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# A Two-Stage Estimator of Instrumental Variable Quantile Regression for Panel Data with Time-Invariant Effects

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## ABSTRACT

This paper proposes a two-stage instrumental variable quantile regression (2S-IVQR) estimation to estimate the time-invariant effects in panel data model. In the first stage, we introduce the dummy variables to represent the time-invariant effects, and use quantile regression to estimate effects of individual covariates. The advantage of the first stage is that it can reduce calculations and the number of estimation parameters. Then in the second stage, we adapt instrument variables approach and 2SLS method. In addition, we present a proof of 2S-IVQR estimator's large sample properties. Monte Carlo simulation study shows that with increasing sample size, the Bias and RMSE of our estimator are decreased. Besides, our estimator has lower Bias and RMSE than those of the other two estimators.

***KEYWORDS:*** *Time-invariant effects; Panel data; Quantile regression; Instrument variables*

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## 1. INTRODUCTION

Panel data not only reflects the individual heterogeneity of cross-section data, but also shows the dynamic information about the time series data. Quantile regression can describe the independent variable $X$ for the dependent variable $Y$ range accurately, and capture systematic influences of covariates on the location, scale and shape of the conditional distribution of the response. Additionally, there is no need for quantile regression model to make any assumptions on the overall distribution, compared with the ordinary least squares method.

Quantile regression model for panel data, can fully describe the conditional distribution of the response variable as well as control the variability. There are three types of estimation of quantile regression for panel data with fixed effects: the penalty estimation, two-step estimation, minimum distance estimation.

The penalty estimation mainly considers adding a penalty item in the objective function. Koenker(2004) proposed a general approach to estimate quantile regression models for longitudinal data employing $l_1$ regularization methods. Lamarche(2010) proved, in theory, that there existing optimal penalty parameter of penalized quantile regression for panel data with fixed effect. Galvao and Montes-Rojas(2010) proposed penalized quantile regression for dynamic panel data, using instrumental variables to resolve the problem of endogeneity. Galvao(2011) also adopted instrumental variables approach to study a quantile regression dynamic panel model with fixed effects. Harding and Lamarche(2014) gave an ℓ1 penalized quantile regression estimator which adapted the Hausman–Taylor instrumental variable approach. Lamarche(2014) proposed a penalized quantile regression estimator for panel data that explicitly considers individual heterogeneity associated with the covariates. Tao, Zhang and Tian(2017) proposed two new panel data instrumental variable estimators that combine the Huasman-Taylor instrumental variables of Huasman and Taylor(1981) and $l_1$ shrinkage of Koenker(2004) to resolve biased parameter estimation problem caused by lagged response variables and random error.

The main idea of the two-step estimation is to eliminate the fixed effect in the first step, use simple quantile regression for transformed data in the second step. Canay(2011) introduced a two-step estimator for panel data quantile regression models. It is noted that the two-step estimation method eliminates the fixed effect in the first step, which can greatly reduce the estimated parameters in quantile regression. Notethatdue to the first step, the two-step estimation cannot adapt the time-invariant independent variables in the panel data model.

Galvao and Wang (2015) developed a new minimum distance quantile regression (MD-QR) estimator for panel data models with fixed effects. The MD-QR is defined as the weighted average of the individual quantile regression slope estimators. However, from the definition of MD-QR we can know, the model cannot adapt the time-invariant independent variables. Besides, MD-QR estimator ignores the endogenous problems. Galvao, Gu and Volgushev(2018) provided new insights on the asymptotic properties of the MD-QR estimator under the two different assumption, the assumption that data within individuals are independent and the assumption that data are dependence across time while maintain independence across individuals. There have been other growing studies on quantile regression for panel data, see e.g., Galvao(2011), Chetverikov, Larsen, and Palmer(2016), Gu and Volgushev(2019), Zhang, Jiang and Feng(2021).

As the two-step estimation and minimum distance estimation fail when there are time-invariant independent variables in the model, we propose a two-stage instrumental variable quantile regression(2S-IVQR) estimation to estimate the time-invariant effects in panel data model. In the first stage, we introduce the dummy variables and perform quantile regression with all the data to estimate effects of individual covariates. Then in the second stage, we adapt instrument variables approach and 2SLS method. Moreover, we study the asymptotic properties of the proposed estimators. Monte Carlo simulation in various parameters sets proves the validity of the proposed approaches. We compare the bias and root mean squared error (RMSE) of the proposed estimators with the QR estimator's of Koenker and Bassett (1978) and the Grouped IVQR estimator's of Chetverikov, Larsen, and Palmer (2016).

