International Journal of Trend in Scientific Research and Development (IJTSRD)
Volume 5 Issue 5, July-August 2021 Available Online: www.ijtsrd.com e-ISSN: 2456 – 6470

An Approach to Mathematically Establish the Practical Use of Assignment Problem in Real Life

Suraj Kumar Sahu

Guest Lecture at Model Degree College Nabarangpur, Odisha, India

ABSTRACT

The assignment problem is a discrete and combinatorial problem where agents are assigned to perform tasks for efficiency maximization or cost (time) minimization. Assignment Problem is a part of human resource project management..

The aim of the current study is to describe the importance of an researching solution method for the classical assignment problem the outcomes from the study show that both classical methods and the Greedy method provides the optimal or near optimal results

How to cite this paper: Suraj Kumar Sahu "An Approach to Mathematically Establish the Practical Use of Assignment Problem in Real Life" Published in International Journal of Trend in Scientific Research and Development (ijtsrd), ISSN: 2456-6470, Volume-5 | Issue-5, August 2021, pp.1815-1820, URL: www.ijtsrd.com/papers/ijtsrd45217.pdf

IJTSRD45217

INTRODUCTION

Assignment problem is to assign a number of origin to the equal number of destination at minimum cost or maximum profit.

The aim of looking into assignment problem is to discover an assignment among two or more sets of elements, which could minimize the total cost of all matched pairs. Relying on the specific structure of the matched sets and the cost function form, the allocation problems can be categorised into quadratic, bottleneck, linear, and multidimensional groups, Hence, every assignment problem has a table or matrix. Normally, the rows are comprised of objects or people to assign, while the columns consist of the things or tasks to be assigned. Meanwhile, the numbers in the table refer to the costs related to every particular assignment. With that, this study presents a review of assignment problem within educational activities, where the problems were classified into timetabling and allocation problems.

TERMINOLOGY

The Assignment problem is one amongst the elemental combinatorial improvement within the branch of optimization in mathematics. It is a particular case of transportation problem where the source has a supply one and every destination has a demand one. The assignment is to be made in such a way so as to maximize (Or minimize)the total effectiveness. In this problem, the jobs must be equal to the number of machines so that each job is assigned to one and only one machine. In the assignment process, the challenge here is to optimally assign the resource to jobs such as all job work is assigned without leaving it out and all resources are engaged in tasks on a job. The allocation to be optimal forms the basis of this assignment problem, in other words, each of the jobs to be assigned to one resource so as to derive the work with lesser time thereby bringing in efficiency. A matrix comprising an array of rows and columns where each entity represents a job and its resources are considered. The problem of assignment arises because the resources those are available such as men, machine shaving a varying degree of efficiency for performing different activities. Along these lines cost, benefit or time of performing distinctive exercises is likewise unique. Consequently, the issue is by what method should the assignments be made in order to upgrade the given goal.

Assignment problem is a special type of linear programming problem which deals with the allocation of the various resources to the various activities on one to one basis. It does it in such a way that the cost or time involved in the process is minimum and profit or sale is maximum. Though there problems can be solved by simplex method or by transportation method but assignment model gives a simpler approach for these problems.

In a factory, a supervisor may have six workers available and six jobs to fire. He will have to take decision regarding which job should be given to which worker. Problem forms one to one basis. This is an assignment problem.

The assignment problem is a fundamental combinatorial optimization problem. In its most general form, the problem is as follows:

The problem instance has a number of *agents* and a number of *tasks*. Any agent can be assigned to perform any task, incurring some *cost* that may vary depending on the agent-task assignment. It is required to perform as many tasks as possible by assigning at most one agent to each task and at most one task to each agent, in such a way that the *total cost* of the assignment is minimized.

Alternatively, describing the problem using graph theory:

The assignment problem consists of finding, in, weighted bipartitr a matching of a given size, in which the sum of weights of the edges is a minimum.

LITERATURE REVIEW
INTRODUCTION

The aim of this section is to elaborate on basic concepts and terminology underlying to study of assignment problem, Here we provide a basic revies of the leteratures to data with the intent of fostering a better understanding of concept and analyses that are used in leter section. We will begin in introduction to assignment model and methods of analysis that have been developed over the past several centuries

Literature review,

An assignment problem is a particular case of transportation problem where the objective is to assign a number of resources to an equal number of activities so as to minimise total cost or maximize total profit of allocation.

