

# Limit States Solution to CSCS Orthotropic Thin Rectangular Plate Carrying Transverse Loads

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## ABSTRACT

The analysis of thin rectangular orthotropic plate with two opposite edges clamped and the other two opposite edges simply supported (CSCS), carrying transverse loads was investigated in this study. The Ritz total potential energy functional was used. The minimization of the total potential energy functional produces the expression for the coefficient of deflection. The coefficient of deflection was used to obtain equation for the maximum lateral load of an orthotropic thin rectangular plate based on allowable deflection. Also, equation for the maximum lateral load of an orthotropic thin rectangular plate based on allowable stress was developed. Developed stiffness coefficients were substituted in the lateral load equations to obtain the maximum lateral load values for a CSCS plate. Numerical examples using permissible deflection of 10mm and yield strength of 250MPa, plate thickness (varying from 5mm to 12.5 mm with 0.5mm intervals) were done to determine the maximum lateral loads corresponding to an orthotropic thin rectangular CSCS plate carrying transverse loads (when  $n_1 = E_y/E_x = 0.7$  and  $n_2 = G/E_x = 0.41$ ) for aspect ratios (b/a) of 1.0, 1.25 and 1.50.

**KEYWORDS:** Transverse Loads, Plates, Orthotropic, Direct Variation, Allowable deflection, Elastic yield stress

**Notations:** a: Length of the plate, b: Width of the plate, w: Deflection equation of the plate, A: Coefficient of deflection of the plate, h: Shape function,  $\sigma_x$ : Normal stress in x - direction,  $\sigma_y$ : Normal stress in y - direction,  $\tau_{xy}$ : Shear stress in x-y direction,  $\epsilon_x$ : Direct strain along x - direction,  $\epsilon_y$ : Direct strain along y - direction,  $\gamma_{xy}$ : Shear Strain on x - y plane,  $\mu_{xy}$ : Poisson ratio on x axis,  $\mu_{yx}$ : Poisson ratio on y axis,  $E_x$ : Elastic modulus in the x direction,  $E_y$ : Elastic modulus in the y direction,  $G_{xy}$ : Shear modulus in the x-y plane,  $\alpha$ : Aspect Ratio = b/a, t: Thickness of the plate, x: Primary axis of the plate, y: Secondary axis of the plate, z: Axis corresponding to the thickness of the plate, C: Clamped Support, S: Simply supported support, R: Non-dimensional parameter equal to x/a, Q: Non-dimensional parameter equal to y/b, q: Transverse load uniformly distributed,  $n_1$ : Ratio of the young modulus in y direction to the young modulus in the x direction,  $n_2$ : Ratio of the shear modulus in x-y plane to the young modulus in the x direction,  $n_3$ : Ratio of the stress in the y direction to the stress in the x direction,  $n_4$ : Ratio of

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the shear stress in x-y plane to the stress in the x direction.

## 1. Introduction

Thin plates are structural elements that their thickness is smaller than its two dimensions. Among practical examples to describe the dimensions of these plates are roof, building windows, flat part of a table, manhole thin covering and panels. Plates are divided into two categories: thin plates with large deflections and thick plates (Boot, 1988 and Krysko, 2011). In thin plates, deflections and deformation of structural elements are usually considered and for ease of work, in structural plates, their deflections are studied under loading conditions.

Geometrically, plates are bounded by either rectilinear or curved boundaries. Plates are widely used in the fields of aerospace, aeronautical, naval, marine, mechanical, architectural, structural, and highway engineering. The structural behavior of plates as a function of the type of loading acting on it can be classified as static flexure, dynamic flexure or buckling. The analysis of the plates for static flexure,

dynamic flexure, and buckling has been extensively done in the works of Osadede et al. (2011), Ezeh et al. (2013), Aginam et al. (2012), Ibearugbulem et al. (2013), Ibearugbulem et al. (2011). Considerable research interest and activities have been generated on the analysis of plates, with varying methods being developed and used for specific cases. The methods for plate analysis can be grouped into two namely: analytical methods and numerical or approximate methods. Analytical methods target to obtain mathematical expressions valid for the entire plate region that identically solve the governing partial differential equations on the entire plate domain. This is usually subject to the geometric and boundary conditions at the plate edges. They are closed form mathematical solutions which exist for a limited number of plate problems, and do not exist for the vast majority of plate problems whether in static flexure, dynamic flexure or buckling (Szilard, 2004; Timoshenko and Woinowsky-Krieger, 1959). Numerical methods came up to address the need for approximate solutions for cases where closed form