The rest of the paper is organized as follows. Section 2 presents two-stage instrumental variable quantile regression (2S-IVQR) estimator for panel data with time-invariant effects. Section 3 is devoted to the asymptotic behavior of the proposed estimators. Section 4 describes the Monte Carlo experiment. Finally, section 5 concludes the paper.

## 2. Model and methods

### 2.1. Basic Model

Consider the panel data model that contains time-varying as well as time-invariant regressors:

$$y_{it} = \eta_i + z_i'\gamma + x_{it}'\beta + \varepsilon_{it}, i = 1,2,\cdots,N; t = 1,2,\cdots,T \quad (1)$$

where

$$\eta_i = \eta + \zeta_i, \quad (2)$$

$x_{it}$is a $k \times 1$ vector of time-varying variables, and $z_i$ is an $m \times 1$ vector of observed individual-specific variables that only vary over the cross-section units $i$. In addition to $z_i$, the outcomes, $y_{it}$, is also governed by unobserved individual specific effects, $\eta_i$.

We consider the following model for the $\tau$th conditional quantile functions of the response of the $t$th observation on the $i$th individual $y_{it}$,

$$Q_{y_{it}}(\tau|x_{it}, z_i, \eta_i) = \eta_i(\tau) + z_i'\gamma(\tau) + x_{it}'\beta(\tau), \quad (3)$$

There has been a lot of research on how to estimate $\beta(\tau)$, see Koenker(2004), Canay(2011), Galvao and Wang (2015), etc. Besides, the estimation method proposed by Canay(2011), Galvao and Wang (2015) can not identify $\gamma(\tau)$. Therefore, we are primarily interested in estimation $\gamma(\tau)$ inthispaper,that is,the focus of the following analysis is on estimation and inference involving the elements of $\gamma(\tau)$.

### 2.2. 2S-IVQR estimator

Chetverikov, Larsen, and Palmer(2016) are interested in estimating $\gamma(\tau)$, they propose the Grouped IVQR estimator. The Grouped IVQR estimator consists of two stages. First, estimate $\tau$th quantile regression of $y_{it}$on $x_{it}$ using the data $\{(y_{it},x_{it}):i = 1,2,\ldots,N\}$by the classical quantile regression(QR) estimator of Koenker and Bassett (1978). Second, estimate a 2SLS regression of intercept term coefficient on $z_i$using instrument variablesto get an estimator $\hat{\gamma}_{Grouped\ IVQR}(\tau)$ of $\gamma(\tau)$.

However, the Grouped IVQR estimator calculates each individual data separately, that is, it needs to calculate $n$ times in the first stage. To simplify computation and improve the estimation accuracy, we propose a two-stage instrumental variable quantile regression estimator(2S-IVQR) for model (3).

The 2S-IVQR estimator can be obtained via the following two stages.

Stage 1 : For each individual $i$ and each quantile $\tau$, estimates $\tau$th quantile regression of $y_{it}$on $x_{it}$ and $\vartheta_i$ using the data $\{(y_{it},x_{it},\vartheta_i):i=1,2,\cdots,N;t=1,2,\cdots,T\}$by the QR estimator:

$$(\hat{\beta}(\tau),\hat{\vartheta}(\tau))=\underset{\beta,\vartheta_i}{\operatorname{argmin}}\sum_{i=1}^{N}\sum_{t=1}^{T}\rho_u(y_{it}-x'_{it}\beta(\tau)-e_i\vartheta_1(\tau)-\cdots-e_i\vartheta_N(\tau))\ (4)$$

where $\vartheta_i(\tau)=\eta_i(\tau)+z'_i\gamma(\tau),\hat{\vartheta}(\tau)=(\hat{\vartheta}_1(\tau),\cdots,\hat{\vartheta}_N(\tau))',e_i=\begin{cases}1 & belong\ to\ individual\ i\\0 & else\end{cases}$,$\rho_\tau(u)=u(\tau-I(u<0))$ is the check function.