The problem of assignment arises because available resources such as men, machines etc. have varying degrees of efficiency for performing different activities, therefore, cost, profit or loss of performing the different activities is different.

Suppose there are n jobs to be performed and n persons are available for doing these jobs. Assume that each person can do each job at a term, though with varying degree of efficiency, let cij be the cost if the i-th person is assigned to the j-th job. The problem is to find an assignment (which job should be assigned to which person one on-one basis) So that the total cost of performing all jobs is minimum, problem of this kind are known as assignment problem.

Hungarian Method for Solving Assignment Problem:

The Hungarian method of assignment provides us with an efficient method of finding the optimal solution without having to make a-direct comparison of every solution. It works on the principle of reducing the given cost matrix to a matrix of opportunity costs.

Opportunity cost show the relative penalties associated with assigning resources to an activity as opposed to making the best or least cost assignment. If we can reduce the cost matrix to the extent of having at least one zero in each row and column, it will be possible to make optimal assignment.

In mathematical terms, the assignment problem is: given an $n \times n$ 'cost' matrix $C = c(i,j)$,

find a permutation π of 1,..., n for which is as small as possible.

Monge 1784

The assignment problem is one of the first studied combinatorial optimization problems.

It was investigated by G. Monge [1784], albeit camouflaged as a continuous problem, and often called a transportation problem.

Monge was motivated by transporting earth, which he considered as the discontinuous, combinatorial problem of transporting molecules. There are two areas of equal acreage, one filled with earth, the other empty. The question is to move the earth from the first area to the second, in such a way that the total transportation distance is as small as possible.

The total transportation distance is the distance over which a molecule is moved, summed over all molecules. Hence it is an instance of the assignment problem, obviously with an enormous cost matrix.

Bipartite matching: Frobenius 1912-1917, K˝onig 1915-1931

Finding a largest matching in a bipartite graph can be considered as a special case of the assignment problem. The fundaments of matching theory in bipartite graphs were laid by Frobenius (in terms of matrices and determinants) and K˝onig. We briefly review their work.

In his article Ub¨ er Matrizen aus nicht negativen Elementen, Frobenius [1912] investigated the decomposition of matrices, which led him to the following 'curious determinant theorem': Egerv´ary 1931

After the presentation by K˝onig of his theorem at the Budapest Mathematical and Physical Society on 26 March 1931, E. Egerv´ary [1931] found a weighted version of K˝onig's theorem.

It characterizes the maximum weight of a matching in a bipartite graph, and thus applies to the assignment problem: Robinson 1949

Cycle reduction is an important tool in combinatorial optimization. In a RAND Report dated 5 December 1949, Robinson [1949] reports that an 'unsuccessful attempt' to solve the traveling salesman problem, led her to the following cycle reduction method for the optimum assignment problem.

Let matrix (ai,j) be given, and consider any permutation π. Define for all i, j a 'length'

li,j by: li,j := aj,π(i) − ai,π(i)

if j 6= π(i) and li,π(i) = ∞. If there exists a negative-length

directed circuit, there is a straightforward way to improve π. If there is no such circuit, then π is an optimal permutation. This clearly is a finite method,

The simplex method

A breakthrough in solving the assignment problem came when Dantzig [1951a] showed that the assignment problem can be formulated as a linear programming problem that automatically has an integer optimum solution. The reason is a theorem of Birkhoff [1946]

stating that the convex hull of the permutation matrices is equal to the set of doubly stochastic matrices — nonnegative matrices in which each row and column sum is equal to 1. Therefore, minimizing a linear functional over the set of doubly stochastic matrices (which is a linear programming problem) gives a permutation matrix, being the optimum assignment. So the assignment problem can be solved with the simplex method

The complexity issue

The assignment problem has helped in gaining the insight that a finite algorithm need not be practical, and that there is a gap between exponential time and polynomial time.