analytical solutions cannot be found. The double trigonometric or Fourier series method was one of the earliest methods of solving the plate problem. The method, applicable to plates with all edges simply supported, assumes that a double Fourier series can be developed to represent any distribution of the applied load  $p(x, y)$ . By assuming that the deflection response can be represented by a double Fourier series of the same form as the loading, which is specifically constructed to satisfy the geometric and force boundary conditions at the simply supported edges, the governing fourth order biharmonic equation for flexural static analysis of thin plates is simplified to an algebraic problem, readily solved to find the unknown generalized displacement parameters. Thus, the internal forces are obtained from the internal force displacement relations. The energy method is another numerical method adopted in the solution to the vast plate problems. This was used extensively in the works of Ibearugbulem et al. (2014).

## 2. Theoretical Background

The Ritz total potential energy functional for an orthotropic thin rectangular plate carrying both in-plane forces and transverse load was obtained by Bertram (2020) in his masters degree work. When only transverse load is considered, the result is as shown in Equation 1.

$$\Pi = \frac{1}{2} \iint \left[ D_x \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + 2B \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 + D_y \left( \frac{\partial^2 w}{\partial y^2} \right)^2 \right] \partial x \partial y - q \iint w \partial x \partial y \tag{1}$$

where  $D_x$ ,  $D_y$  and  $B$  are the flexural rigidity components for an orthotropic plate given as

$$D_x = \frac{E_x t^3}{12(1 - \nu_{xy} \nu_{yx})} ; D_y = \frac{E_y t^3}{12(1 - \nu_{yx} \nu_{xy})} ; B = \frac{E_x \nu_{yx} t^3}{12(1 - \nu_{xy} \nu_{yx})} + \frac{2G t^3}{12}$$

## 3. Methodology

The procedure used in this work is outlined as follows:

### 3.1. Formulation of the Equation for the Maximum Transverse Load for an Orthotropic Thin Rectangular Plate

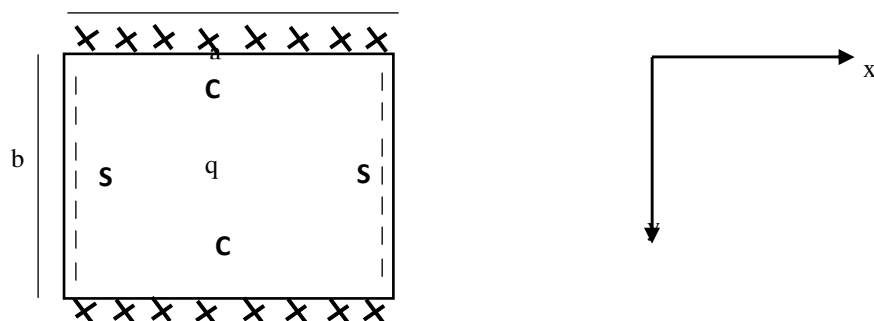


Figure 1: Schematic Representation of CSCS plate carrying transverse uniform load (q).

#### 3.1.1. Equation for the Maximum Transverse Load Parameter based on allowable deflection condition

The Equation of Maximum Transverse Load Parameter based on allowable deflection is obtained by minimizing Equation 1 to obtain the deflection  $w$ , as a product of a coefficient of deflection  $A$  and a shape function  $h$ . Then, by direct variation of the total potential energy functional (that is, differentiating with respect to the coefficient

of deflection and equating to zero). The Equation of Maximum Transverse Load Parameter based on allowable deflection is as shown in Equation 2

$$qa(a/t)^3 \leq \frac{\theta E_x}{12 K_m h(1 - n_1 \mu_{xy}^2)} \tag{2}$$

where  $\theta$  represent the allowable deflection of the plate and  $K_m$  is the ratio of load stiffness coefficient to total material stiffness coefficient given as

$$K_m = \left[ \frac{K_q}{K_T} \right] \tag{3}$$

**3.1.2. Equation for the Maximum Transverse Load Parameter as a function of the elastic yield stress**  
 In an elastic body, for general state of stress, the expression of internal work is given as shown in Equation 4

$$W = \frac{1}{2} [\sigma_x \epsilon_x + \sigma_y \epsilon_y + \tau_{xy} \gamma_{xy}] \tag{4}$$

