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On BD – Algebras

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ABSTRACT

In this paper discuss and investigate a class of algebras which is related to several classes of algebras of interest such as BCIK-algebras and which seems to have rather nice properties without being excessively complicated otherwise. A study on BK-algebra discuss and investigate a class of algebras and define BK-algebra and its properties and also see the Commutative Derived BK-algebra. The notion of D-algebras which is the another use full generalization of BCIK-algebra. In this paper generalization of BCIK/BK/D-algebras to introduce BD-algebra and its properties.

KEYWORDS: *BCIK-algebra, B-algebra, BK-algebra, Commutative, Derived algebra, BD-algebra.*

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1. INTRODUCTION

In [1] S Rethina Kumar defined BCIK – algebra in this notion originated fromtwo different sources: one of them is based on the set theory the other is form the classical and non – classical propositional calculi. They are two important classes of logical algebras, and have applied to many branches of mathematics, such as group theory, functional analysis, probability theory and topology. Also S Rethina Kumar introduced the notion of BCIK-algebra which is a generalization of a BCIK-algebra of a BCIK-algebra [1]. Several properties on BCIK-algebra are investigated in the papers [1-4].In [5] S Rethina Kumardefine BK-algebra a study on BK-algebra discuss and investigate a class of algebras and define BK-algebra and its properties and also see the Commutative Derived BK-algebra. The Present authors [6] introduced the notion of D-algebra which is another useful generalization of BCIK-algebras, and as well as some other interesting relations between D-algebras and oriented digraphs. Recently S Rethinakumar [5] introduce a new notion, called an BK-algebra, which is also a generalization of BCIK/B-algebra, and define the notions of ideals and boundedness in BK-algebra, showing that there is always a maximal ideal in bounded BK-algebra. Furthermore, they constructed quotient BK-algebra via translation ideals and they obtained the fundamental theorem of homomorphism for BK-algebra as consequence. In this paper for constructing proper examples of great variety of Commutative Derived BD-algebras. It should be noted that good examples of some of these properties.

2. BD-algebras

A BD-algebra is a non-empty set X with a constant 0 and a binary "$*$" satisfying the following axioms hold for all $x, y, z \in X$:

1. $0 \in X$,
2. $x * x = 0$,
3. $x * 0 = x$,
4. $0 * x = 0$,
5. $(x * y) * x = 0$,
6. $(x * y) * z = (x * z) * y$,
7. $((x * y) * (x * z)) * (z * y) = 0$,
8. $(x * (x * y)) * y = 0$,
9. $x * y = 0 \text{ and } y * x = 0 \text{ imply } x = y$,
10. $0 * (x * y) = (0 * x) * (0 * y)$,
11. $(x * y) * z = x * \big(z * (0 * y)\big)$.

Example 2.1. Let $X := \{0,1,2\}$ be a set with following table:

$*$	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Then $(X;* ,0)$ is a BD-algebra.

Example 2.2. Let be the set of all real numbers except for a negative integer-n. Define a binary operation $*$ on X by

$$x * y := \frac{n(x-y)}{n+y}.$$

Then $(X;* ,0)$ is a BD-algebra.

Example 2.3. Let $X := \{0,1,2,3,4,5\}$ be a set with the following table:

*	0	1	2	3	4	5
0	0	2	1	3	4	5
1	1	0	2	4	5	3
2	2	1	0	5	3	4
3	3	4	5	0	2	1
4	4	5	3	1	0	2
5	5	3	4	2	1	0

Then $(X;*,0)$ is a BD-algebra[1-5].

Example 2.4. Let $F\langle x,y,z\rangle$be the free group on three elements. Define $u*v := vuv^{-2}$. Thus $u*u = e \ and\ u*e = u$. Also $e*u = u^{-1}$. Now, given $a,b,c \in F\langle x,y,z\rangle$, let

$$w(a,b,c) = ((a*b)*c)(a*(c*(e*b)))^{-1}$$
$$= (cbab^{-2}c^{-2})(b^{-1}cb^2a^{-1}cbcb^2)^{-1}$$
$$= cbab^{-2}c^{-2}b^{-2}c^{-1}b^{-1}c^{-1}ba^{-1}b^{-2}c^{-1}b.$$

Let $N(*)$ be the normal subgroup of $F\langle x,y,z\rangle$ generated by the elements $w(a,b,c)$.

Let $G = F\langle x,y,z\rangle/N(*)$. On G define the operation " $\cdot$ " as usual and define

$\big(uN(*)\big)*\big(vN(*)\big) := (u*v)N(*)$. It follows that

$\big(u(N(*)\big)*\big(uN(*)\big) = eN(*), \big(uN(*)\big)*\big(e*N(*)\big) = uN(*)$ and

$w\big(aN(*),bN(*),cN(*)\big) = w(a,b,c)N(*) = eN(*)$.

