

Duality Theorems of Nonlinear Time-Lag Systems with State Saturation Nonlinearities and Multiple Time Delays

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ABSTRACT

In this paper, the problem of the asymptotic state estimator design for a class of time-lag systems with state saturation nonlinearities is explored. A simple criterion is established to guarantee the asymptotic convergence of the observation error between the observer state estimate and the true state. Such a criterion can be view as the dual to static output feedback controller for a class of time-lag systems with multiple time-varying delays. At last, an upper bound of arbitrary time-varying delays is also derived to assure the observation error can converge asymptotically to zero.

KEYWORDS: Saturation nonlinearities, time-lag systems, dual problem, static output feedback control

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1. INTRODUCTION

Any physical dynamic systems inherently comprise, more or less, some time-lag phenomena for the reason that the energy in the systems propagates with a finite speed. Time-lag systems has been extensively explored in recent years; see, for instance, [1-8] and is often encountered in various areas, such as chemical engineering systems, the rolling mill, the ship stabilization, the aircraft stabilization, the manual control, the nuclear reactor, AIDS epidemic, and systems with lossless transmission lines. Frequently, the existence of the delays in many control systems is a source of oscillation and a source of instability.

On the other hand, saturation nonlinearities frequently appear in most physical systems, e.g., the states are constrained to stay within a bounded set due to physical limitations of the devices or by protection equipment. Furthermore, form practical considerations, it is either impracticable or inappropriate to measure all the elements of the state vector. The state observer has come to take its pride of place in filter theory, system identification,

and control design. Nevertheless, the state observer design of dynamic systems with saturation nonlinearities is in general not as easy as that without saturation nonlinearities. On the basis of the above-mentioned reasons, the observer design of time-lag systems with saturation nonlinearities is actually crucial and significant.

In this paper, the asymptotic state estimator design for a class of time-lag systems with state saturation nonlinearities is investigated. Based on the time-domain approach, the duality between state estimator design and static output feedback design will be provided. Besides, an upper bound of arbitrary time-varying delays is also derived to guarantee the global asymptotic stability of the resulting error system.

2. PROBLEM FORMULATION AND MAIN RESULTS

Nomenclature

\mathcal{R}^n := the n -dimensional real space,

$\mathcal{R}^{m \times n}$:= the set of all real m by n matrices,

A^T := the transpose of the matrix A ,

I := the unit matrix,

$\|A\|$:= the induced Euclidean norm of the matrix A ,

$\lambda_{\max}(Q)$ (res. $\lambda_{\min}(Q)$)::= the maximum (res. minimum) eigenvalue of the symmetric matrix Q ,

$Q > 0$:= the symmetric matrix Q is positive definite,

$\underline{p} := \{1, 2, \dots, p\}$,

$\bar{p} := \{0, 1, 2, \dots, p\}$.

As a start, we consider the following uncertain time-lag system with multiple discrete and distributed time delays:

$$\begin{aligned} \dot{x}(t) &= A_0 x(t) + \sum_{i=1}^p A_i x(t - h_i(t)) \\ &+ \Delta f(x(t - \tau_1(t)), x(t - \tau_2(t)), \dots, x(t - \tau_q(t))), \\ \forall t \geq 0, \end{aligned} \tag{1a}$$

$$x(t) = \theta(t), \quad \forall t \in [-H, 0], \tag{1b}$$

where $x \in \mathbb{R}^n$ is the state vector, the uncertain term $\Delta f \in \mathbb{R}^n$ is a smooth function with $\Delta f(0, 0, \dots, 0) = 0$, $A_i \in \mathbb{R}^{n \times n}, \forall i \in \bar{p}$, $B_i \in \mathbb{R}^{n \times n}, \forall i \in \underline{q}$, $h_i(t) \geq 0, \forall i \in \underline{p}$ and $\tau_i(t) \geq 0, \forall i \in \underline{q}$ are arbitrary delay arguments with $0 \leq h_i(t) \leq H$ and $0 \leq \tau_i(t) \leq H$ for some constant H , and $\theta(t)$ is a given continuous vector-valued initial function. In addition, the uncertain term Δf satisfies

$$\begin{aligned} &\|\Delta f(x(t - \tau_1(t)), x(t - \tau_2(t)), \dots, x(t - \tau_q(t)))\| \\ &\leq \sum_{i=1}^q \|x(t - \tau_i(t))\|. \end{aligned}$$

For the convenience of the sequel, we present the first main result, which is a delay-dependent criterion, for the global asymptotic stability of the systems (1).