If the constitutive relation Equations of strains and stresses are substituted in Equation 4, it gives Equation 5

$$W = \frac{1}{2} \left[ \frac{\sigma_x^2}{E_x} - \mu_{yx} \frac{\sigma_x \sigma_y}{E_y} - \mu_{xy} \frac{\sigma_x \sigma_y}{E_x} + \frac{\sigma_y^2}{E_y} + \frac{\tau_{xy}^2}{G_{xy}} \right] \tag{5}$$

Solving Equation 5 bearing in mind that the internal work must be less than the maximum allowable internal work of the plate, an expression for the Maximum Lateral Load Parameter as a function of the elastic yield stress is obtained as shown in Equation 6

$$q \left( \frac{a}{t} \right)^3 \leq \pm \frac{F_y}{6\rho K_m \sqrt{\left[ (1 - n_2 \mu_{xy}) + \frac{1}{n_1} (n_2^2 - n_1 n_2 \mu_{xy}) + \frac{n_4^2}{n_2} \right]}} \tag{6}$$

where  $\rho$  is as given in Equation 7

$$\rho = - \left[ \frac{d^2 h}{dR^2} + \frac{n_1 \mu_{xy}}{\alpha^2} \left( \frac{d^2 h}{dQ^2} \right) \right] \tag{7}$$

**Formulation of the Ratios  $n_3$  and  $n_4$**

$n_3$  is the ratio of the stress in y-direction to the stress in x-direction, while  $n_4$  is the ratio of the shear stress to the stress in x-direction.

$$n_3 = \frac{\sigma_y}{\sigma_x} \tag{8}$$

$n_3$  is given by

$$n_3 = \frac{n_1 [\mu_{xy} \left( \frac{d^2 h}{dR^2} \right) + \frac{1}{\alpha^2} \left( \frac{d^2 h}{dQ^2} \right)]}{\left[ \left( \frac{d^2 h}{dR^2} \right) + \frac{n_1 \mu_{xy}}{\alpha^2} \left( \frac{d^2 h}{dQ^2} \right) \right]} \tag{9}$$

$$n_4 = \frac{\tau_{xy}}{\sigma_x} \tag{10}$$

$n_4$  is given by

$$n_4 = \frac{-2n_2(1 - n_1 \mu_{xy}^2) \left( \frac{d^2 h}{\alpha dRdQ} \right)}{\left[ \frac{d^2 h}{dR^2} + \frac{\mu_{yx} d^2 h}{\alpha^2 dQ^2} \right]} \tag{11}$$

**3.2. Numerical example**

The material properties of CSCS plate include:

$E_x = 207 \times 10^9 \text{ N/m}^2$ ;  $\mu_{xy} = 0.3$ ;  $0.1 \leq E_y/E_x \leq 1$ ;  $0.385 \leq G/E_x \leq 0.415$ ;  $0.03 \leq \mu_{yx}/\mu_{xy} \leq 0.3$ ;  
 $80 \leq a/t \leq 200$ ;  $\theta = 0.010 \text{ m}$ ;  $F_y = 250 \text{ MPa}$ . Span,  $a = 1 \text{ m}$

**3.2.1. Numerical example based on allowable deflection criteria**

Bertram, 2020 obtained the stiffness coefficient  $K_m$  for a CSCS plate as

$$K_m = \frac{1}{1.142844 + \frac{1.387748\phi_{12}}{\alpha^2} + \frac{5.904735\phi_2}{\alpha^4}} \quad 12$$

$\phi_{12}$  is given as in equation 13

$$\phi_{12} = \left( \frac{n_1 E_x \mu_{xy} t^3}{12(1 - n_1 \mu_{xy}^2)} + \frac{2Gt^3}{12} \right) \times \frac{12(1 - n_1 \mu_{xy}^2)}{E_x t^3} \quad 13$$