Stage 2: Estimate a 2SLS regression of $\hat{\vartheta}_i(\tau)$ on $z_i$using $w_i$as an instrument to get an estimator$\hat{\gamma}(\tau)$ of $\gamma(\tau)$:

$$\hat{\gamma}(\tau)=(Z'P_WZ)^{-1}(Z'P_W\hat{\vartheta}(\tau)),\qquad(5)$$

where $Z=(z_1,\cdots,z_N)',W=(w_1,\cdots,w_N)'$, and $P_W=W(W'W)^{-1}W'$.

As we can see, for fixed $\tau$, the first stage only needcalculate one time by introducing the dummy variable. In addition, the second stage employs the two-stage least squares method to get$\hat{\gamma}_{2S-IVQR}(\tau)$ and$\hat{\gamma}_{2S-IVQR}(\tau)$is a weighted combination of $z_i$ and $w_i$.

## 3. Asymptotic theory

Now we briefly discuss the asymptotic properties of the$\hat{\gamma}_{2S-IVQR}(\tau)$. To establish the asymptotic properties of the $\hat{\gamma}_{2S-IVQR}(\tau)$, we impose the following regularity assumptions.

Assumption 1: Observations are independent across individuals, and for all $i=1,\cdots,N,(x_{it},y_{it})$ are i.i.d. across $t=1,\cdots,T$.

Assumption 2: The set of quantile indices $U$ is a compact set included in (0,1).

Assumption3:(i) For all $\tau\in U$ and $i=1,\cdots,N$, $E[(w_i\gamma(\tau))]=0$.(ii) As$N\to\infty$, $N^{-1}\sum_{i=1}^{N}E[z_iw'_i]\to Q_{zw}$ and$N^{-1}\sum_{i=1}^{N}E[w_iw'_i]\to Q_{ww}$ where $Q_{zw}$ and $Q_{ww}$ are matrices with singular values bounded in absolute value from above by $C_M$and from below by $c_M$. (iii) For all $i=1,\cdots,N$ and $t=1,\cdots,T,y_{it}$is independent of$w_i$conditional on$(x_{it},z_i)$. (iv) For all $i=1,\cdots,N$, $E[||w_i||^{4+c_M}]\le C_M$.

Assumption 4: As $N\to\infty$,$N^{2/3}(\log T)/T\to 0$.

Assumption5: For all $i=1,\cdots,N$, $z_i$satisfy the moment conditions$||z_i||\le C_M$, $||x_{it}||\le C_M$.

Assumption 6: For $\tau_1,\tau_2\in U$ and $i=1,\cdots,N$, $\|\gamma(\tau_2)-\gamma(\tau_1)\|\le C_L\|\tau_2-\tau_1\|$.

Assumption 7:(i) For all $i=1,\cdots,N$, $E\left[\sup_{\tau\in U}|\eta_i(\tau)|^{4+c_M}\right]\le C_M$. (ii)As$N\to\infty$, for $\tau_1,\tau_2\in U$ $N^{-1}\sum_{i=1}^{N}E[\eta_i(\tau_2)\eta_i(\tau_1)w_iw'_i]\to J(\tau_1,\tau_2)$. (iii) For $\tau_1,\tau_2\in U$, $\|\eta(\tau_2)-\eta(\tau_1)\|\le C_L\|\tau_2-\tau_1\|$.

Assumption 8:(i) Let $f_i(\cdot)$ denote the conditional density function of $u_{it}=y_{it}-\eta_i-z'_i\gamma-x'_{it}\beta$ given $(x_{it},z_i)$,for all $\tau\in U$ and $i=1,\cdots,N$,$f_i(\cdot)$ is continuously differentiable, $f_i(\cdot)\le C_f$ and $f_i(0)\ge c_f$. (ii) For $i=1,\cdots,N$,the derivative $f'_i(\cdot)$ satisfy $|f'_i(u)|\le C_f$.