Lemma 1. The system (1) is globally asymptotically stable provided that the following conditions are satisfied.

(i) $A := \sum_{i=0}^p A_i$ is a Hurwitz matrix;

(ii)
$$\begin{aligned} &\sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}^3(P)}} > \left(\sum_{i=1}^p h_i(t) \cdot \|A_i\| \right) \\ &\times \left[\left(\sum_{i=0}^p \|A_i\| \right) + \left(\sum_{i=1}^q k_i \right) \right] + \left(\sum_{i=1}^q k_i \right), \forall t \geq 0, \end{aligned}$$

where $P > 0$ is the unique solution to the Lyapunov equation of $A^T P + PA = -2I$.

Proof. From (1), we have

$$\begin{aligned} \dot{x}(t) &= A_0 x(t) + \sum_{i=1}^p A_i x(t) - \left[\sum_{i=1}^p A_i \left(\int_{t-h_i(t)}^t \dot{x}(z) dz \right) \right] \\ &+ \Delta f(x(t - \tau_1(t)), x(t - \tau_2(t)), \dots, x(t - \tau_q(t))) \\ &= \left(\sum_{i=0}^p A_i \right) x(t) - \left\{ \sum_{i=1}^p A_i \right. \\ &\times \int_{t-h_i(t)}^t \left[\left(\sum_{i=0}^p A_i x(z - h_i(z)) \right) \right. \\ &\left. \left. + \Delta f(x(z - \tau_1(z)), \dots, x(z - \tau_q(z))) \right) \right] dz \left. \right\} \\ &+ \Delta f(x(t - \tau_1(t)), \dots, x(t - \tau_q(t))), \quad \forall t \geq 0, \end{aligned} \tag{2a}$$

$$x(t) = \theta(t), \quad \forall t \in [-H, 0]. \tag{2b}$$

Define the dynamic system

$$\begin{aligned} \dot{y}(t) &= A_0 y(t) + \sum_{i=1}^p A_i y(t) - \left[\sum_{i=1}^p A_i \left(\int_{t-h_i(t)}^t \dot{y}(z) dz \right) \right] \\ &+ \Delta f(y(t - \tau_1(t)), y(t - \tau_2(t)), \dots, y(t - \tau_q(t))) \\ &= \left(\sum_{i=0}^p A_i \right) y(t) - \left\{ \sum_{i=1}^p A_i \right. \\ &\times \int_{t-h_i(t)}^t \left[\left(\sum_{i=0}^p A_i y(z - h_i(z)) \right) \right. \\ &\left. \left. + \Delta f(y(z - \tau_1(z)), \dots, y(z - \tau_q(z))) \right) \right] dz \left. \right\} \\ &+ \Delta f(y(t - \tau_1(t)), \dots, y(t - \tau_q(t))), \quad \forall t \geq 0, \end{aligned} \tag{3a}$$

$$y(t) = x(-H), \quad \forall t \in [-2H, -H] \tag{3b}$$

Furthermore, we define $y_t(s) = y(t + s), \forall s \in [-2H, 0]$, and $\|y_t\|_s := \sup_{-2H \leq r \leq 0} \|y(t + r)\|$. By comparing (3) with (2), it is easy to see that $y(t) = x(t), \forall t \geq 0$. Thus it can be deduced that

