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Concept on Rough Soft Set and Its Application in Decision Making

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ABSTRACT

The notion of full rough soft set, upper Soft set and lower Soft set are introduced here in a different manner. Various properties on rough soft set are studied with examples. We define rough soft subset and rough soft equal set with examples. Moreover rough soft equality relation are introduced and properties are presented. Finally measure of roughness of soft set is defined and an algorithm is presented to solve decision making problem by the application of rough soft set.

KEYWORDS: *rough set, soft set, rough soft set, upper soft set and lower soft set, rough soft subset, rough soft equity relation, measure of roughness*

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1. INTRODUCTION

Methods in classical mathematics are not successful to solve various types of uncertainties problems in economics, engineering, environmental science and social science. Probability theory, fuzzy set theory (Zedah 1965), rough set the-ory (Pawlak 1982) and other mathematical tools are often useful approaches to solve uncertainties. In 1999 Molodtov introduced the concepts of Soft set based on parameters for dealing with uncertainties. Now work on soft set theory are progressing rapidly. Several operations on soft set and theoretical study on the theory of soft set are introduced (Maji et al. 2003). Ali et al (2009) introduced some new operations on soft set and the notion of complement of soft set. Soft set theory, fuzzy set theory and Rough set theory are all mathematical tools to deal with uncertainty or vagueness. Maji (2012) introduced the notion of reduct soft set and described the application of soft set theory to a decision making problem using Rough set. Malodtsov (2004) has shown several applications of soft set in many eld like economics, engineering, medical sciences etc.

The soft set theory now become a very good source of research for many scholars of mathematics and computer science due to its wide range of applicability. Several interesting new concepts such as Rough soft set, Soft rough set are emerged by combining Rough set and soft set. Many researchers has shown several application of this theory in many elds like economics, engineering, medical sciences etc.

In the present work the lower soft set, upper soft set, lower and upper rough approximation operators on soft set are introduced in a different approach. The notion of Rough soft set is introduced in a different approach and few results are investigated in this context. We de ne rough soft subset and rough soft equal set with examples. Rough soft equality relation are studied and properties are presented. Measure of roughness of soft set is de ned and an algorithm is presented to solve decision making problem by the application of rough soft set. The paper is completed with some concluding remarks.

2. Preliminaries:

Difinition 2. 1 (Ali et al 2009):A soft set is a pair (F, A)of non empty nite set U and A where U be an universe of objects, A is a subset of pa-rameter set E and F is a function from A to power set of U. For each x 2 A, F (x) is considered as the set of x-approximate elements in the soft set. we denote F_A= (F, A)

Soft Subset: Let (F,A) and (G,B)be two soft set over U, A,B⊆E;(F,A) is a subset of (G,B)If (i)A ⊆ B

(ii)F(e)⊆ G(A) ∀ e ϵ A

Symbolically written as (F,A) ⊆ (G,B)

Equal Soft set:(F,A) and (G,B)be two soft set over U, A,B ⊆E;(F,A) is equal to (G,B), symbolically written as (F,A) =(G,B)If (F,A) ⊆(G,B) and (G,B)⊆ (F,A)

Null Soft set: A soft set (F,A) over U is called a null soft set denoted by $(\varphi,A)=\varphi_A$ if for each xϵ A, $F(x)=\varphi$

Whole Soft set: A soft set (F,A) over U is called a whole soft set denoted by $(U,A)=U_A$ if for each x ϵ A, F(x)=U

Definition: The restricted Intersection of (F,A) and (G,B)denoted by $(F,A)\cap_R(G,B)$

=(F,A)∩(G,B) is defined as the soft set (H,C), where C=A∩B and H(x)=F(x)∩G(x) ∀xϵ C

Definition: The restricted union of (F,A) and (G,B)denoted by $(F,A) \cup_R (G,B)$

=(F,A) ∪(G,B) is defined as the soft set (H,C), where C=A B and H(x)=F(x)∪G(x) ∀ xϵC

Equivalence relation: A relation R on a nonempty set U is said to be an equivalence relation if for x, y.z ϵ U

1. R is reflexive i.e. (x,x)ϵR ∀xϵ U
2. R is symmetric i.e. (x,y)ϵR ⇒ (y,x)ϵR
3. R is transitive i.e. (x,y)ϵR;(y,z)ϵR ⇒ (x,z)ϵR

Equivalence class: The equivalence class of an element x ϵ U with respect to the equivalence relation R, denoted by $[x]_R$= { yϵ U| yRx }

Definition: Let R be an equivalence relation on U, U be a non empty finite set (Universe).U/R denote the family of equivalence class of R. For any arbitrary set X⊆U and X is the union of some equivalence classes. We approximate X by a pair of Lower and Upper approximations

$\underline{R}X = \{x \in U \mid [x]_R \subseteq X\}$

$\overline{R}X = \{ x \in U \mid [x]_R \cap X \neq \Phi \}$

$\underline{R}X$ and $\overline{R}X$ are called R-lower and R-upper approximations of X with respect to R.