**Theorem 1** Under Assumptions 1-8,

$\sqrt{N}(\hat{\gamma}_{2S-IVQR}(\cdot)$ - $\gamma(\cdot))\to\mathbb{G}(\cdot)$,

where $\mathbb{G}(\cdot)$ is a zero-mean Gaussian process with uniformly continuous sample paths and covariance function $\mathcal{C}(\tau_1,\tau_2)=SJ(\tau_1,\tau_2)S'$,$J(\tau_1,\tau_2)$is defined in Assumption 7, $S=(Q_{zw}Q_{ww}^{-1}Q'_{zw})^{-1}Q_{zw}Q_{ww}^{-1}$where $Q_{zw}$and $Q_{ww}$are defined in Assumption 3.

Remark 1: Under Assumptions 1-8,

$\sqrt{N}(\hat{\gamma}_{2S-IVQR}(\tau)$ - $\gamma(\tau))\to N(0,V)$,

where $V=SJ(\tau,\tau)S'$.

## 4. Monte Carlo

The samples are generated from the following model:

$$y_{it}=x'_{it}\alpha+z'_i\beta+\epsilon_i+v_{it},z_i=\pi w_i+\theta_i+u_i,\epsilon_i=n\theta_i-k,$$

where$v_{it},\theta_i\sim U(0,1),x_{it},w_i,u_i\sim exp(0.25*N(0,1)),\alpha=\beta=2,\pi=n=1,k=0.5$. Notice that $E(\epsilon_i)=E(\epsilon_i|w_i)=nE(\theta_i)-k=0.5-0.5=0$. For the sake of comparing the performance and efficiency between different methods, we compare the Bias and RMSE of the following estimators: fixed effects quantile regression (QR) and grouped IV quantile regression (Grouped IVQR) as in Chetverikov, Larsen, and Palmer(2016).Here we consider two different forms of generation of $z_i$. The first case is $z_i$ being endogenous (correlated with $\epsilon_i$through $\theta_i$). In the second case, $z_i$is exogenous, where we set $z_i=w_i$.In the simulations, we report results considering $\{(N,T)|(25,25),(25,50),(100,25),(100,50)\}$, andquantiles$\tau=\{0.1,0.25,0.5,0.75,0.9\}$. We set the number of replications to 1000.

**Table 1Bias and RMSE of estimators when $z_i$ is endogenous**

| $\tau$ | | | 0.1 | 0.25 | 0.5 | 0.75 | 0.9 | Avg.abs. |
|---|---|---|---|---|---|---|---|---|
| $N = 25, T = 25$ | QR | Bias | 0.3607 | 0.3909 | 0.3754 | 0.3929 | 0.4031 | **0.3846** |
| | | RMSE | 0.0241 | 0.0256 | 0.0247 | 0.0258 | 0.0267 | **0.0254** |
| | Grouped IVQR | Bias | -0.0636 | -0.1002 | -0.1022 | -0.1254 | -0.0464 | **0.0876** |
| | | RMSE | 0.0208 | 0.0399 | 0.0390 | 0.0557 | 0.0389 | **0.0388** |
| | 2S-IVQR | Bias | -0.0662 | -0.0720 | -0.0711 | -0.1186 | 0.0032 | **0.0662** |
| | | RMSE | 0.0208 | 0.0256 | 0.0254 | 0.0599 | 0.0668 | **0.0397** |
| $N = 25, T = 50$ | QR | Bias | 0.3570 | 0.3796 | 0.3864 | 0.4090 | 0.4172 | **0.3899** |
| | | RMSE | 0.0149 | 0.0158 | 0.0161 | 0.0169 | 0.0173 | **0.0162** |
| | Grouped IVQR | Bias | -0.0243 | -0.0468 | -0.0239 | -0.0337 | -0.0438 | **0.0345** |
| | | RMSE | 0.0111 | 0.0126 | 0.0130 | 0.0115 | 0.0111 | **0.0118** |
| | 2S-IVQR | Bias | -0.0265 | -0.0526 | -0.0348 | -0.0282 | -0.0314 | **0.0347** |
| | | RMSE | 0.0078 | 0.0091 | 0.0090 | 0.0082 | 0.0080 | **0.0084** |
| $N = 100, T = 25$ | QR | Bias | 0.3686 | 0.3772 | 0.3787 | 0.4033 | 0.4157 | **0.3887** |
| | | RMSE | 0.0244 | 0.0247 | 0.0247 | 0.0262 | 0.0272 | **0.0254** |
| | Grouped IVQR | Bias | -0.0670 | -0.0236 | -0.0762 | -0.0371 | -0.0772 | **0.0562** |
| | | RMSE | 0.0307 | 0.0504 | 0.0246 | 0.0711 | 0.0372 | **0.0428** |
| | 2S-IVQR | Bias | -0.0544 | -0.0316 | -0.0632 | -0.0321 | -0.0642 | **0.0491** |
| | | RMSE | 0.0268 | 0.0444 | 0.0199 | 0.0645 | 0.0333 | **0.0378** |
| $N = 100, T = 50$ | QR | Bias | 0.3548 | 0.3838 | 0.3832 | 0.4029 | 0.4138 | **0.3877** |
| | | RMSE | 0.0182 | 0.0195 | 0.0194 | 0.0204 | 0.0211 | **0.0197** |
| | Grouped IVQR | Bias | -0.0183 | -0.0451 | -0.0516 | -0.0166 | -0.0192 | **0.0301** |
| | | RMSE | 0.0105 | 0.0181 | 0.0149 | 0.0102 | 0.0100 | **0.0127** |
| | 2S-IVQR | Bias | -0.0166 | -0.0430 | -0.0537 | -0.0120 | -0.0244 | **0.0299** |
| | | RMSE | 0.0101 | 0.0179 | 0.0141 | 0.0090 | 0.0093 | **0.0121** |