$$\begin{aligned} &\left\| - \sum_{i=1}^p A_i \times \int_{t-h_i(t)}^t \left[\left(\sum_{i=0}^p A_i y(z - h_i(z)) \right) \right. \right. \\ &\left. \left. + \Delta f(y(z - \tau_1(z)), \dots, y(z - \tau_q(z))) \right) \right] dz \right\| \\ &\leq \sum_{i=1}^p \|A_i\| \cdot h_i(t) \cdot \left[\left(\sum_{i=0}^p \|A_i\| \cdot \|y_t\|_s \right) \right. \\ &\left. + \left(\sum_{i=1}^q k_i \cdot \|y_t\|_s \right) \right] \\ &= \left\{ \sum_{i=1}^p \|A_i\| \cdot h_i(t) \cdot \left[\left(\sum_{i=0}^p \|A_i\| \right) + \left(\sum_{i=1}^q k_i \right) \right] \right\} \cdot \|y_t\|_s, \end{aligned} \tag{4}$$

$$\forall t \geq 0,$$

Let

$$V(y(t)) = y^T(t) P y(t). \tag{5}$$

The time derivative of $V(y(t))$ along the trajectories of the system (3) is given by

$$\begin{aligned}
 \dot{V}(y(t)) &= y^T(t) [A^T P + PA] y(t) \\
 &\quad - 2y^T(t) P \sum_{i=1}^p A_i \\
 &\quad \times \int_{t-h_i(t)}^t \left[\left(\sum_{i=0}^p A_i y(z-h_i(z)) \right) \right. \\
 &\quad \left. + \Delta f(y(z-\tau_1(z)), \dots, y(z-\tau_q(z))) \right] dz \\
 &\quad + 2y^T(t) P \Delta f(y(z-\tau_1(z)), \dots, \\
 &\quad y(t-\tau_q(t))) \\
 &= -2\|y(t)\|^2 - 2y^T(t) P \sum_{i=1}^p A_i \\
 &\quad \times \int_{t-h_i(t)}^t \left[\left(\sum_{i=0}^p A_i y(z-h_i(z)) \right) + \Delta f \right. \\
 &\quad \left. (y(z-\tau_1(z)), \dots, y(z-\tau_q(z))) \right] dz \\
 &\quad + 2y^T(t) P \Delta f(y(t-\tau_1(t)), y(t-\tau_2(t)), \\
 &\quad \dots, y(t-\tau_q(t))), \forall t \geq 0. \tag{6}
 \end{aligned}$$

Applying (4) to (6) yields

$$\begin{aligned}
 \dot{V}(y(t)) &\leq -2\|y(t)\|^2 + 2\|y(t)\| \cdot \lambda_{\max}(P) \\
 &\quad \cdot \left\{ \sum_{i=1}^p \|A_i\| \cdot h_i(t) \cdot \left[\left(\sum_{i=0}^p \|A_i\| \right) + \left(\sum_{i=1}^q k_i \right) \right] \right\} \\
 &\quad \cdot \|y_t\|_s + 2\|y(t)\| \lambda_{\max}(P) \cdot \left(\sum_{i=1}^q k_i \right) \cdot \|y_t\|_s \\
 &= -2\|y(t)\|^2 + 2\|y(t)\| \cdot \lambda_{\max}(P) \\
 &\quad \cdot \left\{ \sum_{i=1}^p \|A_i\| \cdot h_i(t) \cdot \left[\left(\sum_{i=0}^p \|A_i\| \right) + \left(\sum_{i=1}^q k_i \right) \right] \right. \\
 &\quad \left. + \left(\sum_{i=1}^q k_i \right) \right\} \cdot \|y_t\|_s, \quad \forall t \geq 0. \tag{7}
 \end{aligned}$$

By (ii), there exists a sufficiently small constant $\varepsilon_1 > 0$ such that

$$\begin{aligned}
 \varepsilon_2 &:= 1 - (1 + \varepsilon_1) \sqrt{\frac{\lambda_{\max}^3(P)}{\lambda_{\min}(P)}} \times \left\{ \left(\sum_{i=1}^p h_i(t) \cdot \|A_i\| \right) \right. \\
 &\quad \cdot \left. \left(\sum_{i=0}^p \|A_i\| + \sum_{i=1}^q k_i \right) + \left(\sum_{i=1}^q k_i \right) \right\} > 0, \quad \forall t \geq 0. \tag{8}
 \end{aligned}$$

