On Intuitionistic Fuzzy Transportation Problem Using Pentagonal Intuitionistic Fuzzy Numbers Solved by Modi Method

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ABSTRACT

In this paper a new method is proposed for finding an optimal solution for Pentagonal intuitionistic fuzzy transportation problems, in which the cost values are Pentagonal intuitionistic fuzzy numbers. The procedure is illustrated with a numerical example.

KEYWORDS: *Intuitionistic fuzzy transportation problems, Pentagonal intuitionistic fuzzy numbers, Optimal solution, Proposed Ranking method, Modi Method*

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1. INTRODUCTION

In general, transportation problems are solved with assumptions that the cost, supply and demand are specified in precise manner. However, in many cases the decision maker has no precise information about the coefficient belonging to the transportation problem. Intuitionistic fuzzy set is a powerful tool to deal with such vagueness.

The concept of Intuitionistic Fuzzy Sets (IFSs), proposed by Atanassov in [2], has been found to be highly useful to deal with vagueness. Many authors discussed the solutions of Fuzzy Transportation Problem (FTP) using various techniques. In 1982, O'heigeartaigh [9] proposed an algorithm to solve Fuzzy Transportation Problem with triangular membership function. In 2013, NagoorGani.A and Abbas. S [8], introduced a new method for solving in Fuzzy Transportation Problem. In 2015, A. Thamaraiselvi and R. Santhi [3] introduced Hexagonal Intuitionistic Fuzzy Numbers. In 2015,ThangarajBeaula – M. Priyadharshini [4] proposed. A New Algorithm for Finding a Fuzzy Optimal Solution.

The paper is organized as follows, in section 2, introduction with some basic concepts of Intuitionistic fuzzy numbers, in section 3, introduce Pentagonal Intuitionistic Fuzzy Definition and proposed algorithm followed by a Numerical example using Modi method and in section 4, finally the paper is concluded.

2. PRELIMINARIES

2.1. Definition (Fuzzy set[FS])[10]

Let X be a nonempty set. A fuzzy set $\bar{A}$ of Xis defined as$\bar{A}$={$< x, \mu_{\bar{A}}(x) > / x \in X$}. Where $\mu_{\bar{A}}(x)$ is called membership function, which maps each element of X to a value between 0 and 1.

2.2. Definition (Fuzzy Number[FN])[10]

A fuzzy number is a generalization of a regular real number and which does not refer to a single value but rather to a connected a set of possible values, where each possible value has its weight between 0 and 1. The weight is called the membership function.

A fuzzy number $\bar{A}$ is a convex normalized fuzzy set on the real line R such that

There exists at least one x ∈R with $\mu_{\bar{A}}(x) = 1$.

$\mu_{\bar{A}}(x)\mu_{\bar{A}}(x)$is piecewise continuous.

2.3. Definition (Intuitionistic Fuzzy Set [IFS])[2]
Let X be a non-emptyset. An Intuitionistic fuzzy set $\bar{A}^I$ of X is defined as

$\bar{A}^I$ = {<x,$\mu_{\bar{A}^I}(x), \vartheta_{\bar{A}^I}(x)$ > / x∈ X}. Where $\mu_{\bar{A}^I}(x)$ and $\vartheta_{\bar{A}^I}(x)$ aremembership and non-membership function. Such that $\mu_{\bar{A}^I}(x), \vartheta_{\bar{A}^I}(x)$: X→ [0, 1] and 0≤ $\mu_{\bar{A}^I}(x) + \vartheta_{\bar{A}^I}(x) \leq 1$ for all x∈ X.

2.4. Definition (Intuitionistic Fuzzy Number [IFN])[2]
An Intuitionistic Fuzzy Subset $\bar{A}^I$ = {< x, $\mu_{\bar{A}^I}(x), \vartheta_{\bar{A}^I}(x)$ > / x∈ X} of the real line R is called an Intuitionistic Fuzzy Number, if the following conditions hold,

There exists m ∈ R suct that $\mu_{\bar{A}^I}(m) = 1$ and $\vartheta_{\bar{A}^I}(m) = 0$.

$\mu_{\bar{A}^I}\mu_{\bar{A}^I}(x)$is a continuous function from R → [0,1]

such that 0≤ $\mu_{\bar{A}^I}(x) + \vartheta_{\bar{A}^I}(x)$ ≤1 for all x∈ X.