Hence $(G;*,eN(*))$ is a BD-algebra.

If we let $y := x$, then we have

$(x*x)*z = x*\big(z*(0*x)\big)$.

If we let $z := x$, then we obtain also

$0*x = x*\big(x*(0*x)\big)$, using this it follows that

$0 = x*\big(0*(0*x)\big)$.

Let $X := (0,1,2\}$ be a set with the following left table:

*	0	1	2
0	0	1	0
1	1	0	1
2	0	1	0

*	0	1	2
0	0	1	2
1	1	1	1
2	2	1	2

Let the set $X := \{0,1,2,3\}$ be a set with the following table:

*	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	0	0	1
3	3	0	0	0

Then $(X;*,0)$ satisfies.

Lemma 2.5. If $(X;*,0)$ is a BD-algebra, then $y*z = y*(0*(0*z))$ for any $y,z \in X$.

Proof.

$y*z = (y*z)*0$

$= y*\big(0*(0*z)\big)$.

Lemma 2.6. If $(X;*,0)$ is a BD-algebra then $(x*y)*(0*y) = x$ for any $x,y \in X$.

Proof. From $z = 0*y$, we find that

$(x*y)*(0*y) = x*\big((0*y)*(0*y)\big)$. Hence

$(x*y)*(0*y) = x*0$, so that its follows that

$(x*y)*(0*y) = x$ as claimed.

Lemma 2.7. If $(X;*,0)$ is a BD-algebra then $x*z = y*z \ implies\ x = y$ for any $x,y,z \in X$.

Proof. If $x*z = y*z$, then $(x*z)*(0*z) = (y*z)*(0*z)$ and thus it follows that $x = y$.

Proposition 2.8. If $(X;*,0)$ is a BD-algebra, then

$x*(y*z) = \big(x*(0*z)\big)*y for\ any\ x,y,z \in X$.

Proof. We obtain:

$\big(x*(0*z)\big)*y = x*(y*\big(0*(0*z)\big))$

$= x*(y*z))$.

Lemma 2.9. Let $(X;*,0)$ be a BD-algebra. Then for any $x,y \in X$,

1. $x*y = 0\ implies\ x = y$,
2. $0*x = 0*y\ implies\ x = y$,
3. $0*(0*x) = x$.

Proof.
1. Since $x*y = 0$ implies $x*y = y*y$, it follows that $x = y$.
2. $0*x = 0*y, then\ 0 = x*x = (x*x)*0 = x*\big(0*(0*x)\big) = x*\big(0*(0*y)\big) = (x*y)*0 = x*y, and\ thus\ x = y$.
3. For any $x \in X$, we obtain $0*x = (0*x)*0 = 0*\big(0*(0*x)\big)$, it follows that

$x = 0*(0*x)$ as claimed.

Note: Let $(X;*,0)$ be a BD-algebra and let $g \in X$. Define $g^n := g^{n-1}*(0*g)(n \geq 1)\ and$

$g^0 := 0$. Note that $g^1 = g^0*(0*g) = 0*(0*g) = g$

Lemma 2.10. Let $(X;*,0)$ be a BD-algebra and let$g \in X$.Then $g^n*g^m = g^{n-m}$ where $n \geq m$.

Proof. If X is a BD-algebra,then note that it follows that

$g^2*g = \big(g^1*(0*g)\big)*g = \big(g*(0*g)\big)*g = g\Big(g*\big(0*(0*g)\big)\Big) = g*(g*g) = g*0 = g.$ Assume that$g^{n+1}*g = g^n(n \geq 1)$. Then

$g^{n+2}*g = (g^{n+1}*(0*g))*g$

$= g^{n+1}*\Big(g*\big(0*(0*g)\big)\Big)$

$= g^{n+1}*0.$

$= g^{n+1}.$

Assume $g^n*g^m = g^{n-m}$where $n-m \geq 1$. Then

$g^n*g^{m+1} = (g^n*(g^m*(0*g))$

$= (g^n*g)*g^m$

$= g^{n-1}*g^m$

$= g^{n-(m+1)}, [Since\ n-m-1 \geq 0]$

Proving the lemma.

Lemma 2.11. Let $(X;*,0)$ be a BD-algebra and let $g \in X$. Then $g^m * g^n = 0 * g^{n-m}$ *where* $n > m$.

Proof. If X is a BD-algebra then, we have $g * g^2 = g * \left(g^1 * (0 * g)\right) = (g * g) * g^1 = 0 * g$.