If $\underline{R}X = \overline{R}X$ we say X is $\overline{R}$-definable

We say X is rough with respect to R iff $\underline{R}X \neq \overline{R}X$

The boundary region of X with respect to R denoted by $B_R X = \overline{R}X - \underline{R}X$

it is clear that $\underline{R}X \subseteq X \subseteq \overline{R}X$

3. Rough soft set:

Let U be the set of objects called universe and (F,A) be a soft set over U where A is the set of parameters and F: A →P(U) is a mapping. If R be an equivalence relation on U and (U,R)be a pawlak approximation space w.r.to A.

Define two soft sets $\underline{F}_A$ and $\overline{F}_A$, where $\underline{F}_A$:

A→P(U),$\overline{F}_A$: A→P(U)called lower soft set and upper soft set over U as

$\underline{F}_A(x) = \underline{R}(F(x)) = \{y \in U \mid [y]_R \subseteq F(x)\}$

$\overline{F}_A(x) = \overline{R}(F(x)) = \{y \in U \mid [y]_R \cap F(x) \neq \Phi\}$ ∀xϵ A

Definition: We say (F,A), a rough soft set with respect to parameter x ϵA if $\underline{F}_A(x) \neq F^{-}{}_A(x)$

(F, A) is said to be definable or crip soft set with respect to parameter x ϵ A if $\underline{F}_A(x) = F^{-}{}_A(x)$.

(F, A) is said to be a full rough soft set or rough soft set if F_A(x) ≠F¯A(x) ∀

xϵA and denote rough soft set w.r.to e as RSFA(e)

(F, A) is said to be a full rough soft set or rough

Example 3.1 Table 1 contains some observed symptoms of 6 patients suffering from certain diseases. here U = $\{p_1, p_2, ...p_6\}$,condition attributes = temperature, muscular pain, blood from mouth , decision attributes = *dengue* and R be an equivalence relation 'diagnosis of diseases'.

Table 1

patient	temp.	muscular pain	blood from mouth	dengue
p_1	normal	no	yes	yes
p_2	high	yes	yes	yes
p_3	high	yes	yes	yes
p_4	normal	no	yes	no
p_5	normal	yes	no	yes
p_6	high	no	no	no

IND(R)=$\{\{p_1, p_4\}, \{p_2, p_3\}, \{p_5\}, \{p_6\}\}$

$[p_1]_R = \{p_1, p_4\} = [p_4]_R, [p_2]_R = \{p_2, p_3\} = [p_3]_R, [p_5]_R = \{p_5\}, [p_6]_R = \{p_6\}$

(F,A)be a soft set representing system of patients given below ,where A=$\{e_1, e_2, e_3\}$

and e_1 denotes temp.,e_2 denotes muscular pain,e_3 denotes blood from mouth, F:A →P(U) such that

$F(e_1)=\{p_1,p_3,p_4\}$,$F(e_2)=\{p_2,p_3,p_5\}$,$F(e_3)=\{p_1,p_2,p_3,p_4\}$

$\underline{F_A}(e_1)=\{p_1, p_4\}$,$\overline{F_A}(e_1)=\{p_1, p_2, p_3, p_4\}$

$\underline{F_A}(e_2)=\{p_2, p_3, p_5\}$,$\overline{F_A}(e_2)=\{p_2, p_3, p_5\}$

$\underline{F_A}(e_3)=\{p_1, p_2, p_3, p_4\}$,$\overline{F_A}(e_3)=\{p_1, p_2, p_3, p_4\}$

(F,A) is a rough soft set with respect to parameter e_1 but not e_2 and e_3

Definition: Let (F,A) and (G,B) be two rough soft sets. (G,B) is said to be rough soft subset of (F,A) written as $RSG_B \subseteq RSF_A$ if

(1)$B \subseteq A$

(2)$\underline{G_B}(b)\subseteq \underline{F_A}(b)$,$\overline{G_B}(b)\subseteq\overline{F_A}(b)$ $b\epsilon B$

Defination: Let (F,A) and (G,B) be two rough soft sets, are said to be equal written as $RSG_B = RSF_A$ if

$RSG_B \subseteq RSF_A$ and $RSF_A \subseteq RSG_B$

Example:3.2 In example 3.1 suppose $A=\{e_1, e_2, e_3\}$, $F:A \longrightarrow P(U)$ defined

$F(e_1)=\{p_1,p_3,p_4,p_5\}$,$F(e_2)=\{p_2,p_3,p_4,p_6\}$,$F(e_3)=\{p_1,p_3, p_4\}$