Simplifying Equation 13 gives

$$\phi_{12} = n_1 \mu_{xy} + 2n_2 (1 - n_1 \mu_{xy}^2) \quad 14$$

$\phi_2$  is as given in Equation 15

$$\phi_2 = \frac{E_y}{E_x} = n_1 \quad 15$$

Bertram 2020, obtained  $h_{max}$  for CSCS plate as 0.0195312. If Equations 12 and the value of  $h_{max}$  are substituted into Equation 2, the lateral load parameter equation gives

$$q_a(a/t)^3 \leq \frac{\theta E_x \left( 1.142844 + \frac{1.387748\phi_{12}}{\alpha^2} + \frac{5.904735\phi_2}{\alpha^4} \right)}{12 \times 0.0195312(1 - n_1 \mu_{xy}^2)} \quad 16$$

From Equation 15,  $\phi_2 = n_1$ . Substituting this in Equation 16 and simplifying gives

$$q_a(a/t)^3 \leq \frac{\theta E_x \left( 4.876147 + \frac{5.921073\phi_{12}}{\alpha^2} + \frac{25.193601n_1}{\alpha^4} \right)}{(1 - n_1 \mu_{xy}^2)} \quad 17$$

### Numerical example based on maximum stress condition

The shape function  $h$  for a CSCS plate is given in the work of Ibearugbulem et. al (2014) as

$$h = (R - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \quad 18$$

At midspan  $R = Q = \frac{1}{2}$  (point of maximum deflection)

$$\frac{\partial^2 h}{\partial R^2} = 12 \left[ \left( \frac{1}{2} \right)^2 - \frac{1}{2} \right] \left[ \left( \frac{1}{2} \right)^2 - 2 \left( \frac{1}{2} \right)^3 + \left( \frac{1}{2} \right)^4 \right] = -0.1875 \quad 19$$

$$\frac{\partial^2 h}{\partial Q^2} = \left[ \frac{1}{2} - 2 \left( \frac{1}{2} \right)^3 + \left( \frac{1}{2} \right)^4 \right] \left[ 2 - 12 \left( \frac{1}{2} \right) + 12 \left( \frac{1}{2} \right)^2 \right] = -0.3125 \quad 20$$

$$\frac{d^2 h}{dRdQ} = \left[ 1 - 6 \left( \frac{1}{2} \right)^2 + 4 \left( \frac{1}{2} \right)^3 \right] \left[ 2 \left( \frac{1}{2} \right) - 6 \left( \frac{1}{2} \right)^2 + 4 \left( \frac{1}{2} \right)^3 \right] = 0 \quad 21$$

Substituting Equations 19 and 20 into Equation 7 gives

$$\rho = - \left[ -0.1875 + \frac{n_1 \mu_{xy}}{\alpha^2} (-0.3125) \right] \quad 22$$

$$\rho = 0.1875 \left[ 1 + \frac{1.6667 n_1 \mu_{xy}}{\alpha^2} \right] \quad 23$$

Substituting Equations 19 and 20 into Equation 9 gives

$$n_3 = \frac{n_1 [-0.1875 \mu_{xy} + \frac{1}{\alpha^2} * -0.3125]}{[-0.1875 + \frac{n_1 \mu_{xy}}{\alpha^2} * -0.3125]} \quad 24$$

$$n_3 = \frac{n_1 [\mu_{xy} + \frac{1.6667}{\alpha^2}]}{[1 + \frac{1.6667 n_1 \mu_{xy}}{\alpha^2}]} \quad 25$$

Substituting Equations 19, 20 and 21 into Equation 11 gives

$$n_4 = 0 \quad 26$$

Substituting Equations 12, 23 and 26 into Equation 6 gives

$$q \left( \frac{a}{t} \right)^2 \leq \pm \frac{\left( 1.142844 + \frac{1.387748 \phi_{1z}}{\alpha^2} + \frac{5.904735 \phi_z}{\alpha^4} \right) F_y}{6 \times 0.1875 \left[ 1 + \frac{1.6667 n_1 \mu_{xy}}{\alpha^2} \right] \sqrt{\left[ (1 - n_3 \mu_{xy}) + \frac{1}{n_1} (n_3^2 - n_1 n_3 \mu_{xy}) \right]}} \quad 27$$

