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Multiple Objective LPP and Goal Programming: A Review

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ABSTRACT

This paper studies multiple objective linear programming problems (MOLP) and Goal programming problems (GPP). Evidence of those MOLP and other related examples, remarks with proof were also established. The entire paper is classified and separately discussed based on the topic.

KEYWORDS: *GPP, MOLP, LPP, optimum solution*

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1. Introduction

Linear programming (LP) is a method of mathematical optimization. It has been used extensively in industry solutions in oil refining among other fields. Academic articles, such as Symonds [1955], describing LP solutions for oil refining problems date back to the 1950s. Other articles, including Banae Costa [1990], describe the use of Multi Objective Linear Programming (MOLP) in oil refining.

In MOLP, there are several differing objectives to be optimized and a good compromise solution is sought. However, to this day the public articles concerning the refining industry concentrate on problems where the LP or MOLP problem only covers the perspective a single decision maker. These include the optimization of blending components into final products with several attributes to be optimized. This is similar to the optimization of the operation of a refinery or a group of refineries working towards a mutual goal.

Thus, there is a void of research about using MOLP to describe systems of several decision makers with conflicting goals. This paper aims to create a MOLP model that can be used to Linear programming problem.

In such circumstances, the conflicts between different objective functions are highly fascinating. No refining company is willing to give up their own good for the others.

Therefore, the model solution must be a carefully selected compromise to achieve realism. They request a feasibility study on a mathematical model that could be used to compare the profitability of competing oil refineries. The model should be created on the Spiral Suite1 optimization software. The models in the software are formulated as LP models. This naturally leads to the conclusion that the model designed in this thesis needs to be a MOLP model since the goals of different refining companies must be considered.

This paper is structured as follows. Section 1 introduces related existing work and a preliminary of the MOLPP is followed in section 2. Section 3 contains the multiple objective of LPP. Section 4 deals with multi objective functions based on LPP are discussed in especially simplex method. Section 5 describes the concept of multiple objective goal programming problem and conclusion followed by last chapter.

2. Multiple objective LPP

In this section focus on Multi Objective Optimization (MOO) problem. For MOO problems with connecting objective functions, a completely optimal solution does not always exist. Therefore, Paretooptimality is used as a solution concept. Consider a set of objective functions $f_i(x)$, i = 1; 2; 3………, where $x \in X$ is a vector of decision variables. Each objective function $f_i(x)$ is to be minimized. A point $x^* \in X$ is said to be a Pareto optimal solution if and only if there exists no other $x \in X$ for which $f_i(x) \leq f_i(x^*)$ for all i and $f_i(x) = f_i(x^*)$ for some i [Sakawa et al., 2013]. A point x* is said to be a weakly Pareto optimal solution if and only if there exists no other x for which $f_i(x) < f_i(x^*)$for all i [Sakawa et al., 2013]. Pareto optimality and weak Pareto optimality are presented graphically in Figure 3.1. The Pareto optimal solutions form a Pareto front [Miettinen, 1999]. In Figure 3.1 the Pareto front is the line connecting the labeled Pareto optimal solutions.

Several methods for solving MOO problems exist. These can be classified into four classes which are no preference methods, a posteriori methods, a priori methods, and interactive methods. [Miettinen, 1999]

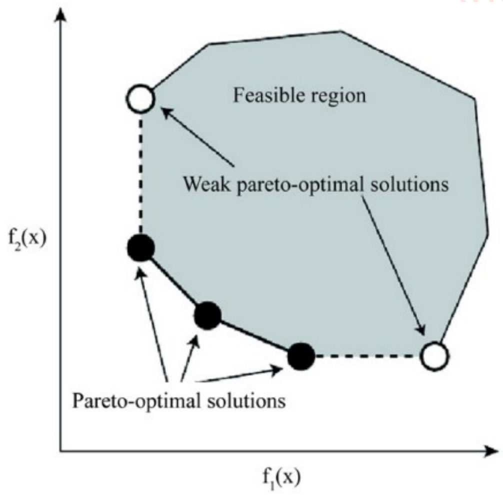

Figure 2.1: Pareto optimal and weak Pareto optimal solutions to a minimization problem with two objective functions

No preference methods are the most simplistic class as they do not assume preference relations between Pareto optimal solutions. The three other classes require a decision maker whose preferences are utilized to form criteria on how to select a preferred solution from a set of Pareto optimal solutions. This can mean, e.g., that preference relations are created between the different conicting objective functions such that an increase in one is deemed more desirable than an equal increase in another.