Also in other disciplines it was recognized that while the assignment problem is a finite problem, there is a complexity issue. In an address delivered on 9 September 1949 at a meeting of the American Psychological Association at Denver, Colorado, Thorndike [1950]

studied the problem of the 'classification' of personnel (being job assignment):

The Hungarian method: Kuhn 1955-1956, Munkres 1957

The basic combinatorial (nonsimplex) method for the assignment problem is the Hungarian method. The method was developed by Kuhn [1955,1956], based on the work of Egerv´ary

[1931], whence Kuhn introduced the name Hungarian method for it.

In an article "On the origin of the Hungarian method"' Kuhn [1991] gave the following reminiscences from the time starting Summer 1953: is the English translation of the paper of Egerv´ary [1931].)

The method described by Kuhn is a sharpening of the method of Egerv´ary sketched above, in two respects: (i) it gives an (augmenting path) method to find either a perfect

matching or sets I and J as required, and (ii) it improves the λi and µj not by 1, but by the largest value possible.

Kuhn [1955] contented himself with stating that the number of iterations is finite, but

Munkres [1957] observed that the method in fact runs in strongly polynomial time.

INTRODUCTION

This method is a classical method for solving assignment problem. It also can be used in other problem this method is based on the principle that if a constantis added to every element of a row/or column then the optimum solution of the resulting assignment problem is the same as the original let us see it by a example

Experimental sample

Five men are available to do five different jobs. From past records, the time (in hour) that each man takes to do each job is known and is given in the following table:

ROW-MEN, COLUMN-JOBS

A	B	C	D	E	
1	1	3	2	3	6
2	2	4	3	1	5
3	5	6	3	4	6
4	3	1	4	2	2
5	1	5	6	5	4

Solution:

Step 1. Subtract the smallest element of each row from every element of the corresponding now, we get the following matrix:

A	B	C	D	E	
1	0	2	1	2	5
2	1	3	2	0	5
3	2	3	0	1	3
4	2	0	3	1	1
5	0	4	5	4	3

Step 2.Subtract the smallest element of each column from every element of the corresponding column, we get the following matrix:

A	B	C	D	E	
1	0	2	1	2	4
2	1	3	2	0	3
3	2	3	0	1	2
4	2	0	3	1	0
5	0	4	5	4	2

Step3.Giving the zero assignment in usual manner and get reduced matrix.

A	B	C	D	E	
1	[0]	2	1	2	4
2	1	3	2	[0]	3
3	2	3	[0]	1	2
4	2	[0]	3	1	0
5	0	4	5	4	2

Since row 5 and column 5 have no assignment we proceed to the next step

Step 4.The minimum numbers of lines drawn in the usual manner are 4

A	B	C	D	E	
1	0	1	0	1	3
2	2	3	2	[0]	3
3	3	3	[0]	1	2
4	3	[0]	3	1	0
5	[0]	3	4	3	1

Step5. Now the smallest of the elements that don not contains line through them is 1. Subtracting this element 1 from the elements that do not have a line through them. adding to every elements that lies at the intersection of two lines and leaving the remaining elements unchanged,

Step6. Again repeating the step 3 we make the zero

assignments in matrix and see that even now the row 1 and column 5do not contain any assignments. Therefore we again repeat step 4 of drawing lines.

Step7. According to our usual manner the minimum number of lines drawn is 4

Step 8.Again repeating step5, we get following matrix

A	B	C	D	E	
1	[0]	0	0	0	2
2	3	3	2	[0]	3
3	3	2	[0]	0	1
4	4	[0]	4	1	0
5	0	2	4	2	[0]

Step 9.Repeating the step 3 we make the zero assignments and get the following option assignments,

1→A,2→D,3→C,4→D,5→E.

So minimal assignment: 1+1+3+1+4=10.

Experimental sample

A trip from chandigarh to Delhi takes six hours by bus. A typical time table of the bus services in both direction is given below.