In the spirit of Theorem 4.2 in [2], with $p(s) = (1 + \varepsilon_1)^2 s$, we suppose that

$$\begin{aligned}
 y^T(t+r) P y(t+r) &< (1 + \varepsilon_1)^2 y^T(t) P y(t), \\
 \forall -2H \leq r \leq 0.
 \end{aligned}$$

It follows that

$$\begin{aligned}
 \lambda_{\min}(P) \|y(t+r)\|^2 &< (1 + \varepsilon_1)^2 \lambda_{\max}(P) \|y(t)\|^2, \\
 \forall -2H \leq r \leq 0.
 \end{aligned}$$

This show that

$$\begin{aligned}
 \|y(t+r)\| &< (1 + \varepsilon_1) \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \|y(t)\|, \\
 \forall -2H \leq r \leq 0. \tag{9}
 \end{aligned}$$

Substituting (9) to (7), it can be shown that

$$\begin{aligned}
 \dot{V}(y(t)) &\leq -2\|y(t)\|^2 + 2\|y(t)\| \cdot \lambda_{\max}(P) \\
 &\quad \cdot \left\{ \left(\sum_{i=1}^p \|A_i\| \cdot h_i(t) \right) \left[\left(\sum_{i=0}^p \|A_i\| \right) + \left(\sum_{i=1}^q k_i \right) \right] \right. \\
 &\quad \left. + \left(\sum_{i=1}^q k_i \right) \right\} \cdot (1 + \varepsilon_1) \cdot \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \cdot \|y(t)\| \\
 &= -2\varepsilon_2 \|y(t)\|^2, \quad \forall t \geq 0. \tag{10}
 \end{aligned}$$

in view of (8). Thus, by Theorem 4.2 in [2] with (5) and (10), we conclude that the system (2) and the system (3) are both globally asymptotically stable. This completes our proof. \square

In this following, we consider nonlinear time-lag systems with state saturation nonlinearities described as

$$\begin{aligned}
 \dot{x}(t) &= A_0 x(t) + \sum_{i=1}^p A_i x(t-h_i(t)) \\
 &\quad + \sum_{i=1}^q [E_i f_{\lambda_1, \lambda_2, \dots, \lambda_n}(x(t-\tau_i(t)))] + Du(t), \\
 \forall t \geq 0, \tag{11a}
 \end{aligned}$$

$$\begin{aligned}
 y(t) &= C_0 x(t) + \sum_{i=1}^p C_i x(t-h_i(t)) \\
 &\quad + \sum_{i=1}^q [H_i f_{\lambda_1, \lambda_2, \dots, \lambda_n}(x(t-\tau_i(t)))] \\
 \forall t \geq 0, \tag{11b}
 \end{aligned}$$

$$x(t) = \theta(t), \quad \forall t \in [-H, 0], \tag{11c}$$

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^r$ is the output vector, $u \in \mathbb{R}^d$ is the input vector, $h_i(t)$'s, $\forall i \in \underline{p}$ and $\tau_i(t)$'s, $\forall i \in \underline{q}$ are arbitrary delay arguments with $0 \leq h_i(t) \leq H$ and $0 \leq \tau_i(t) \leq H$ for some constant H , and $\theta(t)$ is a given continuous vector-valued initial function. Besides, the saturation nonlinearities $f_{\lambda_1, \lambda_2, \dots, \lambda_n}(x) : \mathbb{R}^n \rightarrow \mathbb{R}^n$, with $\lambda_i > 0, \forall i \in \{1, 2, \dots, n\}$, are defined as follows.

$$f_{\lambda_1, \lambda_2, \dots, \lambda_n}(x) := \begin{bmatrix} \text{sat}_{\lambda_1}(x_1) \\ \text{sat}_{\lambda_2}(x_2) \\ \vdots \\ \text{sat}_{\lambda_n}(x_n) \end{bmatrix},$$

with $x := [x_1 \ x_2 \ \dots \ x_n]^T$ and

$$\text{sat}_\lambda(z) := \begin{cases} \lambda & , z \geq \lambda \\ z & , -\lambda < z < \lambda \\ -\lambda & , z \leq -\lambda \end{cases}$$

The following assumption is made on the system (11) throughout this paper.