The membership and non- membership functions of

$\bar{A}^I$ are in the following form

$$\mu_{\bar{A}^I}(x) = \begin{Bmatrix} 0 \; for -\infty < x \leq a_1 \\ f(x) \; for \; a_1 \leq x \leq a_2 \\ 1 \; for \; x = a_2 \\ g(x) \; for \; a_2 \leq x \leq a_3 \\ 0 \; for \; a_3 \leq x < \infty \end{Bmatrix}$$

$$\vartheta_{\bar{A}^I}(x) = \begin{Bmatrix} 1 \; for -\infty < x \leq a_1{}' \\ f'(x) \; for \; a_1{}' \leq x \leq a_2 \\ 0 \; for \; x = a_2 \\ g'(x) \; for \; a_2 \leq x \leq a_3{}' \\ 1 \; for \; a_3{}' \leq x < \infty \end{Bmatrix}$$

Wheref, f', g, g' are functions from R → [0,1] .f and g' are strictly increasing functions and g and f' are strictly decreasing functions with the conditions $0 \leq f(x) + f'(x) \leq 1$ and $0 \leq g(x) + g'(x) \leq 1$.

3. PENTAGONAL INTUITIONISTIC FUZZY NUMBER
3.1. Definition of Pentagonal Intuitionistic Fuzzy Number [PIFN][3]:

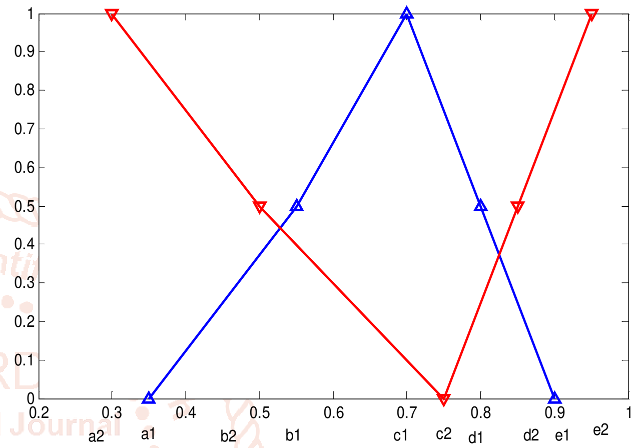
A pentagonal intuitionistic fuzzy number $\bar{A}^I$ is defined as

$\bar{A}^I = \{(a_1,b_1,c_1,d_1,e_1)(a_2,b_2,c_2,d_2,e_2)\}$ Where all $a_1,b_1,c_1,d_1,e_1,a_2,b_2,c_2,d_2,e_2$ are real numbers and its membership function $\mu_{\bar{A}^I}(x)$, non-membership function $V_{\bar{A}^I}(x)$ are given by

$$\mu_{\bar{A}^I}(x) = \begin{cases} 0 & for \; x < a_1 \\ \dfrac{x-a_1}{b_1-a_1} & for \; a_1 \leq x \leq b_1 \\ \dfrac{x-b_1}{c_1-b_1} & for \; b_1 \leq c \leq c_1 \\ 1 & for \; x = c_1 \\ \dfrac{d_1-x}{d_1-c_1} & for \; c_1 \leq x \leq d_1 \\ \dfrac{e_1-x}{e_1-d_1} & for \; d_1 \leq x \leq e_1 \\ 0 & for \quad x > e_1 \end{cases}$$

$$V_{\bar{A}^I}(x) = \begin{cases} 1 & for \; x < a_2 \\ \dfrac{b_2-x}{b_2-a_2} & for \; a_2 \leq x \leq b_2 \\ \dfrac{c_2-x}{c_2-b_2} & for \; b_2 \leq x \leq c_2 \\ 0 & for \; x = c_1 \\ \dfrac{x-c_2}{d_2-c_2} & for \; c_2 \leq x \leq d_2 \\ \dfrac{x-d_2}{e_2-d_2} & for \; d_2 \leq x \leq e_2 \\ 1 & for \; x > e_2 \end{cases}$$

3.2. GRAPHICAL REPRESENTATION OF PENTAGONAL INTUITIONISTIC FUZZY NUMBERS



$\mu_{\bar{A}^I}(x)$ ------ Membership Function

$V_{\bar{A}^I}(x)$ ----- Non - Membership Function

3.3. ARITHMETIC OPERATIONS OF PIFN

Let $\tilde{A}_p^I = (a_1,b_1,c_1,d_1,e_1)(a_2,b_2,c_2,d_2,e_2)$ and $\tilde{B}_p^I = (a_3,b_3,c_3,d_3,e_3)(a_4,b_4,c_4,d_4,e_4)$ be two pentagonal intuitionistic fuzzy numbers, then the arithmetic operations are as follows

