International Journal of Trend in Scientific Research and Development (IJTSRD)
Volume 6 Issue 1, November-December 2021 Available Online: www.ijtsrd.com e-ISSN: 2456 – 6470

Robust Filter Design for a Class of Uncertain Chaotic Systems and Its Circuit Implementation

Yeong-Jeu Sun

Professor, Department of Electrical Engineering, I-Shou University, Kaohsiung, Taiwan

ABSTRACT

In this paper, the concept of robust filter is established and the filter design for a class of uncertain chaotic systems is investigated. Based on differential and integral inequalities, a linear filter is proposed to realize the global exponential stabilization of uncertain chaotic systems. The guaranteed exponential convergence rate can also be correctly estimated. Besides, some numerical simulations with circuit realization are provided to show the capability and feasibility of the obtained result.

KEYWORDS: *Robust filter, uncertain chaotic systems, linear filter, exponential convergence rate*

How to cite this paper: Yeong-Jeu Sun "Robust Filter Design for a Class of Uncertain Chaotic Systems and Its Circuit Implementation" Published in International Journal of Trend in Scientific Research and Development (ijtsrd), ISSN: 2456-6470, Volume-6 | Issue-1, December 2021, pp.1735-1738, URL: www.ijtsrd.com/papers/ijtsrd49111.pdf

IJTSRD49111

1. INTRODUCTION

The development of noise filters has always been an urgent task for engineers and scholars. In the past ten years, related documents have proposed various methods of filtering noise; see, for example, [1]-[8] and the references therein.

In this paper, we will firstly propose a new concept about robust filter and a filter design for a class of uncertain chaotic systems will be developed to guarantee that the resulting signals can converge to zero in some exponential convergence rate. Furthermore, the guaranteed exponential convergence rate of the closed-loop system can be correctly estimated. Finally, some numerical simulations are given to exhibit the effectiveness of the main results.

Throughout this paper, $\Re^n$ denotes the n-dimensional Euclidean space, $\|x\|$ denotes the Euclidean norm of the vector $x \in \Re^n$, $|a|$ denotes the modulus of a real number a, and $\lambda_{\min}(P)$ denotes the minimum eigenvalue of the matrix P with real eigenvalues.

2. PROBLEM FORMULATION AND MAIN RESULTS

Consider the following uncertain chaotic systems with single input described by

$$\dot{x}_1 = \Delta a_8 x_1 + \Delta a_1 x_2 + \Delta a_7 x_3^2, \qquad (1a)$$

$$\dot{x}_2 = \Delta a_2 x_1 + \Delta a_3 x_2 + \Delta a_4 x_3 - \Delta a_6 x_1 x_3 + u, \qquad (1b)$$

$$\dot{x}_3 = \Delta a_5 x_2 + \Delta a_9 x_3 + \Delta a_6 x_1 x_2 - \Delta a_7 x_1 x_3, \qquad (1c)$$

where $x(t) := [x_1(t) \quad x_2(t) \quad x_3(t)]^T \in \Re^{3\times 1}$ is the state vector, $u(t) \in \Re$ is the control input, and $\Delta a_i, \forall i \in \{1,2,\cdots,9\}$ are uncertain parameters with

$$|\Delta a_i| \le \overline{a_i}, \forall i \in \{1,2,3,\cdots,7\}, \qquad (1d)$$

$$\Delta a_8 \le -r_1 < 0, \Delta a_9 \le -r_3 < 0. \qquad (1e)$$

Remark 1: It is worth mentioning that both of Lü chaotic system and Chen chaotic system are special cases of uncertain systems of (1). Besides, the typical state trajectories of Lü chaotic system and Chen chaotic system are depicted in Fig. 1 and Fig. 2, respectively.

Before presenting the main result, we provide a definition as follows.

Definition 1: The system (1) is said to be globally exponentially stable if there exist a control u and positive numbers α and k, such that

$$\|x(t)\| \le k \cdot e^{-\alpha t}, \quad \forall t \ge 0, \; i \in \{1,2,3\}.$$

In this case, the positive number α and control law of u are called the exponential convergence rate and robust filter, respectively.

The goal of this paper is to search a simple robust filter such that the global exponential stabilization of uncertain chaotic systems of (1) can be guaranteed. Besides, an estimate of the exponential convergence rate of such stable systems is investigated.