(A1) There exists a matrix $K \in \mathbb{R}^{d \times r}$ such that

$\tilde{A} := \left(\sum_{i=0}^p A_i \right) + DK \left(\sum_{i=0}^p C_i \right)$ is a Hurwitz matrix and

$$\sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}} > \left(\sum_{i=1}^p h_i(t) \cdot \|A_i + DKC_i\| \right) \cdot \left(\sum_{i=0}^p \|A_i + DKC_i\| + \sum_{i=1}^q \|E_i + DKH_i\| \right) + \left(\sum_{i=1}^q \|E_i + DKH_i\| \right), \quad \forall t \geq 0,$$

where $P > 0$ is the unique solution to the Lyapunov equation of $\tilde{A}^T P + P \tilde{A} = -2I$.

Now we present the first main result for the state estimator of system (11).

Theorem 1. The state estimator of the system (11) is of existence provided that (A1) is satisfied. In this case, a suitable state estimator is given by

$$\begin{aligned} \dot{\hat{x}}(t) &= A_0 \hat{x}(t) + \sum_{i=1}^p A_i \hat{x}(t - h_i(t)) \\ &+ \sum_{i=1}^q [E_i f_{\lambda_1, \lambda_2, \dots, \lambda_n}(\hat{x}(t - \tau_i(t)))] \\ &+ Du(t) - DK[y(t) - \hat{y}(t)], \quad \forall t \geq 0, \end{aligned} \tag{12a}$$

$$\begin{aligned} \hat{y}(t) &= C_0 \hat{x}(t) + \sum_{i=1}^p C_i \hat{x}(t - h_i(t)) \\ &+ \sum_{i=1}^q [H_i f_{\lambda_1, \lambda_2, \dots, \lambda_n}(\hat{x}(t - \tau_i(t)))] \\ &\forall t \geq 0. \end{aligned} \tag{12b}$$

Proof. Define $e(t) = x(t) - \hat{x}(t)$. Then, from (11) and (12), it is easy to see that the error dynamic system is given by

$$\begin{aligned} \dot{e}(t) &= (A_0 + DKC_0)e(t) \\ &+ \sum_{i=1}^p (A_i + DKC_i)e(t - h_i(t)) \\ &+ \sum_{i=1}^q (E_i + DKH_i) [f_{\lambda_1, \lambda_2, \dots, \lambda_n}(x(t - \tau_i(t))) \\ &- f_{\lambda_1, \lambda_2, \dots, \lambda_n}(\hat{x}(t - \tau_i(t)))] , \end{aligned} \tag{13a}$$

with

$$\begin{aligned} &\left\| \sum_{i=1}^q (E_i + DKH_i) [f_{\lambda_1, \lambda_2, \dots, \lambda_n}(x(t - \tau_i(t))) \right. \\ &\quad \left. - f_{\lambda_1, \lambda_2, \dots, \lambda_n}(\hat{x}(t - \tau_i(t)))] \right\| \\ &\leq \sum_{i=1}^q \|E_i + DKH_i\| \cdot \|x(t - \tau_i(t)) - \hat{x}(t - \tau_i(t))\| \\ &= \sum_{i=1}^q \|E_i + DKH_i\| \cdot \|e(t - \tau_i(t))\|, \end{aligned} \tag{13b}$$

in view of the global Lipschitz of $\text{sat}_\lambda(z)$. Thus, by Lemma 1 with (13) and (A1), we conclude that the system (13) is globally asymptotically stable. This completes the proof. \square