$\underline{F_A}(e_1)=\{p_1, p_4, p_5\}$,$\overline{F_A}(e_1)=\{p_1, p_2, p_3, p_4, p_5\}$

$\underline{F_A}(e_2)=\{p_2, p_3, p_6\}$,$\overline{F_A}(e_2)=\{p_1p_2, p_3, p_4, , p_6\}$

$\underline{F_A}(e_3)= p_1, p_4$,$\overline{F_A}(e_3)= p_1, p_2, p_3, p_4$

(F,A) is a rough soft set

suppose $B=\{e_2, e_3\}$, $G:B \rightarrow P(U)$defined by

$G(e_2) =\{p_1, p_2, p_3\}$,$G(e_3) =\{p_1, p_3, p_4\}$

$\underline{G_B}(e_2)=\{p_2, p_3\}$,$\overline{G_B}(e_2)=\{p_1, p_2, p_3, p_4\}$

$\underline{G_B}(e_3)=\{p_1, p_4\}$,$\overline{G_B}(e_3)=\{p_1, p_2, p_3, p_4\}$

(G,B) is also a rough soft set and $B \subseteq A$

$\underline{G_B}(e_2) \subseteq \underline{F_A}(e_2)$,$\overline{G_B}(e_2) \subseteq \overline{F_A}(e_2)$

$\underline{G_B}(e_3) \subseteq \underline{F_A}(e_3)$,$\overline{G_B}(e_3) \subseteq \overline{F_A}(e_3)$

so $RSG_B(e) \subseteq RSF_A(e)$ $\forall e\subseteq B$

Proposition 3.1:If (G,B) is soft subset of (F,A)then $RSG_B \subseteq RSF_A$

Proof: (G,B) is soft subset of (F,A)

so (1)$B \subseteq A$

(2)$G(b)\subseteq F(b)$ $\forall b \subseteq \underline{B}$

$\Rightarrow \underline{G_B}(b) \subseteq \underline{F_A}(b)$,$\overline{G_B}(b) \subseteq \overline{F_A}(b)$ $\forall b \epsilon B$

Therefore $RSG_B(e) \subseteq RSF_A(e)$ $\forall e \epsilon B$

Example:3.3 In example 3.1 suppose $A=\{e_1, e_2, e_3\}$, $F: A \rightarrow P(U)$ defined

$F(e_1) =\{p_1, p_3, p_4, p_5\}$,$F(e_2) =\{p_2, p_3, p_4, , p_6\}$, $F(e_3) =\{p_1, p_3, p_4\}$

suppose $B=\{e_2, e_3\}$, $G:B \rightarrow P(U)$defined by

$G(e_2) =\{p_2, p_3, p_4\}$,$G(e_3) =\{p_3, p_4\}$

here $(G,B) \subseteq (F,A)$ $\forall e \epsilon B$

$\underline{F_A}(e_2)=\{p_2, p_3, p_6\}$,$\overline{F_A}(e_2)=\{p_1p_2, p_3, p_4, p_6\}$

$\underline{G_B}(e_2)=\{p_2, p_3\}$,$\overline{G_B}(e_2)=\{p_1, p_2, p_3, p_4\}$

$\underline{F_A}(e_3)=\{p_1, p_4\}$,$\overline{F_A}(e_3)=\{p_1, p_2, p_3, p_4\}$

$\underline{G_B}(e_3)=\varphi$,$\overline{G_B}(e_3)=\{p_1, p_2, p_3, p_4\}$

$\underline{G_B}(e_2) \subseteq \underline{F_A}(e_2)$;$\overline{G_B}(e_2)\subseteq \overline{F_A}(e_2)$

$\underline{G_B}(e_3) \subseteq \underline{F_A}(e_3)$;$\overline{G_B}(e_3)\subseteq \overline{F_A}(e_3)$ so $RSG_B(e)\subseteq RSF_A(e)$ $e\epsilon B$

Definition: (Measure of roughness of soft set)

Let (F,A) be a soft set over U and (U,R) be a pawlak approximation space w.r.to A. Measure of roughness of (F,A) with respect to parameter $e \epsilon A$ denoted by $RF_A(e)=|\underline{F_A}(e)| / |\overline{F_A}(e)|$ where $0 \leq RF_A(e) \leq 1$

Example:3.4 In example 3.1 $RF_A(e_1)=1/2$;$RF_A(e_2)=1$;$RF_A(e_3)=1$

Proposition 3.2: If F_A= (F,A) is a soft set over the universe X and R be an equivalence relation on X then

1. $\underline{F_A}\subseteq F_A\subseteq \overline{F_A}$
2. $\underline{\varphi_A}=\varphi_A=\overline{\varphi_A}$
3. $\underline{X_A}=X_A=\overline{X_A}$