Table 1 provides the Bias and RMSE of the estimators in the case $z_i$ is endogenous. It is clear that the Bias and RMSE of Grouped IVQR and 2S-IVQR estimators are better than QR estimators.

Moreover, the Bias and RMSE of 2S-IVQR estimator are smaller than those of Grouped IVQR estimator at most quantiles. Meanwhile, we also find that the Bias and RMSE decreases as $N$ and $T$ increases for 2S-IVQR estimators.

**Table 2 Bias and RMSE of estimators when $z_i$ is exogenous**

| $\tau$ | | | 0.1 | 0.25 | 0.5 | 0.75 | 0.9 | Avg.abs. |
|---|---|---|---|---|---|---|---|---|
| $N = 25, G = 25$ | QR | Bias | 0.0254 | 0.0292 | 0.0008 | 0.0131 | -0.0188 | **0.0174** |
| | | RMSE | 0.0139 | 0.0145 | 0.0127 | 0.0139 | 0.0136 | **0.0137** |
| | Grouped IVQR | Bias | -0.0085 | 0.0083 | 0.0274 | 0.0145 | 0.0012 | **0.0120** |
| | | RMSE | 0.0167 | 0.0194 | 0.0212 | 0.0192 | 0.0165 | **0.0186** |
| | 2S-IVQR | Bias | 0.0036 | 0.0069 | -0.0047 | 0.0219 | 0.0027 | **0.0080** |
| | | RMSE | 0.0118 | 0.0136 | 0.0123 | 0.0124 | 0.0121 | **0.0124** |
| $N = 25, G = 50$ | QR | Bias | 0.0135 | -0.0023 | 0.0027 | -0.0078 | -0.0085 | **0.0070** |
| | | RMSE | 0.0098 | 0.0091 | 0.0096 | 0.0101 | 0.0096 | **0.0975** |
| | Grouped IVQR | Bias | 0.0037 | 0.0052 | 0.0033 | 0.0003 | -0.0072 | **0.0039** |
| | | RMSE | 0.0119 | 0.0137 | 0.0142 | 0.0120 | 0.0119 | **0.0127** |
| | 2S-IVQR | Bias | 0.0069 | -0.0040 | 0.0037 | -0.0031 | -0.0029 | **0.0041** |
| | | RMSE | 0.0078 | 0.0086 | 0.0089 | 0.0083 | 0.0086 | **0.0084** |
| $N = 100, G = 25$ | QR | Bias | 0.0308 | -0.0140 | -0.0035 | -0.0161 | 0.0108 | **0.0150** |
| | | RMSE | 0.0141 | 0.0129 | 0.0128 | 0.0128 | 0.0137 | **0.0133** |
| | Grouped IVQR | Bias | 0.0139 | -0.0286 | 0.0073 | -0.0123 | 0.0237 | **0.0172** |
| | | RMSE | 0.0130 | 0.0137 | 0.0145 | 0.0135 | 0.0137 | **0.0137** |
| | 2S-IVQR | Bias | 0.0049 | -0.0166 | -0.0013 | -0.0052 | 0.0179 | **0.0092** |
| | | RMSE | 0.0121 | 0.0119 | 0.0127 | 0.0117 | 0.0125 | **0.0122** |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $N=100, G=50$ | QR | Bias | -0.0032 | 0.0079 | -0.0039 | 0.0036 | -0.0123 | **0.0062** |
| | | RMSE | 0.0087 | 0.0089 | 0.0086 | 0.0094 | 0.0094 | **0.0090** |
| | Grouped IVQR | Bias | -0.0127 | 0.0042 | -0.0053 | 0.0006 | 0.0085 | **0.0062** |
| | | RMSE | 0.0081 | 0.0089 | 0.0100 | 0.0097 | 0.0089 | **0.0091** |
| | 2S-IVQR | Bias | -0.0080 | 0.0052 | -0.0025 | 0.0068 | 0.0029 | **0.0051** |
| | | RMSE | 0.0078 | 0.0076 | 0.0084 | 0.0083 | 0.0082 | **0.0081** |

