

# A Proposed Fuzzy Inventory Management Policy

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## ABSTRACT

Managing inventory under inaccurate or incomplete data considers a complex process to find the optimal policy to reduce total inventory costs. Hence, we resort in this paper to the use of the fuzzy technique to treat this deficiency in the data. This study considers important because it is hoped that on the completion, the study will extend further insights into the conception of stock control measures. Through using automotive service center as a reference point, the study will make an interesting contribution to the understanding of the general and special effects of stock control in other private and public utilities. In this paper, it will be proposed a mathematical model for optimal fuzzy inventory policy to minimize the total inventory cost by using the fuzzy assignment technique and will be applied it on a case study.

**KEYWORDS:** Inventory, Fuzzy, Supply chain, Stock control, Assignment Technique

**How to cite this paper:** El-Sayed Ellaimony | Mohamed Khalil | Ahmed Taha | Mohamed Osman "A Proposed Fuzzy Inventory Management Policy" Published in International Journal of Trend in Scientific Research and Development (ijtsrd), ISSN: 2456-6470, Volume-5 | Issue-3, April 2021, pp.944-952, URL: [www.ijtsrd.com/papers/ijtsrd40005.pdf](http://www.ijtsrd.com/papers/ijtsrd40005.pdf)



IJTSRD40005

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## 1. INTRODUCTION

From the first half of the twentieth century, Inventory systems have received extended attention. Essentially, yet crucial questions such as when replenishing and how much replenishing have been the focus of Stock control. There are several reasons for keeping inventory, from these reasons reducing the risk of change in the rate of demand and supply. As it enables to present a high level of service, which is very important for the corporation to stay in the market, where high inventory levels ensure increasing the customer satisfaction. Inventory management is not only limited to retail stores but also, stocks are spread in the business world. Inventory holding is essential for any company that deals with physical products, especially in-service field.

In any planning field, accurate data must be available as it will entail measures to implement this plan. So, the lack of accurate data needs some relatively complex processes to solve this problem. In this paper, we present a literary review about the research related to the fuzzy inventory system and the methods used to solve this problem. In addition, Explanation the fuzzy assignment technique and the mathematical model that represents the problem of fuzzy inventory system. In the end, the application of this proposed model to some available data for automotive service center.

## 2. Literature Review

**Tanthatamee and Phruksaphanrat**[1] presented a proposed a fuzzy inventory control system for a single

item continuous control system. This model deal with both availability of supply and uncertain demand by using a fuzzy control system, where the conventional stochastic inventory model can determine only an uncertain demand. both demand and availability of supply are described by linguistic terms by using the proposed fuzzy inventory model. It can be considered the proposed model is more flexible than the conventional stochastic inventory model. **Riyadh et al. [2]** suggested a highly advanced dynamic fuzzy approach as an innovation step to identify safety stock level to minimize the total inventory cost and meet with the customer requirements. It can be divided the proposed model to three main steps, firstly: identifying demand uncertainty conditions. Secondly, identify the raw material availability and the final identify of the inventory on hand conditions. He used a MATLAB software to solve this problem as industrial case study, where he success to reduction the safety stock level ranging. **Wei et al. [3]** studied a single-period inventory problem with discrete stochastic demand. Most of their works are based on the expected profit/cost criterion or expected utility criterion. It considered the effect of irrational factor under uncertainty and therefore in- corporate prospect theory into inventory model. Their objective is to maximize the overall value of the prospect, which can be calculated by using the value function and the weighting function. For any given initial inventory level, it can be shown that a state-dependent order-up-to policy is optimal. **Huaming et al. [4]** assumed that the replenishment lead time is