Proof: i) let $y\epsilon \underline{F_A}(x) =\underline{R}(F(x))$
$\Rightarrow [y]_R \subseteq F(x)$ $\forall x\epsilon A$
$\Rightarrow [y]_R \cap F(x) = [y]_R \neq \varphi$
$\Rightarrow y \epsilon R(F(x))$

$\underline{R}(F(x)) \subseteq R(F(x))$

let $y\epsilon \underline{R}(F(x))$
$\Rightarrow [y]_R \subseteq F(x)$
$\Rightarrow y\epsilon F(x)$

$\underline{R}(F(x))\subseteq F(x)$ $\forall\epsilon A$ let $y\epsilon F(x)$
$\Rightarrow y \epsilon [y]_R \cap F(x)$
$\Rightarrow [y]_R \cap F(x) \neq \varphi$
$\Rightarrow y \epsilon R(F(x))$

$F(x)\subseteq R(F(x))$

$\underline{R}(F(x))\subseteq F(x)\subseteq R(F(x))$ $x\epsilon A$

$\underline{R}(\varphi(x)) \subseteq \varphi(x) = \varphi$ by(i) but $\varphi(x)\subseteq \underline{R}(\varphi(x))$

$\underline{R}(\varphi(x)) = \varphi(x)$ $\underline{\varphi_A}=\varphi_A$

Assume $R^{-}(\varphi(x)) \neq \varphi(x)$

then there exist y such that $y \epsilon R(\varphi(x))$

so $[y]_R \varphi(x) \neq \varphi$ but $[y]_R \varphi = \varphi$ which contradict the assumption therefore $R(\varphi(x)) = \varphi(x)$

$\varphi A = \varphi_A$

iii) By (i) $\underline{R}(X(x)) \subseteq \overline{X(x)}$ let $y \subseteq \overline{X(x)}$

$[y]_R \subseteq X$

$y \subseteq \underline{R}(X(x))$ $X(x) \subseteq \underline{R}(X(x))$

so $\underline{R}(X(x)) = X(x)$

By (i) $X(x) \subseteq R(X(x))$

$R(X(x)) \subseteq X(x)$ obvious because X is the universal set so $X(x) = R(X(x))$

therefore $\underline{X_A} = X_A = X_A$

Proposition 3.3 If F_A and G_A are two soft set, ∪ and ∩ denote restricted union and restricted intersection then

1. $F_A \cup G_A = \overline{F_A \cup G_A}$
2. $\underline{F_A \cap G_A} = \underline{F_A} \cap \underline{G_A}$

Proof: i) let $y \subseteq \overline{R}((F \cup G)(x))$
$\Leftrightarrow [y]_R \cap (F \cup G)(x) \neq \varphi$
$\Leftrightarrow ([y]_R \cap F(x)) \cup ([y]_R \cap G(x)) \neq \varphi$
$\Leftrightarrow \{[y]_R \cap F(x) \neq \varphi\} or \{[y]_R \cap G(x) \neq \varphi\}$
$\Leftrightarrow \{y \epsilon R(F(x))\} or \{y \epsilon R(G(x))\}$
$\Leftrightarrow y \epsilon R(F(x)) \cup R(G(x))$

$R((F \cup G)(x)) = R(F(x)) \cup R(G(x))$

$F_A \cup G_A = F_A \cup G_A$

ii) let $y \epsilon \underline{R}((F \cap G)(x))$
$\Leftrightarrow [y]_R \subseteq (F \cap G)(x)$
$\Leftrightarrow \{[y]_R \subseteq F(x)\} and \{([y]_R \subseteq G(x))\}$
$\Leftrightarrow \{y \epsilon \underline{R}(F(x))\} and \{y \epsilon \underline{R}(G(x))\}$
$\Leftrightarrow y \epsilon \underline{R}(F(x)) \cap \underline{R}(G(x))$

$\underline{R}((F \cap G)(x)) = \underline{R}(F(x)) \cap \underline{R}(G(x))$ $\underline{F_A \cap G_A} = \underline{F_A} \cap \underline{G_A}$

Proposition 3.4 If F_A and G_A are two soft set such that $F_A \subseteq G_A$ then

1. $\underline{F_A} \subseteq \underline{G_A}$
2. $F_A \subseteq G_A$

Proof:i] $F_A \subseteq G_A$
$\Rightarrow F_A \cap G_A = F_A$
$\Rightarrow \underline{F_A \cap G_A} = \underline{F_A}$
$\Rightarrow \underline{F_A} \cap \underline{G_A} = \underline{F_A}$ by Proposition 3.2 (i)

$\underline{F_A} \subseteq \underline{G_A}$

ii) $F_A \subseteq G_A$
$\Rightarrow \underline{F_A \cup G_A} = \underline{G_A}$
$\Rightarrow \underline{F_A} \cup \underline{G_A} = \underline{G_A}$
$\Rightarrow F_A \cup G_A = G_A$ by Proposition 3.2 (ii)
$F_A \subseteq G_A$