The Bias and RMSE of the estimators when $z_i$isendogenous are shown in Table 2. The results are similar to those of Table 1. The results of Grouped IVQR and 2S-IVQR estimators are better than those of QR estimators and 2S-IVQR estimator performs better than Grouped IVQR in terms of Avg.abs.bias and Avg. RMSE. In addition, the Bias and RMSE decreases as $N$ and $T$ increases for 2S-IVQR estimators.

## 5. Conclusion

We explore a two-stage approach to instrumental variable quantile regression for panel data with time-invariant effects. In the first stage, the dummy variables are introduced to represent the time-invariant effects. By employing the dummy variables, the number of estimation parameters can be reduced. Then in the second stage, we apply instrument variables approach and 2SLS method. The proposed estimatoris a weighted combination of $z_i$ and $w_i$.Moreover, the asymptotic properties of the proposed estimators are studied in Section 3. Monte Carlo simulation in various parameters sets proves the validity of the proposed approach. Monte Carlo simulation presents that with increasing sample size, the Bias and RMSE of our estimator are decreased. Besides, our estimator has lower Bias and RMSE than those of the QR estimator and the Grouped IVQR estimator.

## 6. Reference

[1] Koenker, R., 2004. Quantile regression for longitudinal data. Journal of Multivariate Analysis, 91(1):74-89.

[2] Lamarche, C., 2010. Robust penalized quantile regression estimation for panel data. Journal of Econometrics, 157(2):396-408.

[3] Galvao, A. F., Montes-Rojas, G., 2010. Penalized quantile regression for dynamic panel Data. Journal of Statistical Planning and Inference, 140(11):3476-3497.

[4] Galvao, A. F., 2011. Quantile regression for dynamic panel data with fixed effects. Journal of Econometrics, 164(1):142-157.

[5] Harding, M., Lamarche, C., 2014. Hausman-Taylor instrumental variable approach to the penalized estimation of quantile panel models. Economics Letters, 124:176-179.

[6] Lamarche, C., 2014. Penalized quantile regression estimation for a model with endogenous fixed effects.

[7] Tao, L., Zhang, Y. J., Tian, M. Z., 2019. Quantile regression for dynamic panel data using Hausman-Taylor instrumental variables. Computational Economics, 53(3):1033-1069.

[8] Hausman, J.A., Taylor, W.E., 1981. Panel data and unobservable individual effects. Econometrica, 49(6):1377-1398.

[9] Canay, I. A., 2011. A simple approach to quantile regression for panel data. Econometrics Journal, 14(3):368-386.

[10] Galvao, A. F., Wang, L., 2015. Efficient minimum distance estimator for quantile regression fixed effects panel data. Journal of Multivariate Analysis, 133:1-26.

[11] Galvao, A. F., Gu, J., Volgushev, S., 2020.On the unbiased asymptotic normality of quantile regression with fixed effects[J]. Journal of Econometrics, 218(1):178-215.

[12] Chetverikov, D., Larsen, B., Palmer, C., 2016. IV quantile regression for group-level treatments, with an application to the distributional effects of trade. Econometrica, 84(2):809-833.