In a posteriori methods, Pareto optimal solutions are generated first and the decision maker selects a satisfactory solution afterwards. A posteriori methods are often computationally heavy. In a priori methods, the decision maker's preferences are surveyed in advance and implemented into the solution method. Interactive methods are iterative in nature. Practically, this means that Pareto optimal solutions are generated and improved based on the input from the decision maker until the decision maker accepts a solution. [Miettinen, 1999]

3. Goal LPP

Goal programming provides a way of striving toward several such objectives *simultaneously*. The basic approach of **goal programming** is to establish a specific numeric goal for each of the objectives, formulate an objective function for each objective, and then seek a solution that minimizes the (weighted) sum of deviations of these objective functions from their respective goals. There are three possible types of goals:

A **lower, one sided goal** sets a *lower limit* that we do not want to fall under (but exceeding the limit is fine).

An **upper, one sided goal** sets an *upper limit* that we do not want to exceed (but falling under the limit is fine).

A **two sided goal** sets a *specific target* that we do not want to miss on either side.

Goal programming problems can be categorized according to the type of mathematical programming model (linear programming, integer programming, nonlinear programming, etc.) that it fits except for having multiple goals instead of a single objective. In this book, we only consider *linear* goal programming those goal programming problems that fit linear programming otherwise (each objective function is linear, etc.) and so we will drop the adjective *linear* from now on.

Another categorization is according to how the goals compare in importance. In one case, called **non preemptive goal programming,** all the goals are of *roughly comparable importance*. In another case, called **preemptive goal programming,** there is a *hierarchy of priority levels* for the goals, so that the goals of primary importance receive first priority attention, those of secondary importance receive second priority attention, and so forth (if there are more than two priority levels).

We begin with an example that illustrates the basic features of non preemptive goal programming and then discuss the preemptive case

3.1 Prototype Example for Nonpreemptive Goal Programming

The DEWRIGHT COMPANY is considering three new products to replace current models that are being discontinued, so their OR department has been assigned the task of determining which mix of these products should be produced. Management wants primary consideration given to three factors: long run profit, stability in the workforce, and the level of capital investment that would be required now for new equipment. In particular, management has established the goals of (1) achieving a long run profit (net present value) of at least $125 million from these products, (2) maintaining the current employment level of 4,000 employees, and (3) holding the capital investment to less than $55 million. However, management realizes that it probably will not be possible to attain all these goals simultaneously, so it has discussed priorities with the OR department. This discussion has led to setting *penalty weights* of 5 for missing the profit goal (per $1 million under), 2 for going over the employment goal (per 100 employees), 4 for going under this same goal, and 3 for exceeding the capital investment goal (per $1 million over). Each new product's contribution to profit, employment level, and capital investment level is *proportional* to the rate of production. These contributions per unit rate of production are shown in Table 1, along with the goals and penalty weights.

Formulation. The Dewright Company problem includes all three possible types of goals: a lower, one sided goal (long run profit); a two sided goal (employment level); and an upper, one sided goal (capital investment). Letting the decision variables x_1, x_2, x_3 be the production rates of products 1, 2, and 3, respectively, we see that these goals can be stated as

$12x_1 + 9x_2 + 15x_3 \geq 125$ Profit goal

$5x_1 + 3x_2 + 4x_3 = 40$ employment goal

$5x_1 + 7x_2 + 8x_3 \geq 55$ investment goal

More precisely, given the penalty weights in the rightmost column of Table 1, let Z be the *number of penalty points* incurred by missing these goals. The overall objective then is to choose the values of x_1, x_2 and x_3 so as to

Minimize Z 5(amount under the long run profit goal) +2(amount over the employment level goal) +4(amount under the employment level goal) +3(amount over the capital investment goal),

TABLE 1 Data for the Dewright Co. non preemptive goal programming problem

Factor	Unit contribution Product 1 2 3	Goal (units)	Penalty Weight
Long run profit employment level capital investment	12 9 15 5 3 4 5 7 8	≥ 125 =40 ≤55	5 2(+), 4(-) 3

where no penalty points are incurred for being over the long run profit goal or for being under the capital investment goal. To express this overall objective mathematically, we introduce some *auxiliary variables* (extra variables that are helpful for formulating the model) y_1, y_2, and y_3, defined as follows:

$y_1 = 12x_1 + 9x_2 + 15x_3 \geq 125$ ((long run profit minus the target)

$y_2 = 5x_1 + 3x_2 + 4x_3 = 40$ (employment level minus the target).

