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Necessary and Sufficient Conditions for the Stability of Fourth-Order and Fifth-Order Systems

Yeong-Jeu Sun

Professor, Department of Electrical Engineering, I-Shou University, Kaohsiung, Taiwan

ABSTRACT

This paper aims at the fourth-order and fifth-order continuous-time systems, respectively, to explore the necessary and sufficient conditions to ensure exponential stability. The main theorem shows that the necessary and sufficient conditions are only simple algebraic inequalities related to the coefficients of the characteristic equation. In other words, the stability of the fourth-order and fifth-order systems can be quickly and easily determined. Finally, we will present several numerical simulations to illustrate the practicability and correctness of the main results.

KEYWORDS: *Exponential stability, necessary and sufficient condition, fourth-order system, fifth-order systems*

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1. INTRODUCTION

Stability analysis has always been one of the topics explored by scholars and engineers engaged in systems engineering. In the past, various methodologies have been proposed to analyze the stability of the system, such as Lyapunov approach, Nyquist stability analysis, time-domain stability analysis, small-signal stability analysis, and large signal stability analysis; see, for instance, [1-7] and the references therein. However, most studies only provide sufficient conditions to ensure the stability of the system, and few results propose the necessary and sufficient conditions to ensure stability. It has proved that necessary and sufficient conditions are not only more important than sufficient conditions, but also more challenging.

As we know, stability criteria that are not easy to calculate or solve too complicated are difficult to be favored by users; on the contrary, fast and convenient stability criteria can provide users with great convenience. In this paper, aiming at the fourth-order and fifth-order continuous-time systems, two sets of simple stability criteria are proposed to ensure that the above two systems achieve exponential stability. It is worth mentioning that the proposed criteria are necessary and sufficient conditions to achieve exponential stability.

2. PROBLEM FORMULATION AND MAIN RESULTS

As a start, we consider the characteristic equation of the fourth-order system:

$$s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0 \qquad (1)$$

with $a_i \in \Re, \forall i \in \{0,1,2,3\}$.

Now, we are in a position to present the first main result for the exponential stability of the system (1).

Theorem 1: A fourth-order continuous-time system, with characteristic equation of (1) is exponentially stable if and only if the following conditions are satisfied:

1. $a_i > 0, \forall i \in \{0,1,2,3\}$;
2. $a_1 a_2 a_3 - a_1^2 - a_0 a_3^2 > 0$.

Proof:

$$a_3>0, a_2>0, a_1>0, a_0>0,$$
$$a_1a_2a_3-a_1^2-a_0a_3^2>0$$

$$\Leftrightarrow \quad a_3>0, a_2>0, a_1>0, a_0>0,$$
$$a_1(a_2a_3-a_1)>a_0a_3^2$$

$$\Leftrightarrow \quad a_3>0, a_2>0, a_1>0, a_0>0, a_2a_3-a_1>0,$$
$$a_1a_2a_3-a_1^2-a_0a_3^2>0$$

$$\Leftrightarrow \quad a_3>0, a_2>0, a_1>0, a_0>0, \frac{a_2a_3-a_1}{a_3}>0,$$
$$\frac{a_1a_2a_3-a_1^2-a_0a_3^2}{a_2a_3-a_1}>0$$

Such a fourth-order continuous-time system is exponentially stable, in view of Routh stability criterion [8] with Routh array:

s^4	1	a_2	a_0
s^3	a_3	a_1	
s^2	$\dfrac{a_2a_3-a_1}{a_3}$	a_0	
s^1	$\dfrac{a_1a_2a_3-a_1^2-a_0a_3^2}{a_2a_3-a_1}$		
s^0	a_0		

This completes the proof. □

Now we present another main result for the exponential stability of the fifth-order continuous-time system.

Theorem 2: A fifth-order continuous-time system, with characteristic equation of

$$s^5+a_4s^4+a_3s^3+a_2s^2+a_1s+a_0=0, \qquad (2)$$

is exponentially stable if and only if the following conditions are satisfied:

1. $a_i>0, \forall i\in\{0,1,2,3,4\}$,
2. $a_4>\max\left\{\frac{a_2}{a_3}, \frac{a_0}{a_1}\right\}$,
3. $b_2(a_2b_1-a_4b_2)-a_0b_1^2>0$,

where $b_1=a_3a_4-a_2$ and $b_2=a_1a_4-a_0$.