Proposition 3.5 If F_A and G_A are two soft set, and denote restricted union and restricted intersection then

1. $\underline{F_A \cup G_A} \supseteq \underline{F_A} \cup \underline{G_A}$
2. $F_A \cap G_A \subseteq F_A \cap G_A$

Proof: i] since $F_A \subseteq F_A \cup G_A$
$\Rightarrow \underline{F_A} \subseteq \underline{F_A \cup G_A}$ by Proposition 3.3 (i)
$G_A \subseteq F_A \cup G_A$
$\Rightarrow \underline{G_A} \subseteq \underline{F_A \cup G_A}$ by Proposition 3.3 (i) hence $\underline{F_A} \cup \underline{G_A} \subseteq \underline{F_A \cup G_A}$

(ii) since $F_A \cap G_A \subseteq F_A$
$\Rightarrow F_A \cap G_A \subseteq F_A$ by Proposition 3.3 (ii)
$F_A \cap G_A \subseteq G_A$
$\Rightarrow F_A \cap G_A \subseteq G_A$ by Proposition 3.3 (ii) hence $F_A \cap G_A \subseteq F_A \cap G_A$

Proposition 3.6 If $F_A = (F, A)$ is a soft set over the universe U and R be an equivalence relation on U then

1. $\underline{F_A} = \underline{F_A}$
2. (ii) $F_A = F_A$
3. (iii) $\underline{F_A} = \underline{F_A}$
4. (iv) $\underline{F_A} = F_A$

Proof: i) $\underline{F_A} \subseteq \underline{F_A}$ by Proposition 3.1
let $y \epsilon \underline{F_A} = \underline{R}(F(x))$
$\Rightarrow [y]_R \subseteq F(x)$
$\Rightarrow \underline{R}[y]_R \subseteq \underline{R}(F(x))$
$\Rightarrow [y]_R \subseteq \underline{R}(F(x))$
$\Rightarrow y \epsilon \underline{R}(\underline{R}(F(x)))$
$\Rightarrow y \epsilon \underline{F_A}$
$\underline{F_A} \subseteq F^{-}A$
hence $\underline{F_A} = \underline{F_A}$

(ii) $F^{-}A \subseteq F^{-}A^{-}$ by Proposition 3.1
let $y \subseteq F^{-}{}_{A^{-}} = R^{-}(R^{-}(F(x)))$

$\Rightarrow [y] \cap R(F(x)) \neq \varphi$
and for $y \epsilon [y]_R$ and $y \epsilon R(F(x))$ hence $[y]_R \cap F(x) \neq \varphi$
$\Rightarrow y \epsilon F_A$
$F_A \subseteq F_A$
(iv) $F_A \subseteq F_A$ by Proposition 3.1
let $y \subseteq F_A = R(F(x))$
$\Rightarrow [y]_R \cap F(x) \neq \varphi$
if $z \epsilon [y]_R$ then $[z]_R \cap F(x) = [y]_R \cap F(x) \neq \varphi$
$\Rightarrow z \epsilon R(F(x))$
hence $[y]_R \subseteq R(F(x))$
$\Rightarrow y \epsilon \underline{R}(R(F(x)))$
$\Rightarrow y \epsilon \underline{F_A}$

$F_A \subseteq \underline{F_A}$

therefore $\underline{F_A} = F_A$

4. Definition :(Rough soft equal relation)

Let (F, A) be a soft set over U and (U,R) be a pawlak approximation space w.r.to A, we define

$F(e_1) \simeq_A F(e_2)$ iff $\underline{F_A}(e_1) = \underline{F_A}(e_2)$ i.e $\underline{R}(F(e_1)) = \underline{R}(F(e_2))$ where $e_1, e_2 \epsilon$ A

$F(e_1) \bar{=}_A F(e_2)$ iff $F_A(e_1) = F_A(e_2)$ i.e. $R(\underline{F}(e_1)) = \underline{R}(F(e_2))$ where $e_1, e_2 \epsilon$ A≈

$F(e_1)\ _A F(e_2)$ iff $\underline{F_A}(e_1) = \underline{F_A}(e_2)$ and $F_A(e_1) = F_A(e_2)$—

The above binary relation are called lower rough soft equal relation, upper

rough soft equal relation and rough soft equal relation respectively.