$y_3 = 5x_1 + 7x_2 + 8x_3 \geq 55$ (capital investment minus the target).

Since each yi can be either positive or negative. we replace each one by the difference of two nonnegative variables:

$y_1 = y_1^+ - y_1^-$, where $y_1^+, y_1^- \geq 0$

$y_2 = y_2^+ - y_2^-$, where $y_2^+, y_2^- \geq 0$

$y_3 = y_3^+ - y_3^-$, where $y_3^+, y_3^- \geq 0$

for any BF solution, these new auxiliary variables have the interpretation

$$y_j^+ = \begin{cases} y_j & \text{if } y_j \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$y_j^- = \begin{cases} y_j & \text{if } y_j \leq 0 \\ 0 & \text{otherwise} \end{cases} \text{ where } |y_j| = y_j$$

So that y_j^+ represent the positive part of the variable y_j^+ and y_j^- its negative part.

Given these new auxiliary variables, the overall objective can be expressed mathematically as,

Minimize $Z = 5y_1^- + 2y_2^+ + 4y_2^- + 3y_3^+$,

which now is a legitimate objective function for a linear programming model. Because there is no penalty for exceeding the profit goal of 125 or being under the investment goal of 55, neither y_1^+ nor y_3^-

should appear in this objective function representing the total penalty for deviations from the goals.

To complete the conversion of this goal programming problem to a linear programming model, we must incorporate the above definitions of the y_j^+ and y_j^- directly into the model. (It is not enough to simply record the definitions, as we just did, because the simplex method considers only the objective function and constraints that constitute the model.) For example, since $y_1^+ - y_1^- = y_1$, the above expression for y_1 gives

$$y_1^+ - y_1^- = y_1 = 12x_1 + 9x_2 + 15x_3 - 125 = 0$$

$$12x_1 + 9x_2 + 15x_3 - (y_1^+ - y_1^-) = 125$$

becomes a legitimate equality constraint for a linear programming model. Furthermore, this constraint forces the auxiliary variables $(y_1^+ - y_1^-)$ to satisfy their definition in terms of the decision variables (x_1, x_2,x_3). Proceeding in the same way for $(y_2^+ - y_2^-)$ and $(y_3^+ - y_3^-)$ we obtain the following linear programming formulation of this goal programming problem.

There are two basic methods based on linear programming for solving preemptive goal programming problems. One is called the sequential procedure, and the other is the streamlined procedure. We shall illustrate these procedures in turn by solving the following example.

Example 3.1

Faced with the unpleasant recommendation to increase the company's workforce by more than 20 percent, the management of the Dewright Company has reconsidered the original formulation of the problem that was summarized in Table 1. This increase in workforce probably would be a rather temporary one, so the very high cost of training 833 new employees would be largely wasted, and the large (undoubtedly well publicized) layoffs would make it more difficult for the company to attract high quality employees in the future.

Consequently, management has concluded that a very high priority should be placed on avoiding an increase in the workforce. Furthermore, management has learned that raising more than $55 million for capital investment for the new products would be extremely difficult, so a very high priority also should be placed on avoiding capital investment above this level.

Based on these considerations, management has concluded that a preemptive goal programming approach now should be used, where the two goals just discussed should be the first priority goals, and the other two original goals (exceeding $125 million in long run profit and avoiding a decrease in the employment level) should be the second priority goals. This reformulation is summarized in Table 2, where a factor of M (representing a huge positive number) has been included in the penalty weights for the first priority goals to emphasize that these goals preempt the second priority goals. (The portions of Table 1 that are not included in Table 2 are unchanged.).

The Sequential Procedure for Preemptive Goal Programming The sequential procedure solves a preemptive goal programming problem by solving a sequence of linear programming models.

At the first stage of the sequential procedure, the only goals included in the linear programming model are the first priority goals, and the simplex method is applied in the usual way. If the resulting optimal solution is unique, we adopt it immediately without considering any additional goals. However, if there are multiple optimal solutions with the same optimal value of Z (call it Z*), we prepare to break the tie among these solutions by moving to the second stage and adding the second priority goals to the model.