From Equation 15,  $\phi_z = n_1$ . Hence, Equation 27 simplifies to

$$q \left( \frac{a}{t} \right)^2 \leq \pm \frac{1.015861 \left( 1 + 1.214293 \frac{\phi_{1z}}{\alpha^2} + \frac{5.166703 n_1}{\alpha^4} \right) F_y}{\left[ 1 + \frac{1.6667 n_1 \mu_{xy}}{\alpha^2} \right] \sqrt{\left[ (1 - n_3 \mu_{xy}) + \frac{1}{n_1} (n_3^2 - n_1 n_3 \mu_{xy}) \right]}} \quad 28$$

The maximum load can readily be calculated from Equation 28

The load that can cause failure need also be checked at the fixed edges where failure might occur first.

At fixed edges  $\left( R = \frac{1}{2}, Q = 0 \right)$  and  $\left( R = \frac{1}{2}, Q = 1 \right)$

At these  $\frac{\partial^2 h}{\partial R^2}$ ,  $\frac{\partial^2 h}{\partial Q^2}$  and  $\frac{d^2 h}{dRdQ}$  as 0, 0.625 and 0 respectively

Hence, from Equation 7,

$$\rho = - \left[ 0 + \frac{n_1 \mu_{xy}}{\alpha^2} (0.625) \right] = \frac{-0.625 n_1 \mu_{xy}}{\alpha^2} \quad 29$$

Also, from Equation 9,

$$n_3 = \frac{n_1 [\mu_{xy}(0) + \frac{1}{\alpha^2} (0.625)]}{\left[ 0 + \frac{n_1 \mu_{xy}}{\alpha^2} (0.625) \right]} = \frac{1}{\mu_{xy}} \quad 30$$

From Equation 11,

$$n_4 = \frac{-2 n_2 (1 - n_1 \mu_{xy}^2) \left( \frac{0}{\alpha} \right)}{\left[ \frac{d^2 h}{dR^2} + \frac{\mu_{yx} (0.625)}{\alpha^2} \right]} = 0 \quad 31$$

Hence, the equation for the maximum load is obtained by substituting Equations 12, 29 and 31 into Equation 6

$$q \left( \frac{a}{t} \right)^2 \leq \pm \frac{\left( 1.142844 + \frac{1.387748 \phi_{1z}}{\alpha^2} + \frac{5.904735 \phi_z}{\alpha^4} \right) F_y}{6 \times \left( \frac{-0.625 n_1 \mu_{xy}}{\alpha^2} \right) \sqrt{\left[ (1 - n_3 \mu_{xy}) + \frac{1}{n_1} (n_3^2 - n_1 n_3 \mu_{xy}) \right]}} \quad 32$$

By substituting Equation 30 into Equation 32 and factorizing, bearing in mind that  $\phi_z = n_1$  gives

$$q \left( \frac{a}{t} \right)^2 \leq \pm \frac{-0.3047587 \left( 1 + \frac{1.214293 \phi_{1z}}{\alpha^2} + \frac{5.166703 n_1}{\alpha^4} \right) F_y}{\left( \frac{n_1 \mu_{xy}}{\alpha^2} \right) \sqrt{\left( \frac{1}{n_1 \mu_{xy}^2} - 1 \right)}} \quad 33$$

#### 4. Results and Discussion

Plate thickness varying from 5mm to 12.5mm (with 0.5mm intervals), allowable deflection of 10mm, yield strength of 250MPa, aspect ratio values varying from 1.0 to 2.25 (with 0.25 intervals) were used for the computation of the maximum Lateral load and the results presented on Table 1. The results presented on Table 1 were plotted for a square plate as shown on Figure 2. From Figure 2 and Table 1, it could be observed that, for plate thickness equal to and less than 10mm, the deflection condition governs the design as the values of the lateral load that will make the plate fail in deflection is the lowest. As the thickness reaches 10.5mm and above, the critical case becomes the stress condition at the clamped edge and this load value now governs the design. At each time, the smallest of the loads that will cause failure governs the design. Also, From Figure 2 and Table 1, it can be seen that in general, the value of the lateral load that will cause failure increases as the plate thickness increases. This is so because the plate tends to sustain more loads when the thickness is increased due to increase



