 0.00001, \mu = 0.2, M = 2000, \pi = 0.01, \phi = 0.0002, \pi_1 = 0.01$. A few first approximations for $X(t), N(t)$ and $F(t)$ are calculated and presented below using LADM as follows.

$$\mathcal{L}\{X_0\} = \frac{1}{w^2}, \mathcal{L}\{N_0\} = \frac{1}{w}, \mathcal{L}\{F_0\} = \frac{1}{w} \qquad (65)$$

$$\mathcal{L}\{X_1\} = \frac{0.05}{w^2} - \frac{0.03}{w^3} + \frac{0.0001}{w^4}$$
$$\mathcal{L}\{N_1\} = \frac{0.00999}{w^2} - \frac{0.0001}{w^3} + \frac{0.000002}{w^4} \qquad (66)$$
$$\mathcal{L}\{F_1\} = \frac{0.00999}{w^2} - \frac{0.0002}{w^3} + \frac{0.000002}{w^4}$$

Taking the inverse Laplace transform of both sides of the above equations, we obtain the solutions of the parameters as follows.

$X(t) = 0.05t - 0.03t^2 + 0.0000333333t^3$

$$N(t) = 0.00999t + 10^{-6}t^2 - 3.33333 \times 10^{-6}t^3 \quad (67)$$

$$F(t) = 0.00999 - 0.0001t^2 + 3.33333 \times 10^{-7}t^3$$

Next., we calculate the $[5/5]$ Pade approximates of the infinite series solution which gives the following rational fraction approximation of the parameters of interest using Mathematica

$$X_{Pade}(t) = \frac{1. + 0.7195405t - 0.365002t^2 + 0.0050t^3 - 0.0000051t^4}{1 - 0.593t + 2.16 \times 10^{-1}\ t^2 + 1.68 \times 10^{-15}t^3 - 1.4 \times 10^{-16}t^4}$$

$$N_{Pade}(t) = \frac{1. + 0.0049t - 0.048t^2 - 0.0333\ ^3 + 1.6649 \times 10^{-8}t^4}{1 - 0.0045t + 6.0408 \times 10^{-17}t^2 + 4.74 \times 10^{-1}\ t^3 + 4.9 \times 10^{-19}t^4}$$

$$F_{Pade}(t) = \frac{1. + 0.0502t - 0.00506t^2 + 0.05308t^3 - 1.6565 \times 10^{-8}t^4}{1 - 0.049t + 6.9697 \times 10^{-14}t^2 - 3.503 \times 10^{-17}t^3 - 3.4 \times 10^{-18}t^4}$$

RESULTS AND DISCUSSION

In this subsection, the results of the problem in Eq. (1) are presented to show the effects of the governing parameters on the model. The effectiveness and accuracy of the numerical methods are displayed in Tables 1-3 and Figures 1-7. The methods give highly accurate results in few steps. The results obtained when compared are consistent with literature

Table 1: Numerical Computations for X(t)

t	LADM	LADM-Padé	4^{th} Order R-K
0	1	1	1
0.2	-1.27322	-1.27369	-1.27310
0.4	-13.01120	-13.1023	-13.1201
0.6	-39.11090	-41.6953	-41.6102
0.8	-92.2623	-134.119	-134.102
1.0	-196.788	-195.23	-194.21
1.2	-392.44	178.808	178.801

Table 2: Numerical computations for N(t)

t	LADM	LADM-Padé	4^{th} Order R-K
0	1	1	1
0.2	19.0903	19.0903	19.0901
0.4	37.3871	37.3872	37.3862
0.6	56.0688	56.0699	56.0700
0.8	75.5955	75.6116	75.6110
1.0	96.867	96.9949	96.9950
1.2	121.397	122.11	122.020

Table 3: Numerical Computations for F(t)

t	LADM	LADM-Padé	4^{th} Order R-K
0	-1.05	-1.05	-1.05
0.2	-1.04715	-1.04715	-1.04715
0.4	-1.02824	-1.02796	-1.02796
0.6	-0.983335	-0.973338	-0.97337
0.8	-0.959682	-0.833489	-.0.833452
1.0	-1.30508	-0.318347	-0.318340
1.2	-3.3568	-5.94349	-5.94340