If Z* = 0, all the auxiliary variables representing the deviations from first priority goals must equal zero (full achievement of these goals) for the solutions remaining under consideration. Thus, in this case, all these auxiliary variables now can be completely deleted from the model, where the equality constraints that contain these variables are replaced by the mathematical expressions (inequalities or equations) for these first priority goals, to ensure that they continue to be fully achieved.

On the other hand, if Z*> 0, the second stage model simply adds the second priority goals to the first stage model (as if these additional goals actually were first priority goals), but then it also adds the constraint that the first stage objective function equals Z* (which enables us again to delete the terms involving first priority goals from the second stage objective function). After we apply the simplex method again, if there still are multiple optimal solutions, we repeat the same process for any lower priority goals.

Example 3.2

We now illustrate this procedure by applying it to the example summarized in Table 2. At the first stage, only the two first priority goals are included in the linear programming model. Therefore, we can drop the common factor M for their penalty weights, shown in Table 2. By proceeding just as for the non preemptive model if these were the only goals, the resulting linear programming model is

Minimize $Z = 2y_2^+ + 3y_3^+$

Table 5.2 : Revised formulation for the Dewright Co. preemptive goal programming problem

Priority Level	Factor	Goal	Penalty Weight
First Priority	Employment level Capital investment	≤40 ≤55	2M 3M
Second priority	Long run profit Employment level	≥125 ≥40	5 4

$5x_1 + 3x_2 + 4x_3 - (y_2^+ - y_2^-) = 40$

$5x_1 + 7x_2 + 8x_3 - (y_3^+ - y_3^-) = 55$

and $x_j \geq 0$, $y_2^+ \geq 0$, $y_2^- \geq 0$ (j=1,2,3; k =2,3).

(For ease of comparison with the nonpreemptive model with all four goals, we have kept the same subscripts on the auxiliary variables.)

By using the simplex method (or inspection), an optimal solution for this linear programming Model has $y_2^+ = 0$ and $y_3^+ = 0$ with Z = 0. So Z* = 0 also., because of there are in numerable solutions for x_1, x_2 x_3 that satisfy the relationships

$5x_1 + 3x_2 + 4x_3 \leq 40$

$5x_1 + 7x_2 + 8x_3 \leq 55$

as well as the non negativity constraints. Therefore, these two first priority goals should be used as constraints hereafter. Using them as constraints will force y_2^+ and y_3^+ to remain zero and thereby disappear from the model automatically.

If we drop y_2^+ and y_3^+ but add the second priority goals, the second stage linear programming model becomes

Minimize $Z = 5y_1^- + 4y_2^-$,

Subject to

$12x_1 - 9x_2 + 15x_3 - (y_1^+ - y_1^-) = 125$

$5x_1 + 3x_2 + 4x_3 + y_2^-) = 40$

$5x_1 + 7x_2 + 8x_3 + y_3^- = 55$

And $x_j \geq 0$, $y_2^+ \geq 0$, $y_k^- \geq 0$ (j=1,2,3; k =1,2,3).

Applying the simplex method to this model yields the unique optimal solution $x_1 = 5$,

$x_2 = 0$, $x_3 = 3\frac{3}{4}$, $y_1^+ = 0$. $y_1^- = 8\frac{3}{4}$, $y_2^- = 0$, and $y_3^- = 0$ with $Z = 43\frac{3}{4}$.

Because this solution is unique (or because there areno more priority levels), the procedure can now stop, with (x_1 , x_2 , x_3) = (5, 0,$3\frac{3}{4}$) as the optimal solution for the overall problem. This solution fully achieves both first priority goals as well as one of the second priority goal (long run profit ≥ 125) by just $8\frac{3}{4}$.

The Streamlined Procedure for Preemptive Goal Programming

Instead of solving a sequence of linear programming models, like the sequential procedure, the streamlined procedure finds an optimal solution for a preemptive goal programming problem by solving just one linear programming model.

Thus, the streamlined procedure is able to duplicate the work of the sequential procedure with just one run of the simplex method. This one run simultaneously finds optimal solutions based just on first priority goals and breaks ties among these solutions by considering lower priority goals. However, this does require a slight modification of the simplex method.

If there are just two priority levels, the modification of the simplex method is one you already have seen, namely, the form of the Big M method illustrated. In this form, instead of replacing M throughout the model by some huge positive number before running the simplex method, we retain the symbolic quantity M in the sequence of simplex tableaux. Each coefficient in row 0 (for each iteration) is some linear function Am + b, where a is the current multiplicative factor and b is the current additive term. The usual decisions based on these coefficients (entering basic variable and optimality test) now are based solely on the multiplicative factors, except that any ties would be broken by using the additive terms. This is how the IOR Tutorial operates when solving interactively by the simplex method (and choosing the Big M method).