Proposition 4.1 The above binary relation are all equivalence relationover A

Exammple: 4.1 In example 3.1 (F, A)be a soft set representing system of patients given below ,where A=$\{e_1, e_2, e_3\}$ and e_1 denotes temp.,e_2 denotes

muscular pain,e_3 denotes blood from mouth, F:A → P(U) such that

$F(e_1) = \{p_1, p_3, p_4\}$,$F(e_2) = \{p_1, p_2, p_4\}$

$\underline{F_A}(e_1) = \{p_1, p_4\}$;$\underline{F_A}(e_2) = \{p_1, p_4\}$

So $F(e_1) \simeq_A F(e_2)$

$F_A(e_1) = \{p_1, p_2, p_3, p_4$;$F_A(e_2) = \{p_1, p_2, p_3, p_4$

so $F(e_1) \bar{=}_A F(e_2)$ hence $F(e_1) \approx_A F(e_2)$

Proposition 4.2 Let F_A be a rough soft set with respect to parameter e ϵA then

1. $F(e_1)\ _A F(e_2)$ iff $F(e_1)\ _A F(e_1) \cup F(e_2)\ _A F(e_2)$
2. $F(e_1) \simeq_A F(e_2)$ iff $F(e_1) \simeq_A F(e_1) \cap F(e_2) \simeq_A F(e_2)$
3. $F(e_1)\ _A F(e_2)$; $F(e_3)\ _A F(e_4) \Rightarrow F(e_1) \cup F(e_3)\ _A F(e_2) \cup F(e_4)$
4. (iv)$F(e_1) \simeq_A F(e_2)$; $F(e_3) \simeq_A F(e_4) \Rightarrow F(e_1) \cap F(e_3) \simeq_A F(e_2) \cap F(e_4)$ (v)$F(e_1) \bar{=}_A F(e_2) \Rightarrow F(e_1) \cup$(U-$F(e_2)) \bar{=}_A$ U
5. $F(e_1) \simeq_A F(e_2) \Rightarrow F(e_1) \cap$(U-$F(e_2)) \simeq_A \varphi$
6. $F(e_1) \subseteq F(e_2)$; $F(e_2)\ _A \varphi \Rightarrow F(e_1)\ _A \varphi$
7. $F(e_1) \subseteq F(e_2)$; $F(e_2) \simeq_A \varphi \Rightarrow F(e_1) \simeq_A \varphi$
8. $F(e_1) \subseteq F(e_2)$; $F(e_1)\ _A U \Rightarrow F(e_2)\ _A U$
9. $F(e_1) \subseteq F(\underline{e_2})$; $F(e_1) \simeq_A U \Rightarrow F(e_2) \simeq_A U$

where e_1,e_2,e_3,e_4 A and $F(e_1),F(e_2),F(e_3),F(e_4)$ U —

Proof:(i) $F(e_1) \bar{=}_A F(e_2)$

$\Rightarrow F_A(e_1) = F_A(e_2)$ i.e.$R(F(e_1)) = R(F(e_2))$ $R(F(e_1) \cup F(e_2)) = R(F(e_1)) \cup R(F(e_2)) = R(F(e_1)) = R(F(e_2))$ $F(e_1) \bar{=}_A F(e_1) \cup F(e_2) \bar{=}_A F(e_2)$

ii) $F(e_1) \simeq_A F(e_2)$

$\Rightarrow \underline{F_A}(e_1) = \underline{F_A}(e_2)$ i.e $\underline{R}(F(e_1)) = \underline{R}(F(e_2))$

$\underline{R}(F(e_1) \cap F(e_2)) = \underline{R}(F(e_1)) \cap \underline{R}(F(e_2)) = \underline{R}(F(e_1)) = \underline{R}(F(e_2))$

$F(e_1) \simeq_A F(e_1) \cap F(e_2) \simeq_A F(e_2)$

iii) $F(e_1) \bar{=}_A F(e_2)$; $F(e_3) \bar{=}_A F(e_4)$

$\Rightarrow R(F(e_1)) = R(F(e_2))$;$R(F(e_3)) = R(F(e_4))$

$R(F(e_1) \cup F(e_3)) = R(F(e_1)) \cup R(F(e_3)) = R(F(e_2)) \cup R(F(e_4)) = R(F(e_2) \cup F(e_4))$

$F(e_1) \cup F(e_3) \bar{=}_A F(e_2) \cup F(e_4)$

iv)$F(e_1) \simeq_A F(e_2)$; $F(e_3) \simeq_A F(e_4)$

$\Rightarrow \underline{R}(F(e_1)) = \underline{R}(F(e_2))$;$\underline{R}(F(e_3)) = \underline{R}(F(e_4))$

$\underline{R}(F(e_1) \cap F(e_3)) = \underline{R}(F(e_1)) \cap \underline{R}(F(e_3)) = \underline{R}(F(e_2)) \cap \underline{R}(F(e_4)) = \underline{R}(F(e_2) \cap$

$F(e_4))$

$F(e_1) F(e_3)\ _A F(e_2) F(e_4)$ v)$F(e_1) \bar{=}_A F(e_2)$

$\Rightarrow R(F(e_1) = R(F(e_2))$

$R(F(e_1) \cup (U - (F(e_2)) = R(F(e_1) \cup R(U - (F(e_2))$

but U=$\underline{F}(e_2) \cup$(U-$F(e_2)$)