Assume that $g * g^n = g^{n-1}$ *where* $(n \geq 1)$. Then

$g * g^{n+1} = g * \left(g^n * (0 * g)\right)$

$= (g * g) * g^n$

$= 0 * g^n$.

Assume that $g^m * g^n = g^{n-m}$ where $n - m \geq 1$. Then

$g^{m+1} * g^n = (g^m * (0 * g)) * g^n$

$= g^m * (g^n * g)$

$= g^m * g^{n-1}$

$= 0 * g^{n-m-1}$,

Proving the lemma.

Theorem 2.12. Let $(X;*,0)$ be a BD-algebra and let $g \in X$. Then

$$g^m * g^n = \begin{cases} g^{m-n} \text{ if } m \geq n, \\ 0 * g^{n-m} \text{ otherwise.} \end{cases}$$

Proposition 2.13. If $(X;*,0)$ is a BD-algebra, then $(a * b) * b = a * b^2$ for any $a, b \in X$.

Proof. $(a * b) * b = a * \left(b * (0 * b)\right) = a * b^2$.

Proposition 2.14. If $(X;*,0)$ is a BD-algebra, then $(0 * b) * (a * b) = 0 * a$ for any $a, b \in X$.

Proof. $(0 * b) * (a * b) = \left((0 * b) * (0 * b)\right) * a = 0 * a$.

3. Commutativity

A BD-algebra $(X;*,0)$ is said to be commutative if $a * (0 * b) = b * (0 * a)$ for any $a, b \in X$.

Proposition 3.1. If $(X;*,0)$ is a commutative BD-algebra, then $(0 * x) * (0 * y) = y * x$ *for any* $x, y \in X$.

Proof. Since X is commutative, $(0 * x) * (0 * y) = y * \left(0 * (0 * x)\right) = y * x$.

Theorem 3.2. If $(X;*,0)$ is a commutative BD-algebra, then $(a * (a * b) = b$ for any $a, b \in X$.

Proof. If X is commutative, $a * (a * b) = \left(a * (0 * b)\right) * a = \left(b * (0 * a)\right) * a = b * (a * a) = b$.

Corollary 3.3. If $(X;*,0)$ is a commutative BD-algebra, then the left cancellation law holds, i.e.,

$a * b = a * b'$ *implies* $b = b'$.

Proof.$b = a * (a * b) = a * \left(a * b'\right) = b'$.

Proposition 3.4. If $(X;*,0)$ is a commutative BD-algebra, then $(0 * a) * (a * b) = b * a^2$ for any

$a, b \in X$.

Proof. If X is a commutative BD-algebra, then

$$(0 * a) * (a * b) = ((0 * a) * (0 * b) * a = (b * a) * a = b * a^2.$$

4. Derived algebra and BD-algebras

Given algebras (i.e., groupoids, binary systems) $(X;*)$ *and* $(X;\circ)$, it is often argued that they are "essentially equivalent" when they are not, and even if it is perfectly clear how proceed from to the other and back again, it is also clear that knowledge of one "implies" knowledge of the other in a complete enough sense as to have the statement that they are "essentially equivalent" survive closer inspection.

We proceed the integers Z, we consider the system $(Z;+,0)$ as an abelian group with identity 0. If we consider the system $(Z;-,0)$, then we can reproduce $(Z;+,0)$ by "defining" $x + y := x - (0 - y)$, and observing that in the first case "0 is the unique element such that $x - 0 = x$ for all x, while in the second case "0 is the unique element such that $x + 0 = x$ for all x".

However, that is by no means all we might have said to identify 0 nor is it necessary what we need it say to identify 0 in this setting.

Let $(X;*,0)$ *and* $(X;\circ,0)$ be algebras. We denote $(X;*,0) \longrightarrow (X;\circ,0)$ if $x \circ y = x * (0 * y)$, for all $x, y \in X$. The algebra $(X;\circ,0)$ is said to be derived from the algebra $(X;*,0)$. Let V be the set of all algebras defined on X and let $\Gamma_d(V)$ be the digraph whose vertices are V and whose described above.

Example 4.1. ([1-5]) If we define $x * y := \max\{0, \frac{x(x-y)}{x+y}\}$ on X, then $(X;*,0)$ is a BD-algebra.

Proposition 4.2. The derived algebra $(X;*,0)$ from BD-algebra $(X;*,0)$ is a left zero semigroup.