The linear programming formulation for the streamlined procedure with two priority levels would include all the goals in the model in the usual manner, but with basic penalty weights of M and 1 assigned to deviations from first priority and second priority goals, respectively. If different penalty weights are desired within the same priority level, these basic penalty weights then are multiplied by the individual penalty weights assigned within the level. This approach is illustrated by the following example.

Example 3.3

For the Dewright Co. preemptive goal programming problem summarized in Table 2, note that (1) different penalty weights are assigned within each of two priority levels and (2) the individual penalty weights (2 and 3) for the first priority goals have been multiplied by M. These penalty weights yield the following single linear programming model that incorporates all the goals.

Minimize $Z = 5y_1^- + 2My_2^+ + 4y_2^- + 3My_3^+$,

Subject to,

$12x_1 + 9x_2 + 15x_3 - (y_1^+ - y_1^-) = 125$

$5x_1 + 3x_2 + 4x_3 - (y_2^+ - y_2^-) = 40$

$5x_1 + 7x_2 + 8x_3 - (y_3^+ - y_3^-) = 55$

and $x_j \geq 0$, $y_k^+ \geq 0$, $y_k^- \geq 0$ (j=1,2,3; k =1,2,3).

Because this model uses M to symbolize a huge positive number, the simplex method can be applied as described and illustrated.

Alternatively, a very large positive number can be substituted for M in the model and the any software package based on the simplex method can be applied. Doing either naturally yields the same unique optimal solution obtained by the sequential procedure.

More than Two Priority Levels. When there are more than two priority levels (say, p of them), the streamlined procedure generalizes in a straightforward way.

The basic penalty weights for the respective levels now areM_1, M_2 , … , M_{p-1} , 1 , where M_1 represents a number that is vastly larger than M_2 ,M_2 is vastly larger than M_3, . . ., and M_{p-1} is vastly larger than 1. Each coefficient in row 0 of each simplex tableau is now to make the necessary decisions, with tie breakers beginning with the multiplicative factor of M_2 and ending with the additive term.

Example 3.4

One of management's goals in a goal programming problem is expressed algebraically as

$3x_1 + 4x_2 + 2x_3 = 60$

Where 60 is the specific numeric goal and the left hand side gives the level achieved toward meeting this goal.

Letting y^+ be the amount by which the level achieved exceeds this goal (if any) and y^- the amount under the goal (if any), show how this goal would be expressed as an equality constraint when reformulating the problem as a linear programming model. If each unit over the goal is considered twice as serious as each unit under the goal, what is the relationship between the coefficients of y^+ and y^- in the objective function being minimized in this linear programming model.

Example 3.5

Montega is a developing country which has 15,000,000 acres of publicly controlled agricultural land in active use. Its government currently is planning a way to divide this land among three basic crops (labeled 1, 2, and 3) next year.

A certain percentage of each of these crops is exported to obtain badly needed foreign capital (dollars), and the rest of each of these crops is used to feed the populace. Raising these crops also provides employment for a significant proportion of the population. Therefore, the main factors to be considered in allocating the land to these crops are (1) the amount of foreign capital generated, (2) the number of citizens fed, and (3) the number of citizens employed in raising these crops.

The following table shows how much each 1,000 acres of each crop contributes toward these factors, and the last column gives the goal established by the government for each of these factors.

Factor	Contribution per 1,000 Acres Crop: 1 2 3	Goal
Foreign capital Citizens fed Citizens employed	\$3,000 \$5,000 \$4,000 150 75 100 10 15 12	≥ \$70,000,000 ≥ 1,750,000 = 200,000

In evaluating the relative seriousness of not achieving these goals, the government has concluded that the following deviations from the goals should be considered equally undesirable: (1) each \$100 under the foreign capital goal, (2) each person under the citizens fed goal, and (3) each deviation of one (in either direction) from the citizens employed goal.

(a) Formulate a goal programming model for this problem.

(b) Reformulate this model as a linear programming model.

(c) Use the simplex method to solve this model.