$R(\underline{U}) = R(F(e_2) \cup (U - F(e_2))) = R(F(e_2)) \cup R(U - F(e_2))$

so $R(F(\underline{e_1}) \cup (U - F(e_2))) = R(F(e_1)) \cup R(U - F(e_2)) = R(F(e_2)) \cup R(U - F(e_2)) = R(U)$

$F(e_1) \cup$(U-$F(e_2)) \bar{=}_A$ U

vi) $F(e_1) \simeq_A F(e_2)$

$\Rightarrow \underline{R}(F(e_1) = \underline{R}(F(e_2))$

but $\varphi = F(e_2) \cap$(U-$F(e_2)$)

$\underline{R}(\varphi) = \underline{R}(F(e_2) \cap (U - F(e_2))) = \underline{R}(F(e_2)) \cap \underline{R}(U - F(e_2))$

so $\underline{R}(F(e_1) \cap (U - F(e_2))) = \underline{R}(F(e_1)) \cap \underline{R}(U - F(e_2)) = \underline{R}(F(e_2)) \cap \underline{R}(U - F(e_2)) = \underline{R}(\varphi)$

$F(e_1) \cap$(U-$F(e_2)) \simeq_A \varphi$

vii) $\underline{F}(e_1) \subseteq F(\underline{e_2})$; $F(e_2)\ _A \varphi$ i.e.$R(F(e_2) = R(\varphi)$

$\Rightarrow R(F(e_1) \subseteq R(F(e_2)) = R(\varphi)$

but $\varphi \subseteq F(e_1) \Rightarrow R(\underline{\varphi}) \subseteq R(F(e_1)$ therefore $R(F(e_1) = R(\varphi)$

$F(e_1)\ _A \varphi$ viii)similarly as per (vii)

ix)$F(e_1) \subseteq F(\underline{e_2}) \Rightarrow R(\underline{F}(e_1) \subseteq R(F(e_2))$ $F(e_1) \bar{=}_A U \Rightarrow R(F(e_1) = R(U)$

$\Rightarrow \underline{R}(F(e_1) \subseteq R(F(e_2))$

$\Rightarrow R(U) \subseteq R^{-}(F(e_2))$

but $F(e_2) \subseteq U$

$R(F(e_2)) \subseteq R(U) \Rightarrow$

therefore $R(F(e_2)) = R(U)$ $F(e_2)$ $_A U$ x) similarly as per (ix)

5. Application of rough soft sets in decision making:

Let U be the set of objects called universe ,R be an equivalence relation on U and (U,R) be a pawlak approximation space w.r.to A where A is the set of parameters. Suppose (F.A) be a soft set over U and F is a mapping from A to P (U)

The decision algorithm for rough soft set as follows

1. Input pawlak approximation space (U, R)
2. Input soft set (F, A)
3. compute $\underline{F}_A(e_i)$ and $\overline{F}_A(e_i)$ for each $e_i \in A$
4. compute measure of roughness of (F,A)w. r. to parameter $e_i \in A$ where $RF_A(e_i) = |\underline{F}_A(e_i)| / |\overline{F}_A(e_i)|$
5. Find maximum value $RF_A(e_k)$ of $RF_A(e_i)$ where $RF_A(e_k)$= Max $RF_A(e_i)$ 6)The decision is $F(e_k)$

Example: 5 A scientist detected five materials denoted by e_1, e_2, e_3, e_4, e_5.After

experiment he comes to know all the materials contain one or more molecules out of molecules $m_1, m_2, m_3, m_4, m_5, m_6, m_7$.He want to choice the closest to the molecular structures out of five materials. According to chemical proper- ties he observed, some molecules are equivalent. Suppose A= $\{e_1, e_2, e_3, e_4, e_5\}$ and U=$\{m_1, m_2, m_3, m_4, m_5, m_6, m_7\}$

Let R denote the molecules whose chemical properties are equivalent .According to scientist, R=$\{\{m_1, m_3\}; \{m_2, m_4, m_6\}; \{m_5, m_7\}\}$

$[m_1]_R=\{m_1,m_3\}=[m_3]_R$ $[m_2]_R=\{m_2,m_4,m_6\}=[m_4]_R=[m_6]_R$ $[m_5]_R= \{m_5, m_7\}=[m_7]_R$

Consider (U, R) be the pawlak approximation space where R is an equivalence relation on U.Each kind of materials containing molecules $F(e_1) = \{m_1, m_3, m_4\}$