dependent on both lot size of the buyer and production rate of the vendor, an integrated production inventory model is presented. The decision-making interaction of lead time between a buyer and a vendor in the integrated inventory model is analyzed. In terms of the model, a solution procedure has been developed to obtain the efficient ordering strategy for a manufacturing company. **Jong et al. [5]** allowed the backorder rate as a control variable to widen applications of a continuous review inventory model. Moreover, it presented a new form for backorder rate that was dependent on the amount of shortages and backorder price discounts. Besides, it also treated the ordering cost as a decision variable. Hence, it developed an algorithmic procedure to find the optimal inventory policy by mini-max criterion. Finally, a numerical example is also given to illustrate the results. **Tripathy and Pattnaik [6]** investigated an instantaneous production plan to obtain an optimal ordering policy wherein demand exceeds supply, all items were subjected to inspection and defective items were discarded. The unit cost of production was inversely related to both process reliability and demand rate. Under reasonable conditions, maximum positive cost savings were generated when the process reliability increased. The market was able to absorb virtually any quantity of the product rolled out from the production line. This situation was typical of a technologically advanced product entering the growth phase of its life cycle. Since the demand for the product is high, the manufacturer will increase production to meet it, which will result in lower unit cost of production because production overheads are spreaded over the items. **Oseph et al. [7]** considered a multi-level/multi-machine lot sizing problem with flexible production sequences, where the quantity and combination of items required to produce another item need not be unique. The problem is formulated as a mixed-integer linear program and the notion of echelon inventory is used to construct a new class of valid inequalities, which are called echelon cuts. Numerical results show the computational power of the echelon cuts in a branch-and-cut algorithm. **Liberopoulos [8]** developed some analytical results on the tradeoff between FG inventory and advance demand information ADI for a model of a single-stage, make-to-stock supplier who uses an order base stock replenishment policy to meet customer orders that arrive a fixed time in advance of their due dates. **Zhou et al [9]** studied a single-product periodic-review inventory system with multiple types of returns. The serviceable products used to fulfill stochastic customer demand can be either manufactured/ordered, or remanufactured from the returned products, and the objective was to minimize the expected total discounted cost over a finite planning horizon. **Yuan et al. [10]** managed with uncertain single period inventory problem. In a traditional single period inventory problem assumed that the mark demand is a random variable. But this paper concerned with a single period inventory problem under two main assumptions, firstly: the market demand is an uncertain variable and the secondly, a setup cost and the initial stock known. Under these assumptions the optimal inventory for uncertain single period is obtained. **Dai et al. [11]** introduced an integration between a location inventory problem into a supply chain network and develop an optimization model for perishable products with fuzzy capacity and carbon emissions constrains. The proposed model is formulated by mixed integer non-linear

programming. He used a hybrid genetic algorithm and hybrid harmony search to solve this problem to attain a minimization total cost. The results of numerical experiments demonstrate that the proposed algorithms can effectively deal with problems under different conditions and these two algorithms have their own advantages.

The above review has shown that almost the efforts which have been done is directed towards the studying of inventory control with the assumption data with some applications, whereas real date has given little considerations. Various authors have exposed their work for the system modeling with some applications; their contributions were limited. However, the review has established that, there is a need for more investigation on the inventory control with the real data. Consequently, there is a general attention to establish suitable concept by which the inventory control with respect of planning (scheduled) maintenance has to be established.

### 3. Inventory Models

There are many forms from the inventory models but in this section, we will present basic Economic Order Quantity (EOQ) model without shortages and lot size model with shortages only and after that we will present a proposed fuzzy inventory model to minimize the total inventory cost using the assignment technique.

#### 3.1. The Basic EOQ Model

The basic EOQ model searches about the economic quantity that be purchased and when make this order through the following assumptions.

##### Assumptions (Basic EOQ Model)

1. A known constant demand rate of units per unit time.
2. The order quantity (Q) to replenish inventory arrives all at once just when desired, namely, when the inventory level drops to zero.
3. Planned shortages are not allowed.

According to assumption 2, there usually is a lag between when an order is placed and when it arrives in inventory. The amount of time between the placement of an order and its receipt is referred to as the lead-time. The inventory level at which the order is placed is called the reorder point. To satisfy assumption 2, this reorder point needs to be set at the product of the demand rate and the lead-time. Thus, assumption 2 is implicitly assuming a constant lead-time as shown in Figure 1

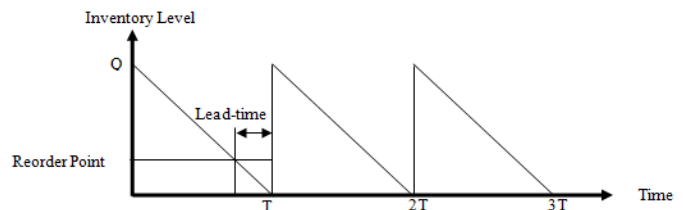


Figure 1 Basic Economic Order Quantity

The total inventory cost expressed is

Total Inventory Cost= Ordering Cost + Purchase Cost + Carrying Cost

$$TC = \frac{D.K}{Q} + D.P + \frac{H.Q}{2} \quad (1)$$

Where

TC: Total inventory cost

D: Demand per year

K: Setup cost per order  
 H: Carrying cost per unit  
 P: purchase cost per unit  
 Q: required quantity per order