(d) Now suppose that the government concludes that the importance of the various goals differs greatly so that a preemptive goal programming approach should be used. In particular, the first priority goal is citizens fed ≥ 1,750,000, the second priority goal is foreign capital ≥ \$70,000,000, and the third priority goal is citizens employed = 200,000. Use the goal programming technique to formulate one complete linear programming

model for this problem.

(e) Use the streamlined procedure to solve the problem as formulated in part (d).

(f) Use the sequential procedure to solve the problem as presented in part (d).

3.2 A Cure for Cuba

Fulgencio Batista led Cuba with a cold heart and iron fist greedily stealing from poor citizens, capriciously ruling the Cuban population that looked to him for guidance, and violently murdering the innocent critics of his politics. In 1958, tired of watching his fellow Cubans suffer from corruption and tyranny, Fidel Castro led a guerilla attack against the Batista regime and wrested power from Batista in January 1959.

Cubans, along with members of the international community, believed that political and economic freedom had finally triumphed on the island. The next two years showed, however, that Castro was leading a Communist dictatorship—killing his political opponents and nationalizing all privately held assets.

The United States responded to Castro's leadership in 1961 by invoking a trade embargo against Cuba. The embargo forbade any country from selling Cuban products in the United States and forbade businesses from selling American products to Cuba. Cubans did not feel the true impact of the embargo until 1989 when the Soviet economy collapsed. Prior to the disintegration of the Soviet Union, Cuba had received an averageof $5 billion in annual economic assistance from the Soviet Union.

With the disappearance of the economy that Cuba had almost exclusively depended upon for trade, Cubans had few avenues from which to purchase food, clothes, and medicine. The avenues narrowed even further when the United States passed the Torricelli Act in 1992 that forbade American subsidiaries in third countries from doing business with Cuba that had been worth a total of $700 million annually.

Since 1989, the Cuban economy has certainly felt the impact from decades of frozen trade. Today poverty ravages the island of Cuba. Families do not have money to purchase bare necessities, such as food, milk, and clothing. Children die from malnutrition or exposure. Disease infects the island because medicine is unavailable. Optical neuritis, tuberculosis, pneumonia, and influenza run rampant among the population.

Few Americans hold sympathy for Cuba, but Robert Baker, director of Helping Hand, leads a handful of tender souls on Capitol Hill who cannot bear to see politics destroy so many human lives. His organization distributes humanitarian aid annually to needy countries around the world.

Mr. Baker recognizes the dire situation in Cuba, and he wants to allocate aid to Cuba for the coming year. Mr. Baker wants to send numerous aid packages to Cuban citizens. Three different types of packages are available. The basic package contains only food, such as grain and powdered milk. Each basic package costs $300, weighs 120 pounds, and aids 30 people. The advanced package contains food and clothing, such as blankets and fabrics. Each advanced package costs $350, weighs 180 pounds, and aids 35 people.

The supreme package contains food, clothing, and medicine. Each supreme package costs $720, weighs 220 pounds, and aids 54 people. Mr. Baker has several goals he wants to achieve when deciding upon the number and types of aid packages to allocate to Cuba. First, he wants to aid at least 20 percent of Cuba's 11 million citizens. Second, because disease runs rampant among the Cuban population, he wants at least 3,000 of the aid packages sent to Cuba to be the supreme packages. Third, because he knows many other nations also require humanitarian aid, he wants to keep the cost of aiding Cuba below $20 million.

Mr. Baker places different levels of importance on his three goals. He believes the most important goal is keeping costs down since low costs mean that his organization is able to aid a larger number of needy nations. He decides to penalize his plan by 1 point for every $1 million above his $20 million goal. He believes the second most important goal is ensuring that at least 3,000 of the aid packages sent to Cuba are supreme packages, since he does not want to see an epidemic develop and completely destroy the Cuban population.

He decides to penalize his plan by 1 point for every 1,000 packages below his goal of 3,000 packages. Finally, he believes the least important goal is reaching at least 20 percent of the population, since he would rather give a smaller number of individuals all they need to thrive instead of a larger number of individuals only some of what they need to thrive. He therefore decides to penalize his plan by 7 points for every 100,000 people below his 20 percent goal.

Mr. Baker realizes that he has certain limitations on the aid packages that he delivers to Cuba. Each type of package is approximately the same size, and because only a limited number of cargo flights from the United States are allowed into Cuba, he is only able to send a maximum of 40,000 packages. Along with a size limitation, he also encounters a weight restriction.