$F(e_2) = \{m_1, m_2, m_4\}$

$F(e_3) = \{m_3, m_4, m_6\}$

$F(e_4) = \{m_2, m_5, m_7\}$

$F(e_5) = \{m_1, m_3, m_5, m_6, m_7\}$ respectively

Consider (F, A) denote soft set on U defined on above $\underline{F}_A(e_1)=\{m_1, m_3\}$ $\underline{F}_A(e_2)=\varphi$

$\underline{F}_A(e_3)=\varphi$ $\underline{F}_A(e_4)=\{m_5, m_7\}$

$\underline{F}_A(e_5)=\{m_1, m_3, m_5, m_7\}$

$\overline{F}_A(e_1)=\{m_1, m_2, m_3, m_4, m_6\}$

$\overline{F}_A(e_2)=\{m_1, m_2, m_3, m_4, m_6\}$

$\overline{F}_A(e_3)=\{m_1, m_2, m_3, m_4, m_6\}$

$\overline{F}_A(e_4)=\{m_2, m_4, m_5, m_6, m_7\}$

$\overline{F}_A(e_5)=\{m_1, m_2, m_3, m_4, m_5, m_6, m_7\}$

$RF_A(e_1)=| \underline{F}_A(e_1) | / | F^{-}_A(e_1) | =2/5=.4$ $RF_A(e_2)=0/5=0$

$RF_A(e_3)=0/5=0$ $RF_A(e_4)=2/5=.4$ $RF_A(e_5)=4/7=.57$

Thus maximum value of $RF_A(e_i)$ is $RF_A(e_5)$

that means $F(e_5)$is the anticipated material. i.e. e_5 is the closest one in all of the materials.

Remarks:

1. It is a new decision making method for rough soft sets.
2. In this method we will get which is the best parameter of given soft set (F,A)
3. We obtain the most expected material on soft set w.r.t. an equivalence relation on the universe U in example 5

In this method, decision makers have adapted various criterions in order to reach the conformation suggesting actual situation.

4. I hope this method may be outfit for our real life decisions making.

Remarks: The decision parameter carries distinct importance to various people. It is a difficult task to solve decision making problem. To obtain decision making parameters in a decision making problem, we can use application of rough soft sets. By using above algorithm, we will obtain the key parameter which is necessary for decision making. In this method decision maker will select the goods/candidates/materials by using the key parameters.

6. Conclusion:

Combination of two approaches rough set and soft set named rough soft set theory is a mathematical tool for dealing with uncertainties. We have introduced the approximation of soft set in pawlak approximation space. We define rough soft subset and rough soft equal set with examples. Rough soft equality relation is studied and properties are presented. Measure of roughness of soft set is defined and an algorithm is presented to solve decision making problem by the application of rough soft set.

7. REFERENCES:

[1] Z. Pawlak; Rough sets, Theorical Aspects of Reasoning about Data, Kluwer Academic Publishers.

[2] D. Molodtsov; Soft set theory first results, Computer and mathematics with applications (1999) 13-31

[3] P. K. Majhi, R. Biswas, A. R. Roy; Soft set theory, Computer and mathematics with applications (2003), 555-562.

[4] M. I. Alli, F. Feng, X. Liu, W. K. Min, M. Shabir; On Some new operations in soft set theory, computer mathematics with applications 57 (2009), 1547-1553

[5] Z. Pawlak; Rough sets, International journal of Computer and information science, vol-11, pp. 341-356, 1982

[6] D. Molodtsov ; The theory of soft sets, URSS publication, moscow 2004

[7] F. Feng, C. X. Li, B. Davvaz and M. I. Alli ; Soft sets combined with fuzzy sets and rough sets a tentative approach, Soft computing(2010), 899-911

[8] F. Feng, Y. Li and V. Leoreanu Fotea, Application of Level Soft set in decision making based on interval valued Fuzzy Soft sets , J comput Math Appl 60(6)2010, 1756-1767

[9] Y. Jiang, Y. Tang and Q. Chan, An adjustable approach to intuitionistic Fuzzy Soft sets based decision making , Appl Math Model 35(2), 2011, 824- 836

[10] P. K. Majhi, A neutrosophic Soft set approach to a decision making problem , Annual of fuzzy mathematics and informatics vol. 3 No 2(april 2012) 313-319

[11] A. K. Ray and P. K. Majhi , A Fuzzy soft set theoretic approach to decision making problems, Journal of Computational and applied mathematics vol 203 no 2 pp412-418, 2007

[12] Nasef and M. Elsayed , molodsov soft set theory and its application in decision making, IJESI, VOL-6 issue 2 feb 17 pp 86-90

[13] P. K. Majhi , A. R. Roy, R. Biswas , An application of soft sets in a decision making problem, compute math appl. 44(2002) 1077-1083

[14] Maharana M., Mohanty D. (2021) An application of soft set theory in decision making problem by parameterization reduction , Soft Computing (2021) 25:3989–3992