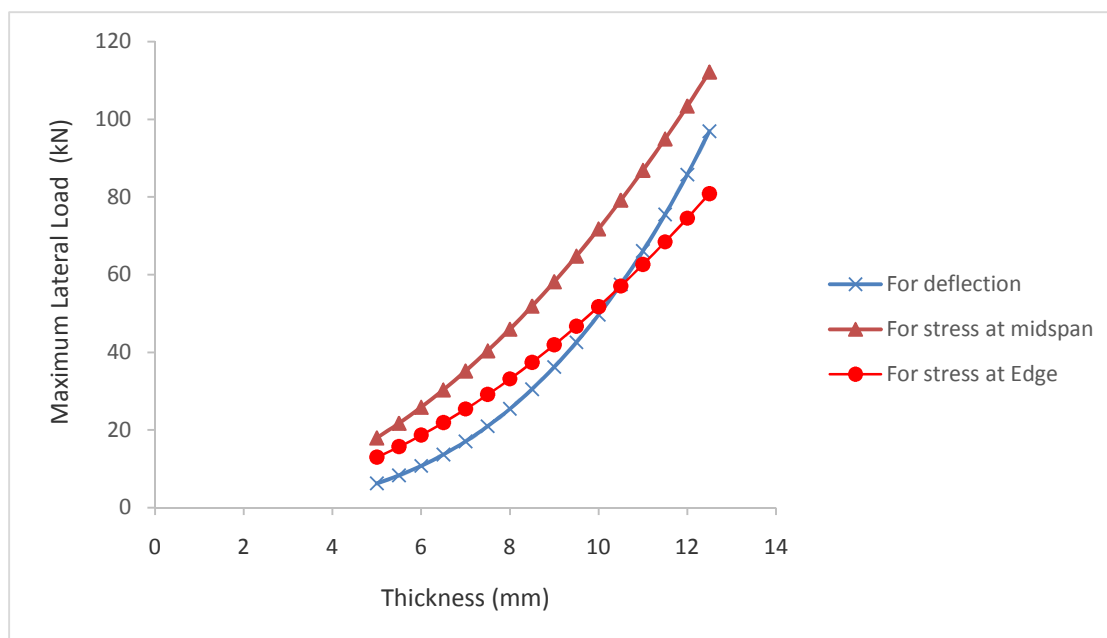
in the stiffness of the plate. It can also be seen from Table 1, that for aspect ratios higher than 1.0, the design is governed always by the deflection case as the maximum lateral load that would cause failure is always smallest for the deflection condition.

**Table 1a: Maximum Lateral load (q) for permissible deflection of 10mm, span of 1m and yield strength of 250 MPa of thin rectangular orthotropic CCCC plate in bending (when  $n_1 = E_y/E_x = 0.5$ ,  $n_2 = G/E_x = 0.4$ ,  $\mu_{xy} = 0.3$ ) for aspect ratios (b/a) of 1.0, 1.25 and 1.50**

t (mm)	q (KN/m <sup>2</sup> )								
	b/a = 1.0			b/a = 1.25			b/a = 1.50		
	Deflection Criteria	Yield stress criteria		Deflection criteria	Yield stress criteria		Deflection criteria	Yield stress criteria	
	Midspan	Midspan	Edge	Midspan	Midspan	Edge	Midspan	Midspan	Edge
5	6.20046505	17.9395998	12.936593	3.65755652	13.0870967	11.923581	2.64701936	10.6405748	12.426112
5.5	8.25281899	21.7069158	15.653278	4.86820773	15.835387	14.427533	3.52318277	12.8750955	15.035596
6	10.7144036	25.8330237	18.628694	6.32025767	18.8454193	17.169957	4.57404945	15.3224277	17.893602
6.5	13.6224217	30.3179237	21.862842	8.03565167	22.1171934	20.150852	5.81550154	17.9825714	21.00013
7	17.0140761	35.1616156	25.355723	10.0363351	25.6507095	23.370219	7.26342113	20.8555266	24.35518
7.5	20.9265696	40.3640995	29.107335	12.3442533	29.4459676	26.828058	8.93369034	23.9412933	27.958753
8	25.3971049	45.9253755	33.117678	14.9813515	33.5029676	30.524368	10.8421913	27.2398715	31.810848
8.5	30.4628848	51.8454434	37.386754	17.9695752	37.8217095	34.45915	13.0048061	30.7512612	35.911465
9	36.1611122	58.1243033	41.914562	21.3308696	42.4021933	38.632403	15.4374169	34.4754624	40.260604
9.5	42.5289898	64.7619553	46.701101	25.0871802	47.2444191	43.044129	18.1559058	38.412475	44.858266
10	49.6037204	71.7583992	51.746373	29.2604522	52.3483868	47.694325	21.1761549	42.5622992	49.70445
10.5	57.4225069	79.1136351	57.050376	33.8726309	57.7140965	52.582994	24.5140463	46.9249349	54.799156
11	66.0225519	86.827663	62.613111	38.9456618	63.3415481	57.710134	28.1854622	51.5003821	60.142384
11.5	75.4410583	94.9004829	68.434578	44.5014902	69.2307416	63.075745	32.2062846	56.2886407	65.734135
12	85.7152289	103.332095	74.514776	50.5620613	75.381677	68.679828	36.5923956	61.2897109	71.574408
12.5	96.8822665	112.122499	80.853707	57.1493206	81.7943544	74.522383	41.3596775	66.5035925	77.663203