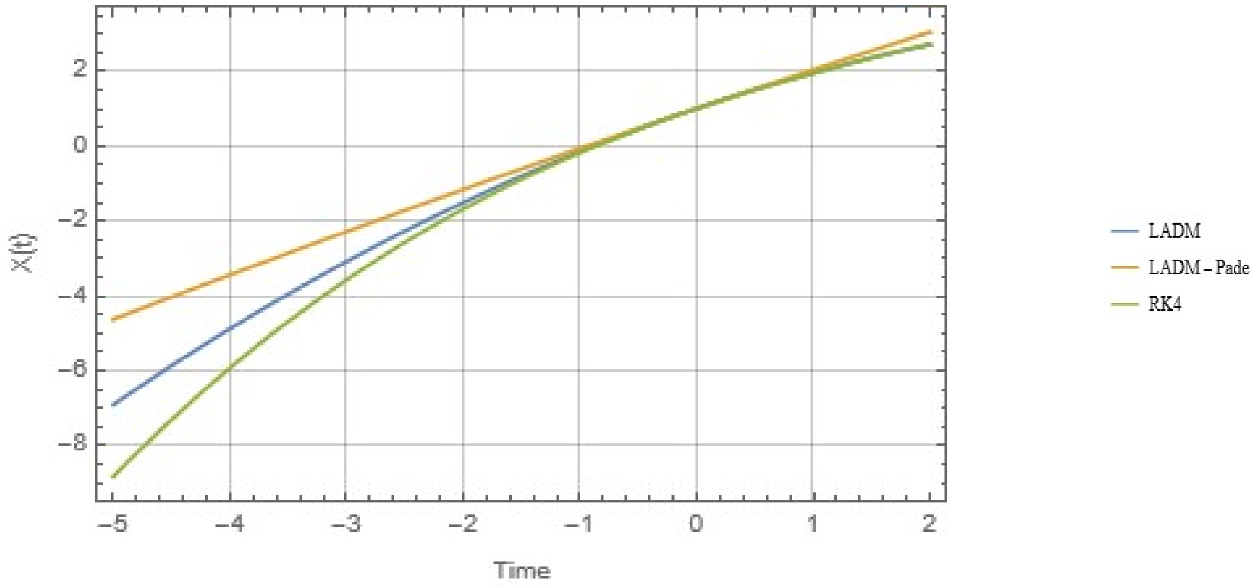

Figure 1 Computation of Atmospheric CO_2 Against Time

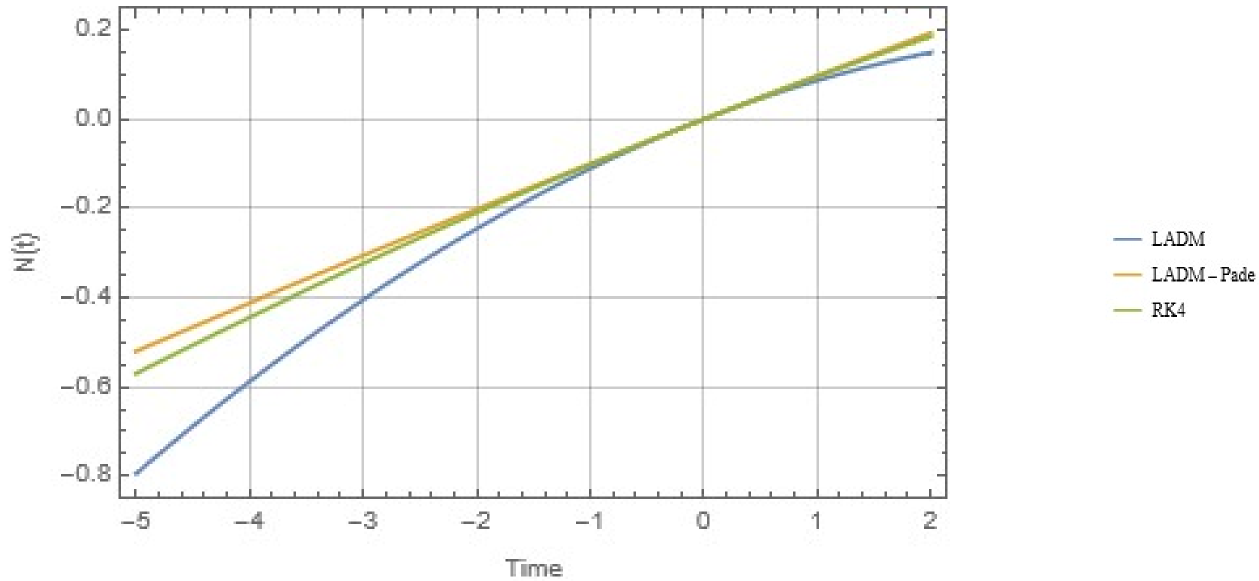

Figure 2 Computations of Human Population Against Time

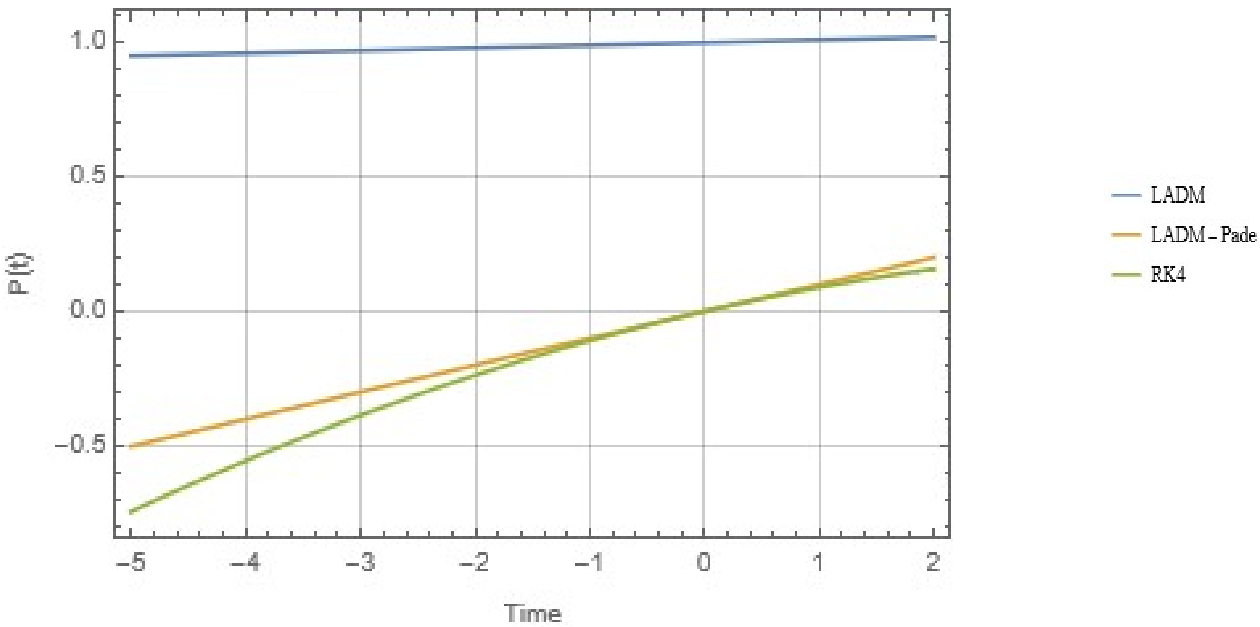

Figure 3 Computation of Forest Biomass Against Time

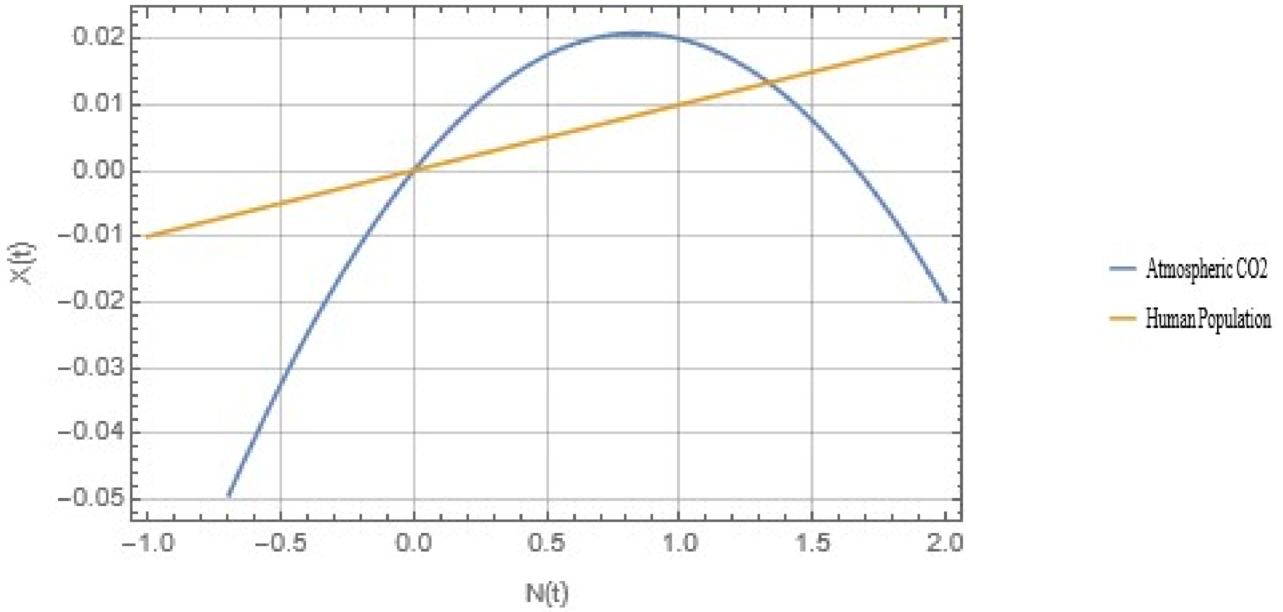