A. ADDITION:

$\bar{A}_P{}^I + \bar{B}_P{}^I = (a_1 + a_3, b_1 + b_3, c_1 + c_3, d_1 + d_3, e_1 + e_3)(a_2 + a_4, b_2 + b_4, c_2 + c_4, d_2 + d_4, e_2 + e_4)$

B. Subtraction:

$\bar{A}_P{}^I - \bar{B}_P{}^I = (a_1 - e_3, b_1 - d_3, c_1 - c_3, d_1 - b_3, e_1 - a_3)(a_2 - e_4, b_2 - d_4, c_2 - c_4, d_2 - d_4, e_2 - a_4)$

3.4. RANKING OF PIFN BASED ON ACCURACY FUNCTION

Accuracy function of a pentagonal intuitionistic fuzzy number $\tilde{A}_p^I = (a_1,b_1,c_1,d_1,e_1)(a_2,b_2,c_2,d_2,e_2)$ is defined as

$$H\left(\tilde{A}_p^I\right) = (a_1+a_2+b_1+b_2+c_1+c_2+d_1+d_2+e_1+e_2)/5$$

3.5. MODI METHOD

There are many methods to find the basic feasible solution, Modi method is heuristic method. The advantage of this method is that it gives an initial solution which is nearer to an optimal solution. Here in this paper Modi method is suitably modified and used to solving Intuitionistic Fuzzy transportation problem.

PROPOSED ALGORITHM

Step –1: In Octagonal Intuitionistic Fuzzy transportation problem (OIFN) the quantities are reduced into an integer using the ranking method called accuracy function.

Step – 2: For an initial basic feasible solution with m + n -1 occupied cells, calculate u_i and v_j for rows and columns. The initial solution can be obtained by any of the three methods discussed earlier.

To start with, any ofu_i's orv_j'sis assigned the value zero. It is better to assign zero for a particular u_i orv_j . Where there are maximum numbers of allocations in a row or column respectively, as it will reduce arithmetic work considerably. Then complete the calculation of u_i's andv_j's for other rows and columns by using the relation.$C_{ij} = u_i + v_J$ for all occupied cells (i,j)

Step – 3: For unoccupied cells, calculate opportunity cost by using the relationship

$d_{ij} = C_{ij} - (u_i + v_J)$ for all iand j.

Step – 4: Examine sign of each d_{ij} .

1. If d_{ij} >0, then current basic feasible solution is optimal.
2. If d_{ij} = 0, then current basic feasible solution will remain unaffected but an alternative solution exists.

If one or more d_{ij} <0, then an improved solutions can be obtained by entering unoccupied cell (i,j) in the basis. An unoccupied cell having the largest negative value of d_{ij} is chosen for entering into the solution mix (new transportation schedule).

	B_1	B_2	B_3	B_4	**Supply**
A_1	(4,9,12,15,17) (6,9,13,14,16)	(2,4,8,13,15) (1,3,8,12,14)	(9,10,13,14,15) (8,11,12,16,17)	(3,4,5,10,12) (1,2,6,9,13)	11
A_2	(3,6,9,12,15) (2,5,8,11,14)	(4,7,9,12,15) (3,6,9,11,14)	(2,4,7,10,12) (3,5,7,9,11)	(10,11,13,15,17) (12,14,15,16,17)	13
A_3	(13,14,15,16,17) (15,16,17,18,19)	(9,10,13,14,16) (11,14,15,16,17)	(4,7,9,12,15) (3,6,9,11,14)	(16,18,20,22,24) (17,19,21,23,25)	19
Demand	6	10	12	15	

Step – 5: Construct a closed path (or loop) for the unoccupied cell with largest negative opportunity cost. Start the closed path with the selected unoccupied cell and mark a plus sign (+) in this cell, trace a path along the rows (or columns) to an occupied cell, mark the corner with minus sign (-) and continue down the column (or row) to an occupied cell and mark the corner with plus sign (+) and minus sign (-) alternatively, close the path back to the selected unoccupied cell.

Step – 6: Select the smallest quantity amongst the cells marked with minus sign on the corners of closed loop. Allocate this value to the selected unoccupied cell and add it to other occupied cells marked with plus signs and subtract it from the occupied cells marked with minus signs.

Step – 7: Obtain a new improved solution by allocating units to the unoccupied cell according to step – 6 and calculate the new total transportation cost.

Step – 8: Test the revised solution further for optimality. The procedure terminates when all $d_{ij} \geq 0$, for unoccupied cells.