**Table 1b: Maximum Lateral load (q) for permissible deflection of 10mm, span of 1m and yield strength of 250 MPa of thin rectangular orthotropic CSCS plate in bending (when  $n_1 = E_y/E_x = 0.5$ ,  $n_2 = G/E_x = 0.4$ ,  $\mu_{xy} = 0.3$ ) for aspect ratios (b/a) of 1.75, 2.0 and 2.25**

t (mm)	q (KN/m <sup>2</sup> )								
	b/a = 1.75			b/a = 2.0			b/a = 2.25		
	Deflection Criteria	Yield stress criteria		Deflection criteria	Yield stress criteria		Deflection criteria	Yield stress criteria	
	Midspan	Midspan	Edge	Midspan	Midspan	Edge	Midspan	Midspan	Edge
5	2.16385019	9.29233565	13.826076	1.90104362	8.49245715	15.865279	1.74396526	7.98710113	18.420377
5.5	2.88008461	11.2437261	16.729552	2.53028906	10.2758731	19.196988	2.32121776	9.66439237	22.288656
6	3.73913313	13.3809633	19.909549	3.28500338	12.2291383	22.846002	3.01357197	11.5014256	26.525343
6.5	4.75397888	15.7040472	23.366068	4.17659284	14.3522526	26.812322	3.83149168	13.4982009	31.130437
7	5.93760493	18.2129779	27.099108	5.2164637	16.645216	31.095947	4.78544068	15.6547182	36.103939
7.5	7.3029944	20.9077552	31.10867	6.41602223	19.1080286	35.696878	5.88588276	17.9709775	41.445848
8	8.86313039	23.7883793	35.394754	7.78667468	21.7406903	40.615115	7.14328171	20.4469789	47.156165
8.5	10.630996	26.85485	39.957359	9.33982732	24.5432012	45.850657	8.56810133	23.0827223	53.234889
9	12.6195743	30.1071675	44.796485	11.0868864	27.5155612	51.403504	10.1708054	25.8782077	59.682021
9.5	14.8418485	33.5453317	49.912133	13.0392582	30.6577703	57.273658	11.9618577	28.8334351	66.497561
10	17.3108016	37.1693426	55.304303	15.208349	33.9698286	63.461117	13.9517221	31.9484045	73.681508
10.5	20.0394166	40.9792002	60.972994	17.605565	37.451736	69.965881	16.1508623	35.223116	81.233862
11	23.0406769	44.9749045	66.918206	20.2423125	41.1034926	76.787951	18.5697421	38.6575695	89.154624
11.5	26.3275653	49.1564556	73.13994	23.1299978	44.9250983	83.927327	21.2188253	42.251765	97.443794
12	29.9130651	53.5238533	79.638196	26.280027	48.9165532	91.384008	24.1085758	46.0057025	106.10137
12.5	33.8101593	58.0770978	86.412973	29.7038066	53.0778572	99.157995	27.2494572	49.9193821	115.12736



**Figure 2: Graph of Maximum transverse load against thickness for a square plate**

### Data Availability Statement

All data that support the findings of this study are available from the corresponding author upon reasonable request.

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