Figure 4 Computation of Amount of CO2 Against Human Population

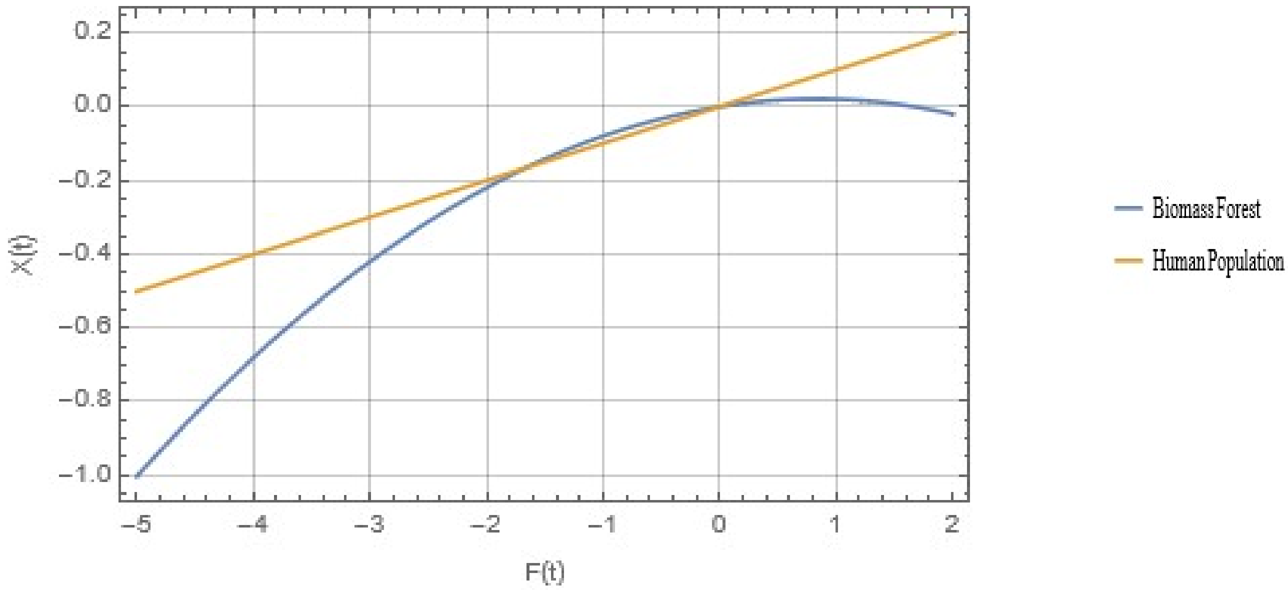

Figure 5 Computation of Amount of CO2 Against Forest Biomass

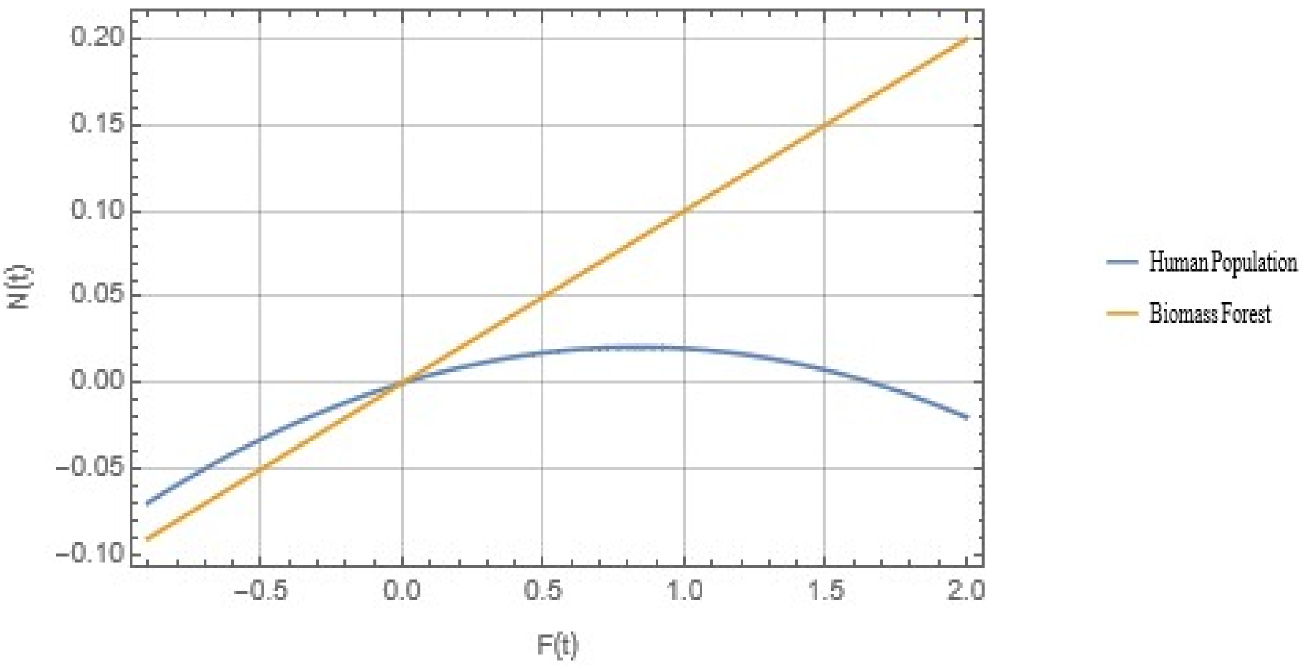

Figure 6 Human population Against Forest Biomass

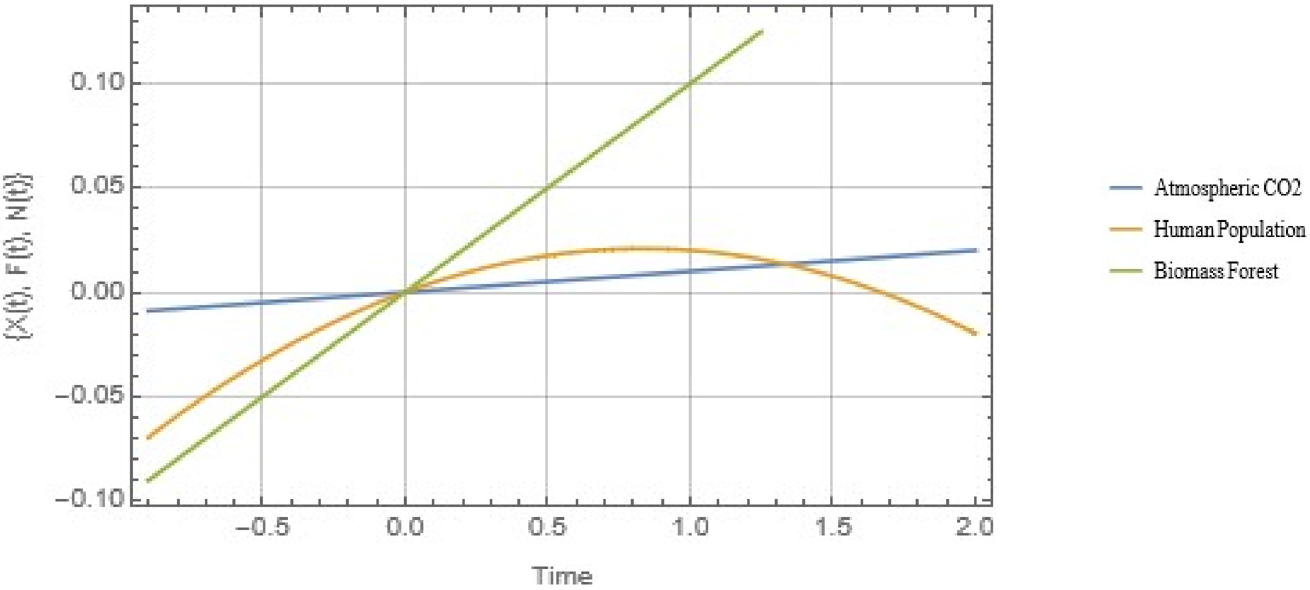

Figure 7 Computation of Atmospheric CO2, Human Population and Forest Biomass against Time

CONCLUDING REMARKS

In this study, the approximate analytical solution of the mathematical model describing the dynamics of carbon dioxide in the atmosphere is solved using the fusion of Laplace transform and Adomian decomposition method (LADM). The validity, accuracy, flexibility, and effectiveness of the method is demonstrated by obtaining the exact solution of the parameters of interest subject to the initial condition. The solution obtained shows the MADM is effective and convenient.

Furthermore, MADM is a promising tool to effectively both linear and nonlinear PDEs. The benchmark solution is a ready reference for further works in the crime model.

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