The main theorem of this paper is stated as follows

Theorem 1: The uncertain chaotic systems of (1) is globally exponentially stabilizable at the zero equilibrium point by the linear filter

$$u = -rx_2, \qquad (2a)$$

with

$$r > \bar{a}_3 + \frac{(\bar{a}_1 + \bar{a}_2)^2}{4r_1} + \frac{(\bar{a}_4 + \bar{a}_5)^2}{4r_3}. \qquad (2b)$$

Meanwhile, the guaranteed exponential convergence rate is given by $\lambda_{\min}(P)$, with

$$P := \begin{bmatrix} r_1 & \frac{-(\bar{a}_1+\bar{a}_2)}{2} & 0 \\ \frac{-(\bar{a}_1+\bar{a}_2)}{2} & r - \bar{a}_3 & \frac{-(\bar{a}_4+\bar{a}_5)}{2} \\ 0 & \frac{-(\bar{a}_4+\bar{a}_5)}{2} & r_3 \end{bmatrix}. \qquad (3)$$

Proof. It can be readily obtained that $\det([r_1]) > 0$, $\det\left(\begin{bmatrix} r_1 & \frac{-(\bar{a}_1+\bar{a}_2)}{2} \\ \frac{-(\bar{a}_1+\bar{a}_2)}{2} & r_2 - \bar{a}_3 \end{bmatrix}\right) > 0$, and $\det(P) > 0$, in view of (1d), (1e), and (2). This implies that the matrix of P is positive definite. Let

$$W(t) := x_1^2(t) + x_2^2(t) + x_3^2(t). \qquad (4)$$

The time derivative of $W(t)$ along the trajectories of the closed-loop system (1) with (2)-(4), is given by

$$\begin{aligned} \frac{dW(t)}{dt} &= 2x_1[\Delta a_8 x_1 + \Delta a_1 x_2 + \Delta a_7 x_3^2] \\ &\quad + 2x_2[\Delta a_2 x_1 + \Delta a_3 x_2 + \Delta a_4 x_3 \\ &\quad - \Delta a_6 x_1 x_3 - r x_2] \\ &\quad + 2x_3[\Delta a_5 x_2 + \Delta a_9 x_3 + \Delta a_6 x_1 x_2 \\ &\quad - \Delta a_7 x_1 x_3] \\ &\le -2r_1 x_1^2 + 2(|\Delta a_1| + |\Delta a_2|)|x_1||x_2| + 2|\Delta a_3| x_2^2 \\ &\quad + 2(|\Delta a_4| + |\Delta a_5|)|x_2||x_3| - 2r_3 x_3^2 - 2r x_2^2 \\ &\le -2r_1 x_1^2 + 2(\bar{a}_1 + \bar{a}_2)|x_1||x_2| + 2\bar{a}_3 x_2^2 \\ &\quad + 2(\bar{a}_4 + \bar{a}_5)|x_2||x_3| - 2r_3 x_3^2 - 2r x_2^2 \\ &= -2r_1 x_1^2 + 2(\bar{a}_1 + \bar{a}_2)|x_1||x_2| \\ &\quad + 2(\bar{a}_4 + \bar{a}_5)|x_2||x_3| - 2r_3 x_3^2 \\ &\quad - 2(r - \bar{a}_3) x_2^2 \\ &= -2x^T P x \\ &\le -2\lambda_{\min}(P)\|x\|^2 \\ &= -2\lambda_{\min}(P) W(t), \; \forall t \ge 0. \end{aligned}$$

Therefore, we have

$$\begin{aligned} & e^{2\lambda_{\min}(P)t} \cdot \dot{W}(t) + 2\alpha e^{2\lambda_{\min}(P)t} W(t) \\ &= \frac{d}{dt}\left[e^{2\lambda_{\min}(P)t} \cdot W(t)\right] \le 0, \; \forall t \ge 0. \end{aligned}$$

It follows that

$$\begin{aligned} & \int_0^t \frac{d}{dt}\left[e^{2\lambda_{\min}(P)t} \cdot W(t)\right] dt \\ &= e^{2\lambda_{\min}(P)t} \cdot W(t) - W(0) \\ &\le \int_0^t 0 \, dt = 0, \quad \forall t \ge 0. \end{aligned} \qquad (5)$$

Thus, from (4) and (5), it can be readily obtained that $\|x(t)\|^2 = W(t) \le e^{-2\lambda_{\min}(P)t} W(0), \quad \forall t \ge 0.$

Consequently, we conclude that
$\|x(t)\| \le e^{-\lambda_{\min}(P)t} \sqrt{W(0)}, \quad \forall t \ge 0.$

This completes the proof. □

3. CIRCUIT IMPLEMENTATION WITH NUMERICAL SIMULATIONS

Example 1: Consider the system (1) with

$$\bar{a}_1 = 36, \; \bar{a}_2 = 0, \; \bar{a}_3 = 28, \; \bar{a}_4 = \bar{a}_5 = 1, \qquad (6a)$$

$$\bar{a}_6 = \bar{a}_7 = 2, \; r_1 = 35, \; r_3 = 3. \qquad (6b)$$

It can be readily obtained that

$$38 > \bar{a}_3 + \frac{(\bar{a}_1 + \bar{a}_2)^2}{4r_1} + \frac{(\bar{a}_4 + \bar{a}_5)^2}{4r_3}$$