Proof:

$$a_i>0, \forall i\in\{0,1,2,3,4\}, \qquad a_4>\max\left\{\frac{a_2}{a_3}, \frac{a_0}{a_1}\right\},$$
$$b_2(a_2b_1-a_4b_2)-a_0b_1^2>0$$

$$\Leftrightarrow \quad a_i>0, \forall i\in\{0,1,2,3,4\}, b_1>0, b_2>0,$$
$$b_2(a_2b_1-a_4b_2)-a_0b_1^2>0$$

$$\Leftrightarrow \quad a_i>0, \forall i\in\{0,1,2,3,4\}, b_1>0, b_2>0,$$
$$a_2b_1-a_4b_2>\frac{a_0b_1^2}{b_2}$$

$$\Leftrightarrow \quad a_i>0, \forall i\in\{0,1,2,3,4\}, b_1>0, b_2>0,$$
$$a_2b_1-a_4b_2>0,$$
$$b_2(a_2b_1-a_4b_2)-a_0b_1^2>0$$

$$\Leftrightarrow \quad a_i>0, \forall i\in\{0,1,2,3,4\}, b_1>0,$$
$$a_2b_1-a_4b_2>0, b_2(a_2b_1-a_4b_2)-a_0b_1^2>0$$

$$\Leftrightarrow \quad a_i>0, \forall i\in\{0,1,2,3,4\}, \frac{b_1}{a_4}>0,$$
$$\frac{a_2b_1-a_4b_2}{b_1}>0,$$
$$\frac{b_2(a_2b_1-a_4b_2)-a_0b_1^2}{a_4(a_2b_1-a_4b_2)}>0$$

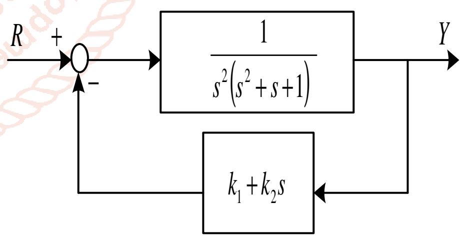
Such a fifth-order continuous-time system is exponentially stable, in view of Routh stability criterion with Routh array:

s^5	1	a_3	a_1
s^4	a_4	a_2	a_0
s^3	$\dfrac{b_1}{a_4}$	$\dfrac{b_2}{a_4}$	
s^2	$\dfrac{a_2b_1-a_4b_2}{b_1}$	a_0	
s^1	$\dfrac{(a_2b_1-a_4b_2)b_2-a_0b_1^2}{a_4(a_2b_1-a_4b_2)}$		
s^0	a_0		

This completes the proof. □

3. NNMERICAL SIMULATIONS

Example 1: Consider the following feedback control system:



Obviously, the characteristic equation of above system is $s^4+s^3+s^2+k_2s+k_1=0$. Consequently, by Theorem 1, we conclude that the above system is exponentially stable if and only if

$$k_1>0, k_2>0, \text{ and } k_2-k_2^2-k_1>0. \qquad (3)$$

In case of $(k_1,k_2)=(0.2, 0.5)$, the conditions of (3) are satisfied, so the system is exponentially stable and the unit step response of such a system is shown in Figure 1. On the other hand, in case of $(k_1,k_2)=(0.3, 0.5)$, the conditions of (3) are not satisfied, so the system is not exponentially stable and the unit step response of such a system is shown in Figure 2.

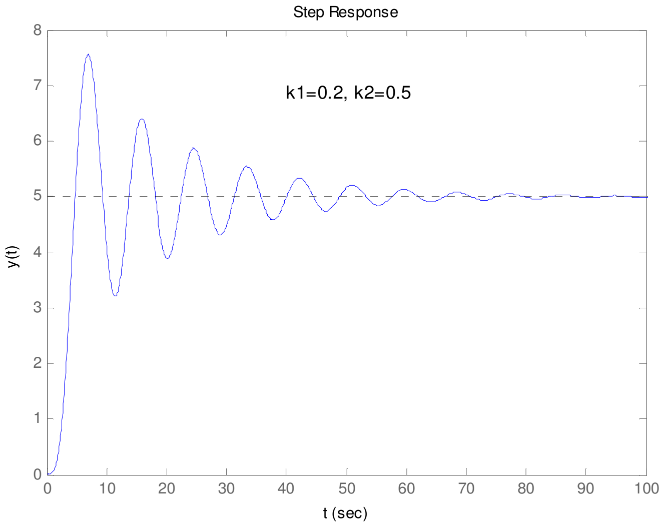


Figure 1: Unit step response of Example 1 with $(k_1, k_2) = (0.3, 0.5)$.