3.6. Numerical Example:

Consider a 3×4Pentagonal Intuitionistic Fuzzy Number.

Table 1 –Reduced Table

	B_1	B_2	B_3	B_4	**Supply**
A_1	21	16	25	13	11
A_2	17	18	14	28	13
A_3	32	27	18	41	19
Demand	6	10	12	15	43

ΣDemand = Σ Supply

Table 2–VAM Method

	B_1	B_2	B_3	B_4	**Supply**
A_1	21	16	25	[11] 13	11
A_2	[6] 17	[3] 18	14	[4] 28	13
A_3	32	[7] 27	[12] 18	41	19
Demand	6	10	12	15	43

The problem is a balanced transportation problem, using the proposed algorithm; the solution of the problem is as follows

Applying accuracy function on pentagonal intuitionistic fuzzy number [(4, 9, 12, 15, 17) (6, 9, 13, 14, 16)], we have

$$H\left(\tilde{A}^I_p\right) = (4+9+12+15+17+6+9+13+14+16)/5 = 21$$

Similarly applying for all the values, we have the following reduced table.

Applying VAM method, Table corresponding to initial basic feasible solution is

Since the number of occupied cell m+n-1= 6 and are also independent. There exists a non- negative basic feasible solution.

The initial transportation cost = (11×13) + (6×17) + (3×18) + (4×28) + (7×27) + (12×18) = Rs 816.

To find the optimal solution:

Applying the Modi method- $u_i + v_j = c_{ij}$

We get number $u_1 = 0$. The remaining number can be obtained as given below

$$u_i + v_j = c_{i\ j} \quad u_1 = 0$$

$$c_{14} = u_1 + v_4 \Rightarrow 13 = 0 + v_4 \Rightarrow v_4 = 13$$

$$c_{21} = u_2 + v_1 \Rightarrow 17 = 15 + v_1 \Rightarrow v_1 = 2$$

$$c_{22} = u_2 + v_2 \Rightarrow 18 = 15 + v_2 \Rightarrow v_2 = 3$$

$$c_{24} = u_2 + v_4 \Rightarrow 28 = u_2 + 18 \Rightarrow u_2 = 15$$

$$c_{32} = u_3 + v_2 \Rightarrow 27 = u_3 + 3 \Rightarrow u_3 = 24$$

$$c_{33} = u_3 + v_2 \Rightarrow 18 = 24 + v_3 \Rightarrow v_3 = -6$$

$$Z_{11} - C_{11} = u_1 + v_1 - c_{11} = 0 + 2 - 21 = -19$$

$$Z_{12} - C_{12} = u_1 + v_2 - c_{12} = 0 + 3 - 16 = -13$$

$$Z_{13} - C_{13} = u_1 + v_3 - c_{13} = 0 + (-6) - 25 = -31$$

$$C_{23} - C_{23} = u_2 + v_3 - c_{23} = 15 + (-6) - 14 = -15$$

$$Z_{31} - C_{31} = u_3 + v_1 - c_{31} = 24 + 2 - 32 = -6$$

$$Z_{34} - C_{34} = u_3 + v_4 - c_{34} = 24 + 13 - 41 = -4$$

$d_{ij} = c_{ij} - (u_i + v_j)$ For each empty cell and enter at the bottom right corner of the cell.

Applying MODI method, table corresponding to optimal solution is

Table 3 - MODI Method

	B_1	B_2	B_3	B_4	**Supply**
A_1	[-19] 21	[-13] 16	[-31] 25	[11] 13	11
A_2	[6] 17	[3] 18	[-5] 14	[4] 28	13
A_3	[-6] 32	[7] 27	[12] 18	[-4] 41	19
Demand	6	10	12	15	43

Since all $d_{ij} > 0$ the solution in optimum and unique. The solution is given by

$$x_{14} = 11, x_{21} = 6, x_{22} = 3, x_{24} = 4, x_{32} = 7, x_{33} = 12$$

The optimal solution is = $11 \times 13 + 6 \times 17 + 3 \times 18 + 4 \times 28 + 7 \times 27 + 12 \times 18$

= Rs. 816.

4. CONCLUSION:

In this paper, we discussed finding optimal solution for pentagonal intuitionistic transportation problem. We have used accuracy function ranking method and Modi method to find the optimal solution. Modi method has easily to understand and the solution is very nearer to optimum solution. In future research we propose generalized pentagonal intuitionist fuzzy numbers to deal problems and handling real life transportation problem having intuitionistic fuzzy numbers.

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