Consequently, by Theorem 1 with the choice $r = 38$, we conclude that the system (1) with (6) and $u = -38x_2$, is globally exponentially stable. In this case, from (4), the guaranteed exponential convergence rate is given by

$$\lambda_{\min}(P) = 0.2946.$$

The typical state trajectories of the feedback-controlled system are depicted in Fig. 3. Besides, the control signal and the electronic circuit to realize such a control law are depicted in Fig. 4 and Fig. 5, respectively.

4. CONCLUSION

In this paper, the concept of robust filter has been established and the filter design for a class of uncertain chaotic systems has been studied. Based on differential and integral inequalities, a linear filter has been developed to realize the global exponential stabilization of uncertain chaotic systems. The guaranteed exponential convergence rate can also be correctly estimated. Besides, some numerical simulations with circuit realization have been provided to show the capability and feasibility of the obtained result.

ACKNOWLEDGEMENT

The author thanks the Ministry of Science and Technology of Republic of China for supporting this work under grant MOST 109-2221-E-214-014. Furthermore, the author is grateful to Chair Professor Jer-Guang Hsieh for the useful comments.

REFERENCES

[1] M. S. Bakr, “Triple-mode microwave filters with arbitrary prescribed transmission zeros,” IEEE Access, vol. 9, pp. 22045-22053, 2021.

[2] G. Chaudhary and Y. Jeong, “Arbitrary prescribed flat wideband group delay absorptive microstrip bandpass filters,” IEEE Transactions on Microwave Theory and Techniques, vol. 69, pp. 1404-1414, 2021.

[3] X. Chen, Y. Wang, and Q. Zhang, “Ring-shaped D-band E-plane filtering coupler,” IEEE Microwave and Wireless Components Letters, vol. 31, pp. 953-956, 2021.

[4] K. Ding, J. Wu, and L. Xie, “Minimum-degree distributed graph filter design,” IEEE Transactions on Signal Processing, vol. 69, pp. 1083-1096, 2021.

[5] M. Fan, K. Song, L. Yang, and G.G. Roberto, “Frequency-reconfigurable input-reflectionless bandpass filter and filtering power divider with constant absolute bandwidth,” IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 68, pp. 2424-2428, 2021.

[6] R. V. Snyde, S. Bastioli, and G. Macchiarella, “The extracted-zero: A practical solution for transmission zeros in wideband filters,” IEEE Microwave and Wireless Components Letters, vol. 31, pp. 1043-1046, 2021.

[7] S. P. Talebi, S. Werner, V. Gupta, and Y. F. Huang, “On stability and convergence of distributed filters,” IEEE Signal Processing Letters, vol. 28, pp. 28-32, 2021.

[8] X. Zhu and G.G. Roberto, “Exploiting parasitic capacitances in 3-D inductors to design RF CMOS quasi-elliptic-type broad-band bandpass filters,” IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 68, pp. 3128-3133, 2021..

[9] K. D. Xu, S. Lu, Y. J. Guo, and Q. Chen, “High-order mode of spoof surface plasmon polaritons and its application in bandpass filters,” IEEE Transactions on Plasma Science, vol. 49, pp. 269-275, 2021.

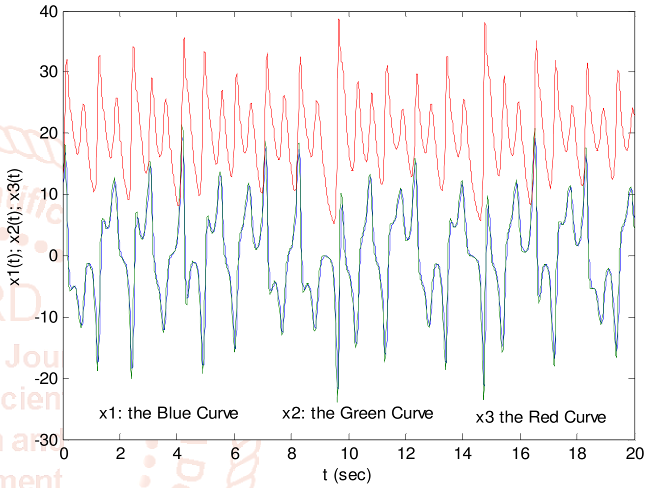

Figure 1: Typical state trajectories of Lü chaotic system.