He cannot ship more that 6 million pounds of cargo. Finally, he has a safety restriction. When sending medicine, he needs to ensure that the Cubans know how to use the medicine properly. Therefore, for every 100 supreme packages, Mr. Baker must send one doctor to Cuba at a cost of $33,000 per doctor.

(a) How many basic, advanced, and supreme packages should Mr. Baker send to Cuba?

(b) Mr. Baker reevaluates the levels of importance he places on each of the three goals. To sell his efforts to potential donors, he must show that his program is effective. Donors generally judge the effectiveness of a program on the number of people reached by aid packages.

Mr. Baker therefore decides that he must put more importance on the goal of reaching at least 20 percent of the population. He decides to penalize his plan by 10 points for every half a percentage point below his 20 percent goal. The penalties for his other two goals remain the same. Under this scenario, how many basic, advanced, and supreme packages should Mr. Baker send to Cuba? How sensitive is the plan to changes in the penalty weights? (c) Mr. Baker realizes that sending more doctors along with the supreme packages will improve the proper use and distribution of the packages' contents, which in turn will increase the effectiveness of the program.

He therefore decides to send one doctor with every 75 supreme packages. The penalties for the goals remain the same as in part (b). Under this scenario, how many basic, advanced, and supreme packages should Mr. Baker send to Cuba?.

(d) The aid budget is cut, and Mr. Baker learns that he definitely cannot allocate more than $20 million in aid to Cuba. Due to the budget cut, Mr. Baker decides to stay with his original policy of sending one doctor with every 100 supreme packages. How many basic, advanced, and supreme packages should Mr. Baker send to Cuba assuming that the penalties for not meeting the other two goals remain the same as in part (a)?.

(e) Now that the aid budget has been cut, Mr. Baker feels that the levels of importance of his three goals differ so much that it is difficult to assign meaningful penalty weights to deviations from these goals.

Therefore, he decides that it would be more appropriate to apply a preemptive goal programming approach (which will ensure that his budget goal is fully met if possible), while retaining his original policy of sending one doctor with every 100 supreme packages. How many basic, advanced, and supreme packages should Mr. Baker send to Cuba according to this approach?

3.3 Airport Security

Shortly after the tragic events of September 11, 2001, the United States Congress enacted emergency legislation to give the Department of Transportation primary responsibility for providing security at over 400 major U.S. airports. The Transportation Security Administration was then created within the Department of Transportation to carry out this responsibility.

A leading OR consultant in the airline industry, Adeline Jonasson, has been hired by the Transportation Security Administration to head up a task force on airport security. The specific charge to the task force is to investigate what advanced security technology should be developed and used at airport checkpoints to maximize the effectiveness with which passengers can be screened within budget constraints. Even prior to 2001, airline passengers had become familiar with the two basic types of systems used to check each passenger at a security checkpoint. One is a portal that can detect concealed weapons as the passenger walks through. The other is a screening system that scans the passenger's carry on luggage.

Various proposals have been made for advanced security technology that would improve these two systems. Adeline's task force now needs to make recommendations on which direction to go for the next generation of these systems. The task force has been told that the functional requirement for the new portal system is that it must be able to detect even one ounce of explosives and hazardous liquids as well as metallic weapons being concealed by a passenger. The technology needed to do this includes quadrupole resonance (closely related to magnetic resonance technology used by the medical industry) and magnetic sensors.

There are various ways to design the portal with this technology that would satisfactorily meet the functional requirement.

However, the designs would differ greatly in the frequency with which false alarms would occur as well as in the purchase cost and maintenance cost for the portal. The frequency of false alarms is a key consideration since it substantially affects the efficiency with which the passengers can be processed. Even more importantly, a high frequency of false alarms greatly decreases the alertness of the security personnel for detecting the relatively rare terrorists who are actually concealing destructive devices. The most basic version of the portal system that satisfactorily meets the functional requirement would have an estimated purchase price of $90,000 and, on the average, would incur an annual maintenance cost of $15,000.

The drawback of this version is that it would generate a false alarm for approximately 10 percent of the passengers. This false alarm rate can be reduced by using more expensive versions of the system. Each additional $15,000 in the cost of the portal system would lower the false alarm rate 1 percent and also would increase the annual maintenance cost by $1,500. The most expensive version would cost $210,000, so it would have a false alarm rate of only 2 percent of the customers as well as an annual maintenance cost of $27,000.