In this following, we consider the time-lag system with multiple time-varying delays:

$$\begin{aligned} \dot{x}(t) &= A_0 x(t) + \sum_{i=1}^p A_i x(t - h_i(t)) \\ &+ \sum_{i=1}^q E_i f_i(x(t - \tau_i(t))) + Du(t), \\ &\forall t \geq 0, \end{aligned} \tag{14a}$$

$$\begin{aligned} y(t) &= C_0 x(t) + \sum_{i=1}^p C_i x(t - h_i(t)) \\ &+ \sum_{i=1}^q H_i f_i(x(t - \tau_i(t))), \quad \forall t \geq 0, \end{aligned} \tag{14b}$$

$$x(t) = \theta(t), \quad t \in [-H, 0], \tag{14c}$$

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^r$ is the output vector, $u \in \mathbb{R}^d$ is the input vector, $A_i \in \mathbb{R}^{n \times n}, \forall i \in \bar{p}, E_i, H_i \in \mathbb{R}^{n \times n}, \forall i \in \underline{q}, h_i(t)$'s, $\forall i \in \underline{p}$ and $\tau_i(t)$'s, $\forall i \in \underline{q}$ are arbitrary delay arguments with $0 \leq h_i(t) \leq H$ and $0 \leq \tau_i(t) \leq H$ for some constant H , and $\theta(t)$ is a given continuous vector-valued initial function. Moreover, the smooth vector-valued functions f_i 's, $\forall i \in \underline{q}$ satisfy

$$\|f_i(x(t - \tau_i(t)))\| \leq \|x(t - \tau_i(t))\|, \quad \forall i \in \underline{q}.$$

Now we present another main result for the global stabilizability of system (14).

Theorem 2. The system (14) is static output feedback stabilizable provided that (A1) is satisfied. In this case, a suitable static output feedback is given by $u(t) = Ky(t)$.

Proof. From (14) with $u(t) = Ky(t)$, the feedback controlled system is given by

$$\begin{aligned} \dot{x}(t) = & (A_0 + DKC_0)x(t) \\ & + \sum_{i=1}^p (A_i + DKC_i)x(t - h_i(t)) \\ & + \sum_{i=1}^q (E_i + DKH_i)f_i(x(t - \tau_i(t))), \end{aligned} \quad (15a)$$

with

$$\begin{aligned} & \left\| \sum_{i=1}^q (E_i + DKH_i)f_i(x(t - \tau_i(t))) \right\| \\ & \leq \sum_{i=1}^q \|E_i + DKH_i\| \cdot \|x(t - \tau_i(t))\|. \end{aligned} \quad (15b)$$

Thus, by Lemma 1 with (15) and (A1), we conclude that the system (15) is globally asymptotically stable. This completes our proof. \square

Remark 1. Note that

1. the structure of the error dynamic system of (13) is the same as that of the feedback controlled system of (15);
2. the criterion to guarantee the existence of state estimator of system (11) is the same as that to guarantee the existence of output feedback controller of system (14).

Consequently, the criterion of (A1) can be view as the duality between state estimator and output feedback for the system (11) and (14), respectively.

Remark 2. Based on Theorem 1, an upper bound of arbitrary time-varying delays without destroying the state estimator is given by $H < \bar{H}$, where

$$\bar{H} = \begin{cases} \frac{\alpha_4 - \alpha_3}{\alpha_1(\alpha_2 + \alpha_3)}, & \text{if } \alpha_1(\alpha_2 + \alpha_3) \neq 0 \wedge \alpha_4 > \alpha_3, \\ \infty, & \text{if } \alpha_1(\alpha_2 + \alpha_3) = 0 \wedge \alpha_4 > \alpha_3, \end{cases}$$

$$\alpha_1 = \sum_{i=1}^p \|A_i + DKC_i\|, \quad \alpha_2 = \sum_{i=0}^p \|A_i + DKC_i\|,$$

$$\alpha_3 = \sum_{i=1}^q \|E_i + DKH_i\|, \quad \alpha_4 = \sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}^3(P)}}.$$

3. CONCLUSION

In this paper, the problem of the state estimator design for a class of time-lag systems with state saturation nonlinearities has been investigated. A simple criterion has been established to guarantee the asymptotic convergence of the observation error between the observer state estimate and the true state. It has been shown that such a criterion can be view as the dual to static output feedback controller for a class of time-lag systems with multiple time-varying delays. Besides, an upper bound of arbitrary time-varying delays has been derived to guarantee the observation error can converge asymptotically to zero. Nevertheless, the duality

between state estimator and dynamic output feedback for more general time-lag systems still remains unanswered. This constitutes an interesting future research direction.

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