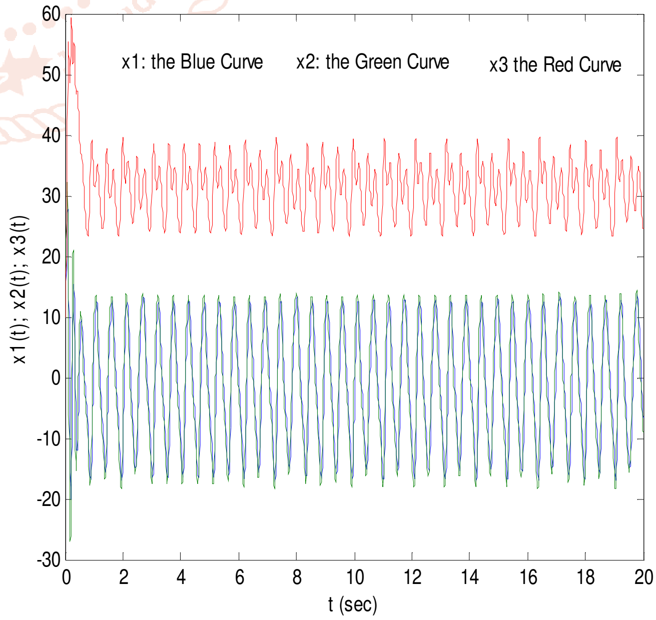

Figure 2: Typical state trajectories of Chen chaotic system.

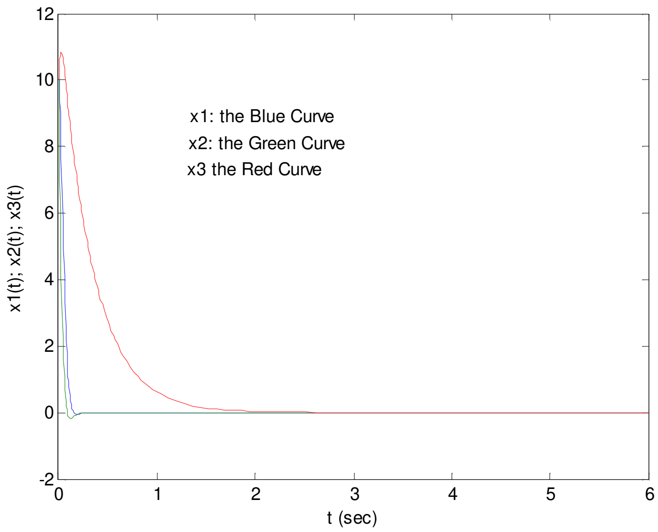

Figure 3: Typical state trajectories of the feedback-controlled system of (1) with (6) and $u = -38x_2$.

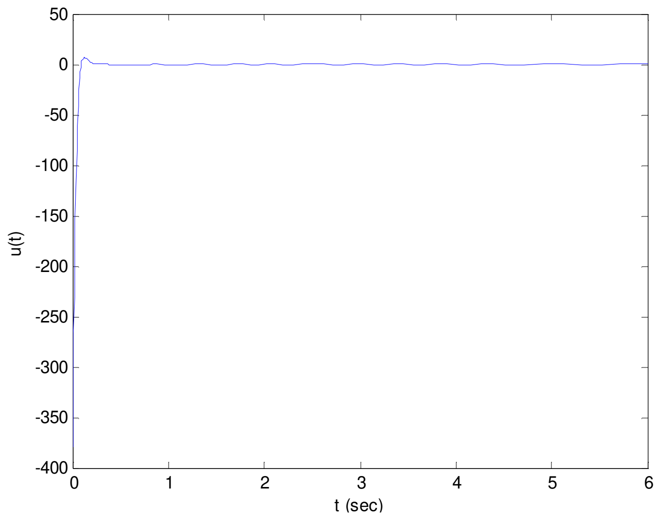

Figure 4: The control signals of $u = -38x_2$.

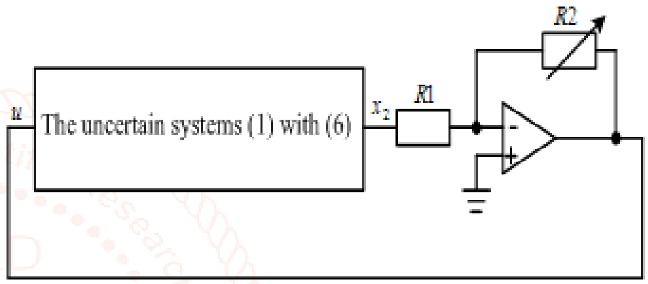

Figure 5: The diagram of implementation of Example 1, where $R1 = 10k\Omega$ and $R2 = 380k\Omega$.