Regarding the new screening system for carry on luggage, the functional requirement is that it must clearly reveal suspicious objects as small as the smallest Swiss army knife. The technology needed to do this combines X ray imaging, a thermal neutron scanner, and computer tomography imaging (which compares the density and other physical properties of any suspicious objects with known high risk materials). It is estimated that the most basic version that satisfactorily meets this functional requirement would cost $60,000 plus an annual maintenance cost of $9,000. As with the most basic portal system, the drawback of this version is that it isn't sufficiently discriminating between suspicious objects that actually are destructive devices and those that are harmless.

Thus, this version would generate false alarms for approximately 6 percent of the customers. In addition to wasting time and delaying passengers, such a high false alarm rate would make it very difficult for the screening operator to pay sufficient attention when the far more unusual true alarms occur. However, more expensive versions of the screening system would be considerably more discriminating.

In particular, each additional $30,000 in the cost of the system would enable a reduction of 1 percent in the false alarm rate, while also increasing the annual maintenance cost by $1,200. Thus, the most expensive version, costing $150,000, would decrease the false alarm rate to 3 percent and incur an annual maintenance cost of $12,600. The task force has been given two budgetary guidelines. First Budgetary Guideline: Plan on a total expenditure of $250,000 for both the portal system and the screening system for carry on luggage at each security checkpoint.

Second Budgetary Guideline: Plan on holding down the average total maintenance costs for the two systems at each security checkpoint to no more than $30,000. These budget guidelines prohibit using the most expensive versions of both the portal system and the screening system for carryon baggage.

Therefore, the task force needs to determine which financially feasible combination of versions for the two systems will maximize the effectiveness with which passengers can be screened. Doing this requires first obtaining input from the top management of the Transportation Security Administration regarding what the measure of effectiveness should be and then what management's goals and priorities are for achieving substantial effectiveness and meeting the budgetary guidelines. Fortunately, Adeline already has had extensive discussions with top management to obtain its guidance on these matters. These discussions led to the adoption of a clear policy that was approved all the way up to the Secretary of Transportation (who also informed the chairmen of the Congressional oversight committees of this action).

The policy establishes the following order of priorities.

Priority 1: The functional requirement for each of the two new systems must be met. (This is satisfied by all the versions under consideration by the task force.)

Priority 2: The total false alarm rate for both systems should not exceed 0.1 per passenger.

Priority 3: Meet the first budgetary guideline.

Priority 4: Meet the second budgetary guideline.

Now that it has obtained all the needed managerial input, the task force is ready to begin its analysis.

(a) Identify the two decisions to be made, and define a decision variable for each one.

(b) Describe why this problem is a preemptive goal programming problem by giving quantitative expressions for each of the goals in terms of the decision variables defined in part (a).

(c) Draw a single two dimensional graph where the two axes correspond to the decision variables defined in part (a). Consider each of the goals in order of priority and use the quantitative expression obtained in part (b) for this goal to draw a plot on this graph that graphically displays the values of the decision variables that fully satisfy this goal. After completing this for all the goals, use this graph to determine the optimal solution for this preemptive goal programming problem.

(d) Use a linear programming software package (such as the Excel Solver, MPL/CPLEX, LINDO, or LINGO) to formulate and solve this preemptive goal programming problem.

(e) If it is possible to fully satisfy all the goals except the lowest priority goal, one can quickly solve a preemptive goal programming problem by formulating and solving a linear programming model that includes all the goals except the last one as constraints and then uses the objective function to strive toward the lowest priority goal. Formulate and solve such a linear programming model for this problem on a spreadsheet. What would be the interpretation for the preemptive goal programming problem if this linear programming model had no feasible solutions?

(f) Perform some post optimality analysis by determining how far the total false alarm rate per passenger can be reduced (perhaps even below the goal) by ignoring the second budgetary guideline but fully meeting the first one.

(g) What additional post optimality analysis do you feel should be performed in order to provide top management with the information needed to make a sound judgment decision about the best trade off between (1) the total false alarm rate per passenger, (2) the total expenditure for the two new security systems per security checkpoint, and (3) the total annual maintenance cost for these two systems per security checkpoint.

Conclusion & Suggestion

In this paper we concentrate and study on some special banach and real function based on algebra. It is a different approach in banach algebra based on algebra and we are also discussed about the cartesian product, spectra banach algebra and sets determining real function algebras and their cartesian product. This paper to give good platform of the young researchers of this field and also it is useful to improve the qualitatively in future research.

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