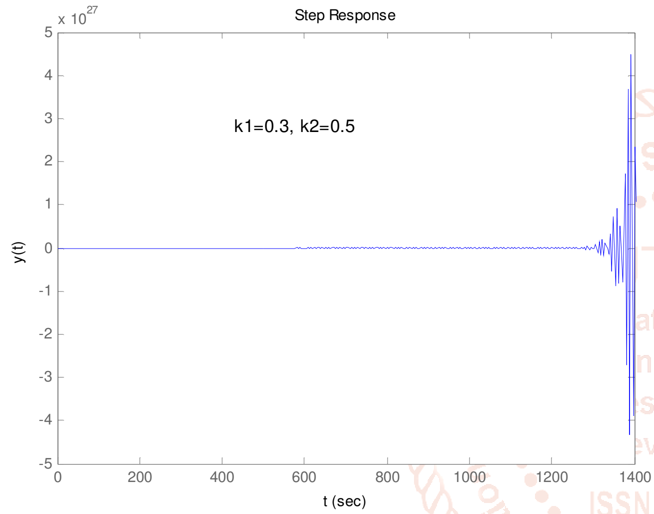
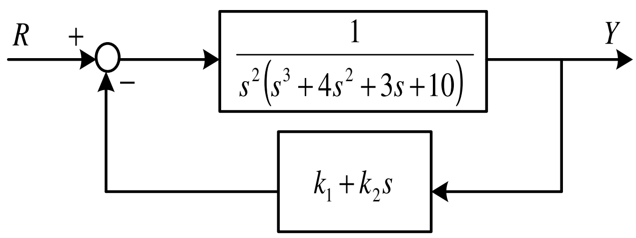


Figure 2: Unit step response of Example 1 with $(k_1, k_2) = (0.2, 0.5)$.

Example 2: Consider the following feedback control system:



Clearly, the characteristic equation of above system is $s^5+4s^4+3s^3+10s^2+k_2 s+k_1=0$. Consequently, by Theorem 2, we conclude that the above system is exponentially stable if and only if

$$k_1>0,\, k_2>0,\, 4>\max\left\{\frac{10}{3}, \frac{k_1}{k_2}\right\}, \qquad (4a)$$

and

$$(4k_2-k_1)(20-16k_2+4k_1)-4k_1>0. \qquad (4b)$$

In case of $(k_1, k_2)=\left(1, \frac{7}{8}\right)$, the conditions of (4) are satisfied, so the system is exponentially stable and the unit step response of such a system is shown in Figure 3. On the other hand, in case of $(k_1, k_2)=\left(\frac{25}{4}, \frac{35}{16}\right)$, the conditions of (4) are not satisfied, so the system is not exponentially stable and the unit step response of such a system is shown in Figure 4.

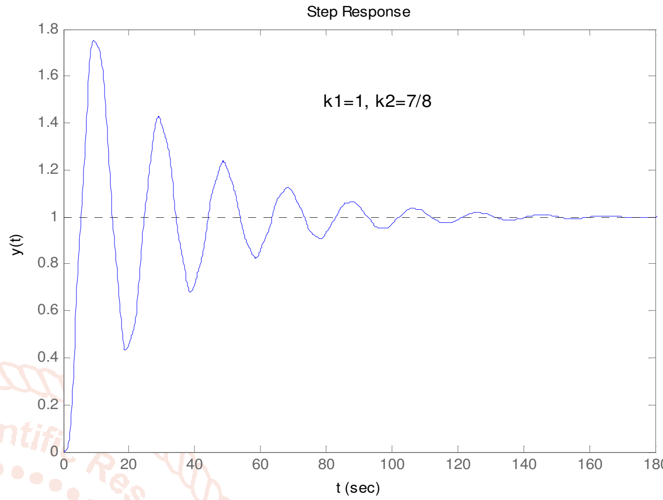


Figure 3: Unit step response of Example 2 with $(k_1, k_2)=\left(1, \frac{7}{8}\right)$.

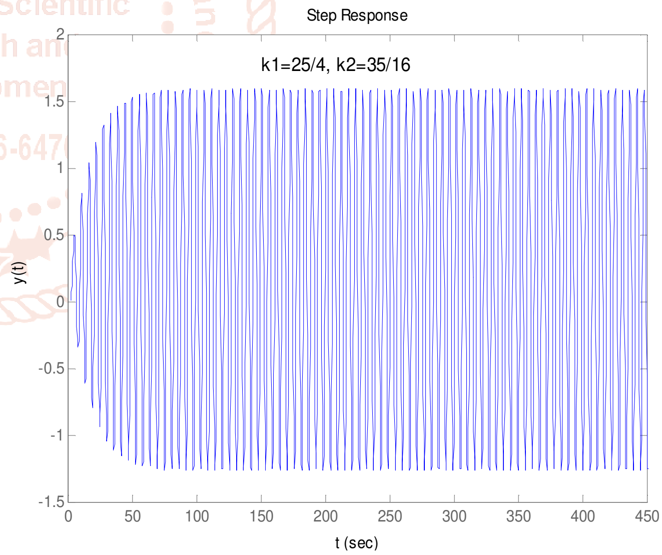


Figure 4: Unit step response of Example 2 with $(k_1, k_2)=\left(\frac{25}{4}, \frac{35}{16}\right)$.

4. CONCLUSION

In this paper, the stability analysis of fourth-order and fifth-order continuous-time systems has been explored. The main theorems have showed that the necessary and sufficient conditions are only simple algebraic inequalities related to the coefficients of the characteristic equation. In other words, the stability of the fourth-order and fifth-order systems can be quickly and easily determined. Finally, several numerical simulations have been presented to illustrate the practicability and correctness of the main results.

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