Proof. Let $(X;*,0)$ be a BD-algebra and let $(X;*,0) \longrightarrow (X;\circ,0)$. Then $x \circ y = x * (0 * y)$,

For any $x, y \in X$. Since$(X;*,0)$is a BD-algebra, $x * (0 * y) = x * 0 = x, i.e., x \circ y = x$, proving that $(X;*,0)$ is a left zero semigroup.

Notice that such an arrow in $\Gamma_d(V)$ can always be constructed, but it is not true that a backward arrow always exists. For example, since every BCIK-algebra $(X;*,0)$ is a BD-algebra, we have $(X;*,0) \longrightarrow (X;\circ,0)$ where $(X;\circ,0)$ is a left zero semigroup. Assume that $(X;\circ,0) \longrightarrow (X;*,0)$, where $(X;*,0)$ is a non-trivial BCIK-algebra. Then$x * y = x \circ (0 \circ y)$, for any $x, y \in X$. Since $(X;\circ)$ is a left zero semigroup, we have $x, y = x$ for any $x, y \in X$, contradicting that $(X;*,0)$ is a BCIK-algebra.

The most interesting result in this context may be:

Theorem 4.3. Let $(X;*,0)$ be a BD-algebra. If $(X;*,0) \longrightarrow (X;\circ,0)$, i.e., if $x \circ y = x * (0 * y)$, then $(X;\circ,0)$ is a group.

Proof. If $(X;*,0) \longrightarrow (X;\circ,0)$, then $x \circ y = x * (0 * y)$, for any $x, y \in X$. By $0 * (0 * x) = x$ *for any* $x \in X$, i.e., $x = 0 \circ x$. Since $x \circ 0 = x * (0 * 0) = x * 0 = x$, 0 acts like an identity element of X. Routine calculations show that $(X;\circ,0)$ is a group.

Proposition 4.4. The derived algebra from a group is that group itself.

Proof. Let $(X;*,0)$ be a group with identity 0. If $(X;*,0) \longrightarrow (X;\circ,0)$, then $x \circ y = x * (0 * y) = x * y$, Since 0 is the identity, for any $x, y \in X$. This proves the proposition.

Thus, we can use the $\longrightarrow$mechanism to proceed from the BD-algebra to the groups, but since groups happen to be sinks in this graph, we cannot use $\longrightarrow$ mechanism to return from groups to BD-algebra. This does not mean that there are no other ways to do so, but it does argue for the observation that BD-algebras are not only "different", but in a deep sense "non-equivalent", and from the point of view of the digraph $\Gamma_d(V)$ the BD-algebra is seen to be a predecessor of the group.

Given a group $(X;\cdot,e)$, if we define $x * y := x \cdot y^{-1}$, then $(X;*,0=e)$ is seen to be a BD-algebra, and furthermore, it also follows that $(X;*,0=e) \rightarrow (X;\cdot,e)$, since $x*(e*y) = x\cdot(e\cdot y^{-1})^{-1} = x\cdot(y^{-1})^{-1} = x\cdot y$.

The problem here is that there is not a formula involving only $(X;\cdot,e)$ which produces $x*y$, i.e., we have to introduce $(X;\cdot,-^{-1},e)$ as the type to describe a group to permit us to perform this task. In fact, we may use this observation as another piece of evidence that BD-algebra $(X;*,0)$ are not "equivalent" to groups $(X;*,0)$. If we introduce the mapping $x \rightarrow 0*x$ is not a new item which needs to be introduced, while in the case of groups it is.

The difficulty we are observing in the situation above is also visible in the case of "the subgroup test". If$(X;\cdot,e)$is an infinite group, and if $\emptyset \neq S \subseteq X$, then if S is closed under multiplication it is not the case that S need a subgroup. Indeed, the rule is that if $x,y \in S$, then also $x\cdot y^{-1} \in S$. From what we have already seen, $x\cdot y^{-1}$ is precisely the element $x*y$ if

$(X;*,e) \rightarrow (X;\cdot,e)\, in \Gamma_d(V)$. Thus we have the following "subgroup test" for BD-algebra: $\neq S \subseteq X$is a sub algebra of the BD-algebra $(X;*,0)$, precisely when $x,y \in S$ implies $x,y \in S$.

Also, suppose $(X;*,0) \rightarrow (X;\cdot,e=0) in\, \Gamma_d(V)$ where it is given that $(X;\cdot,e)$ is a group. Then it is not immediately clear that $(X;*,0)$must be a uniquely defined BD-algebra, even if we know that there is at least one BD-algebra with this property.

5. Conclusion

In this paper introduce BD-algebra (commutative) derived algebra and BD-algebra.

In our future study some useful properties of thisa complete ideal in extended in various algebraic structure of Q-algebras, subtraction algebras, and so forth.

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