Dept from chandigarh	Rout Number	Arrival at delhi	Arrival at chandigarh	Rout number	Dept from delhi
06.00	A	12.00	11.30	1	05.30
07.30	B	13.30	15.00	2	09.30
11.30	C	17.30	21.00	3	15.00
19.00	D	01.00	00.30	4	18.30
00.30	E	06.30	06.00	5	00.00

The cost of providing this services by the transport company depend upon the time spent by the bus crew(driver and conductor) away from their places in addition to services time. There are five crews. There is a conductor that every crew should be provided with more than 4 hours of, must before the return trip again and should not wait for more than 24 hours for the return trip. The company has residential facilities for the crew at chandigarh as well as at delhi. Find the optimal services lines connections?

Solution

All the crew members are asked to reside at chandigarh than the waiting time at delhi for different services line connection can be calculate as

	1	2	3	4	5
A	17.5	21	3	6.5	12
B	16	19.5	1.5	5	10.5
C	12	15.5	21.5	1	6.5
D	4.5	8	14	17.5	23
E	23	2.5	8.5	12	17.5

If the crew as assumed to stay at delhi then the waiting time for different services line connection can be collected as

	1	2	3	4	5
A	18.5	15	9	5.5	0
B	20	16.5	10.5	7	1.5
C	0	20.5	14.5	11	5.5
D	7.5	4	22	18.5	13
E	13	9.5	35	0	18.5

Now the minimum waiting time of different rout connection by choosing minimum values out of the waiting time given,

	1	2	3	4	5
A	17.5	15	9	5.5	12
B	16	16.5	10.5	5	10.5
C	12	15.5	14.5	11	5.5
D	4.5	8	14	17.5	13
E	13	9.5	8.5	12	17.5

Now we the Hungarian method to find the optimal solution

Step-1 (Row penalty)

Subtract the minimum element in each row from other element of the row

	1	2	3	4	5
A	12	9.5	3.5	0	6.5
B	11	11.5	5.5	0	5.5
C	6.5	10	9	5.5	0
D	0	3.5	9.5	13	8.5
E	4.5	1	0	3.5	9

Step-2 (Column penalty)

After row penalty, subtract the minimum element in each column from other element of the respective column.

	1	2	3	4	5
A	12	8.5	3.5	0	6.5
B	11	10.5	5.5	0	5.5
C	6.5	9	9	5.5	0
D	0	2.5	9.5	13	8.5
E	4.5	0	0	3.5	9

Step-3 (Assignment of zeros)

	1	2	3	4	5
A	12	8.5	3.5	[0]	6.5
B	11	10.5	5.5	0*	5.5
C	6.5	9	9	5.5	[0]
D	[0]	2.5	9.5	13	8.5
E	4.5	[0]	0*	3.5	9

Here the number of assignment of number of raw so we go to the next step

1. Tick that row where the assignment is not taken
2. Then tick that columb where 0 is occure
3. Then tick the assigned row in that columb
4. Now line out the marked column and unmarked raw.

	1	2	3	4	5
A	8.5	5	[0]	0*	3
B	7.5	7	2	[0]	2
C	6.5	9	9	9	[0]
D	[0]	2.5	9.5	16.5	8.5
E	4.5	[0]	0*	7	9

No of assignment =No of column

CREW	Residencies	Rout no	Waiting time
1	Chandigarh	d-1	4.5
2	Delhi	e-2	9.5
3	Delhi	a-3	9.0
4	Chandigarh	b-4	5.0
5	Delhi	c-5	5.5

Total=33.5

Optimal solution =33.5

Summary

After analysis of this Paper very efficiently we have got the minimum job allocation and minimum time for a crew assignment problem. This is the classical method for solving every assignment problem. It also can be widely used in other problem

Reference

[1] Faudzi, Syakinah, Syariza Abdul-Rahman, and RosshairyAbdRahman. 2018. An assignment problem and its application in education

[2] Aissi, Hassene, Cristina Bazgan, and Daniel Vanderpooten. 2005. Complexity of the min-max and min-max regret assignment problems. Operations Research Letters 33

[3] Hindawi Advances in Operations Research Volume 2018, Wikipedia assignment problem

[4] Saumyaranjanjena linear programming Kalyani publishers

[5] Internet research assignment problem

[6] LyleRamshaw, Robert E. Tarjan (2012). "On minimum-cost assignments in unbalanced bipartite graphs" (PDF). HP research labs.

[7] For publication of the a foreside paper in the June Issue