Then the optimal economic order Quantity ( $Q^*$ ) =  $\sqrt{\frac{2DK}{H}}$

The optimal cycle time (T) =  $Q^*/D = \sqrt{\frac{2K}{HD}}$

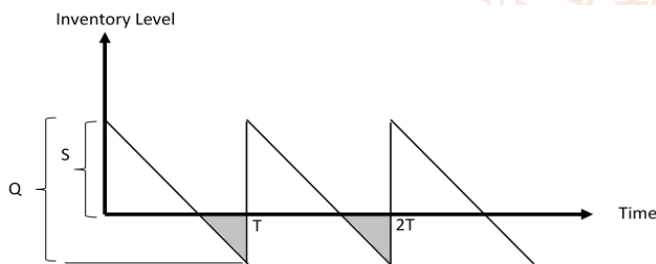
**3.2. The EOQ Model with Planned Shortages**

One of the problems of any inventory manager is the existence of an inventory shortage (sometimes referred to as a stock out)—demand that cannot be met currently because the inventory is consumed. This causes a variety of headaches, including dealing with customer’s dissatisfaction and having extra record keeping to arrange for filling the demand later (backorders) when the inventory can be replenished. By assuming that planned shortages are not allowed, the basic EOQ model presented above satisfies the common desire of managers to avoid shortages as much as possible. (Nevertheless, unplanned shortages can still occur if the demand rate and deliveries do not stay on schedule.)

The EOQ model with planned shortages addresses this kind of situation by replacing only the third assumption of the basic EOQ model by the following new assumption.

*Planned shortages now are allowed. When a shortage occurs, the affected customers will wait for the product to become available again. Their backorders are filled immediately when the order quantity arrives to replenish inventory*

Under these assumptions, the form of inventory levels over time has the appearance shown in Fig.2. The saw-toothed appearance is the same as in Fig. 1. However, now the inventory levels extend down to negative values that reflect the number of units of the product that are backordered.



**Figure 2 Economic Order Quantity Model with Planned Shortages**

The total inventory cost expressed is

Total Inventory Cost= Ordering Cost + Purchase Cost + Carrying Cost + Shortage Cost

$$TC = \frac{D.K}{Q} + D.P + \frac{H.S^2}{2D} + \frac{C(Q-S)^2}{2Q} \quad (2)$$

Where

C: is the shortage cost per unit

In this model there are two decision variable (S and Q), so the optimal values are found by the partial derivatives  $\partial T/\partial S$  and  $\partial T/\partial Q$  equal to zero. Thus,

$$S^* = \sqrt{\frac{2DK}{H}} \sqrt{\frac{C}{C+H}}, Q^* = \sqrt{\frac{2DK}{H}} \sqrt{\frac{C+H}{C}} \quad (3)$$

The Optimal Cycle time  $T^*$  is given by

$$T^* = \frac{Q^*}{D} = \sqrt{\frac{2K}{DH}} \sqrt{\frac{C+H}{C}} \quad (4)$$

**3.3. Mathematical Model of Inventory problem for multi-items.**

In this section, we will illustrate how the proposed model for solving the inventory problem is built as an assignment model to minimize the total inventory cost. Fig 3 illustrates that above the diagonal of the table represents the carrying or holding cost for the item, while under the diagonal of the table represents the Shortage or backorder cost.

period	1	2	.	.	M
1					
2				Holding cost	
.					
.	Backorder cost				
M					
Demand	$d_1$	$d_2$	.	.	$d_M$

**Fig. 3 Inventory cost problem as assignment problem model**

**Model assumptions**

1. The demand varies with the time and known.
2. Purchase price per any unit is constant though the plan period
3. The required units to satisfy demand in a particular period can be acquired at any time including the backorders.
4. Backorder cost may vary with the time.
5. The replenishment lead-time is known with certainty so that delivery can be timed to occur accordingly.
6. The unit variable cost does not depend upon replenishment quantity i.e. no quantity discounts are permitted.
7. If the required quantity is disbursed in the same period, then the carrying (holding) and backorder (shortage) cost equal zero.