[13] Gu, J., Volgushev, S., 2019. Panel data quantile regression with grouped fixed effects. Journal of Econometrics, 213(1):68-91.

[14] Zhang, Y., Jiang, J., Feng, Y., 2021. Penalized quantile regression for spatial panel data with fixed effects. Communication in Statistics-Theory and Methods,1-13.

[15] Koenker, R., Bassett, G., 1978. Regression Quantile. Econometrica, 46(1):33--50.

## 7. Appendix

**Proof of Theorem 1** Denote $\tilde{\gamma}(\tau) = (Z'P_W Z)^{-1}(Z'P_W \vartheta(\tau))$, $\hat{Q}_{zw} = \frac{Z'W}{N}$, $\hat{Q}_{ww} = \frac{W'W}{N}$, then

$\sqrt{N}(\hat{\gamma}_{2S-IVQR}(\tau) - \tilde{\gamma}(\tau)) = (\hat{Q}_{zw}\hat{Q}_{ww}^{-1}\hat{Q}'_{zw})^{-1}\hat{Q}_{zw}\hat{Q}_{ww}^{-1}(W'(\hat{\vartheta}(\tau) - \vartheta(\tau))/\sqrt{N})$.(6)

By Assumptions 3(iv) and 5 and Chebyshev's inequality, we have

$\frac{1}{N}\sum_{i=1}^{N}(z_i w'_i - E[z_i w'_i]) \xrightarrow{p} 0$.

Observe that $N^{-1}\sum_{i=1}^{N} E[z_i w'_i] \to Q_{zw}$ by Assumption 3, it suffices to prove

$\hat{Q}_{zw} = N^{-1}\sum_{i=1}^{N} z_i w'_i \xrightarrow{p} Q_{zw}$.

Similarly, it can be obtained

$$\hat{Q}_{ww} = N^{-1}\sum_{i=1}^{N} w_i w'_i \xrightarrow{p} Q_{ww}$$

Thus,

$$\hat{S} = (\hat{Q}_{zw}\hat{Q}_{ww}^{-1}\hat{Q}'_{zw})^{-1}\hat{Q}_{zw}\hat{Q}_{ww}^{-1} \xrightarrow{p} (Q_{zw}Q_{ww}^{-1}Q'_{zw})^{-1}Q_{zw}Q_{ww}^{-1} = S \qquad (7)$$

According to Chetverikov, Larsen, and Palmer(2016),

$$S(\tau) = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\hat{\vartheta}_i(\tau) - \vartheta_i(\tau)\right) w'_i = o_p(1)(8)$$

uniformly over $\tau \in U$.

Observe that(7) and(8)are satisfied, by (6)we have

$\sqrt{N}(\hat{\gamma}_{2S-IVQR}(\tau) - \tilde{\gamma}(\tau)) = o_p(1)$.(9)

Next, we show $\sqrt{N}(\tilde{\gamma}(\cdot) - \gamma(\cdot)) \to \mathbb{G}(\cdot)$. Note that

$\sqrt{N}(\tilde{\gamma}(\cdot) - \gamma(\cdot)) = \hat{S} \cdot \frac{1}{\sqrt{N}}\sum_{i=1}^{N} w_i \eta_i(\tau)$.

and $\hat{S} \xrightarrow{p} S$ by (7). In addition, Lemma 3 of Chetverikov, Larsen, and Palmer(2016) implies that

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N} w_i \eta_i(\tau) \to \mathbb{G}^0(\cdot)$$

where $\mathbb{G}^0(\cdot)$ is a zero-mean Gaussian process with uniformly continuous sample paths and covariance function $J(\tau_1,\tau_2)$, $J(\tau_1,\tau_2)$ is defined in Assumption 7. Therefore, by Slutsky's theorem,

$\sqrt{N}(\tilde{\gamma}(\cdot) - \gamma(\cdot)) \to \mathbb{G}(\cdot)$,

where $\mathbb{G}(\cdot)$ is a zero-mean Gaussian process with uniformly continuous sample paths and covariance function $\mathcal{C}(\tau_1,\tau_2) = SJ(\tau_1,\tau_2)S'$. Combining (9) gives the asserted claim and completes the proof of Theorem 1.