Fig. 4 represents the sequence of the inventory cost problem that means it can makes the order at period  $i$  this quantity may be put in the store for multi-periods. In this case the cost will take in consideration called carrying cost. On the other hand if the responsible person makes order in period  $i$  but this order is received in another planned time say in period  $j$ . In this case the cost becomes the backorder cost.

The inventory problem can be formulated as an assignment model. The inventory problem can be considered as transportation model with multi sources and multi demand. The source of the each period must be attained all required in planned horizon and the demand of each period is already known. The objective function is minimization the total inventory cost whereas the total inventory cost equals the summation of the carrying (holding) cost, backorder (shortage) cost and ordering (setup) cost. The first constraint set is put to attain the required demand through the planned horizon. The second constraint set to be confirmed that all required demand at each period will deliver. On the other hand, the third constraint is put to confirm the order or not.

Item No. n	Period					
Period	1	2	3	.	.	m
1	0	$h_{12}^n$	$h_{13}^n$	.	.	$h_{1m}^n$
Item No. 2	Period					
Period	1	2	3	.	.	m
1	0	$h_{12}^2$	$h_{13}^2$	.	.	$h_{1m}^2$
Item No.1	Period					
Period	1	2	3	.	.	m
1	0	$h_{12}^1$	$h_{13}^1$	.	.	$h_{1m}^1$
2	$b_{21}^1$	0	$h_{23}^1$	.	.	$h_{2m}^1$
.	.	.	.	.	.	.

**Fig4The inventory Cost matrix for all items**

**The objective function for Minimization of the total inventory cost**

$$\begin{aligned}
 & \text{Min } \sum_{i=1}^m \sum_{j=i+1}^m \sum_{k=1}^n h_{ijk} x_{ijk} + \sum_{i=2}^m \sum_{j=1}^{i-1} \sum_{k=1}^n b_{ijk} x_{ijk} + \sum_{i=1}^m s_i y_i \\
 & \text{Subject to} \\
 & \sum_{j=1}^M x_{ijk} \leq Q_k \cdot y_i \quad i = 1, 2, \dots, M \ \& \ k = 1, 2, \dots, N \\
 & \sum_{i=1}^M x_{ijk} \leq d_{jk} \quad j = 1, 2, \dots, M \ \& \ k = 1, 2, \dots, N \\
 & y_i = 0, 1 \quad i=1, 2, \dots, M \\
 & x_{ijk} \geq 0, \text{ Integer}
 \end{aligned} \tag{5}$$

Where

$x_{ijk}$  represents the number of units acquired in period  $i$  for demand in period  $j$  (associated with  $x$ , is used. the holding cost when  $i < j$ . and backordering cost when  $i > j$ ) for item  $k$

$h$  represents the holding cost (in EGP) per unit per period.

$s$  represents setup cost or ordering cost (in EGP).

$M$  represents the length of planning horizon. (i.e. the number of periods in planning horizon).

$N$  represents the number of items.

$b$  represents the backorder cost (in EGP) per unit per period,

$d$  represents the number of the required units,

$= 0$  if  $x_{ijk} = 0$ . (i.e., if no replenishment is made in period  $i$ . for all  $j$ & all item  $k$ )

$y_i$   $\left\{ \begin{aligned} &= 1 \text{ if } x_{ijk} > 0. \text{ (i.e., if replenishment is made in period } i. \text{ for all } j \& \text{ all item } k) \end{aligned} \right.$

$Q =$  a large number  $\geq d_1 + d_2 + \dots + d_M$ .

**3.4. Mathematical Model of Fuzzy Inventory problem for multi-items.**

In this section, we present the proposed model of fuzzy inventory problem for the fuzzy cost parameters and fuzzy required quantity parameters in each period. In practical case this parameters really is changed from time to another and cannot take them as a deterministic value with the time.



$$\text{Min } \sum_{i=1}^m \sum_{j=i+1}^m \sum_{k=1}^n \tilde{h}_{ijk} x_{ijk} + \sum_{i=2}^m \sum_{j=1}^{i-1} \sum_{k=1}^n \tilde{b}_{ijk} x_{ijk} + \sum_{i=1}^m \tilde{c}_i y_i$$

Subject to

$$\left. \begin{aligned} \sum_{j=1}^M x_{ijk} &\leq \tilde{Q}_k \cdot y_i \quad i = 1, 2, \dots, M \ \& \ k = 1, 2, \dots, N \\ \sum_{i=1}^M x_{ijk} &\leq \tilde{d}_{jk} \quad j = 1, 2, \dots, M \ \& \ k = 1, 2, \dots, N \end{aligned} \right\} \quad (6)$$

$$y_i = 0, 1 \quad i=1, 2, \dots, M$$

$x_{ijk} \geq 0$ , Integer

To solve this model must transform the fuzzy form to crisp form as ref [12]

#### 4. Member ship Function

There are many ships for the member ship function that describe the fuzzy number from this forms

##### 4.1. Generalized fuzzy number

A generalized fuzzy number  $\tilde{A}$  is a fuzzy set defined on  $\mathbb{R}$  whose membership function satisfies the following properties

1.  $\tilde{\mu}_A(x)$  is a continuous mapping from  $\mathbb{R}$  to  $[0, 1]$ .
2.  $\tilde{\mu}_A(x) = 0, -\infty < x \leq a$ .
3.  $\tilde{\mu}_A(x)$  is strictly increasing on  $[a, b]$ .
4.  $\tilde{\mu}_A(x) = 1, b \leq x \leq c$ .
5.  $\tilde{\mu}_A(x)$  is strictly decreasing on  $[c, d]$ .
6.  $\tilde{\mu}_A(x) = 0, d \leq x < \infty$ .

where  $a, b, c, d$  are real numbers[12].

##### 4.2. Trapezoidal fuzzy number

A fuzzy set  $\tilde{A}$  defined on  $\mathbb{R}$  is called trapezoidal fuzzy number (as shown in Fig. 5) and is denoted by  $\tilde{A} = (a, b, c, d)$  if the membership function of  $\tilde{A}$  is given by



Figure 5 Trapezoidal fuzzy number

Note If  $b$  and  $c$  are equal, then the trapezoidal fuzzy number becomes a triangular Fuzzy number as shown in Fig. 6 and is denoted as  $\tilde{A} = (a, b, d)$ .

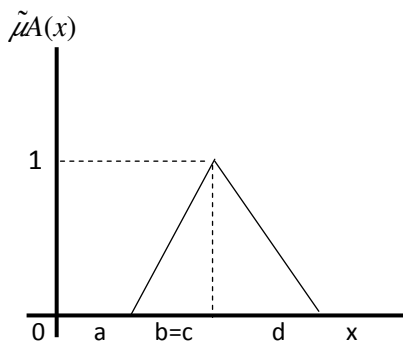


Figure 6 Triangle membership function.

#### 5. Application

A case study was conducted to apply the inventory model for a service center for a well-known brand of vehicles in Egypt. This service center performs regular maintenance for specific brand car model. This application displays the inventory policy to provide the maintenance department in the service center with the spare parts required for the periodic

maintenance schedule for the brand car being studied. The service center makes a purchase order by the required quantities from the spare parts every two months (i.e. six purchase orders per year).

### 5.1. Collected Data

The automotive service station is interested in determining when to receive a batch of spare parts and how many quantities from spare parts in each batch taken in consideration several costs will be presented later.

From the back history of fast-moving items for many years that require the provision of spare parts necessary to carry out regular maintenance at the service station of a car. We selected a brand type of the car that enter the service station to apply it. The collected data is for six periods that represent a year; we can summarize the average required quantities from different spare parts  $\pm 10\%$  as shown in Table 1. On the other hands, the cost related to the holding cost, ordering cost, or backorder cost are considered as fixed values, and the fuzzy parameters are required quantity per period where the data not deterministic values.

**Table 1 The average required spare parts for carry out service to each item per period  $\pm 10\%$**

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Oil Filter	145	170	175	180	130	135
Spark Plug	260	270	320	320	260	240
Air Filter	70	70	80	80	70	60
Engine belt	30	40	30	40	40	30
Fuel Filter	170	200	200	210	180	160
Front brake pads	430	510	525	550	450	400
Rear brake pads	330	400	375	420	420	300

The service station policy prohibits deliberate planning of deficiency of any of its components. However, there are occasional shortages of some spare parts, and it is estimated that every spare part that is unavailable cause extra costs an amount of money per month. This deficiency cost includes the additional cost of supplying replacement parts from the local market at an otherwise higher price, the loss of interest due to the increased cost of sales, the cost of keeping additional records, etc. On the other hands, each time a payment is requested, a setup cost is incurred. This cost includes "processing" costs, administrative costs, transportation, record keeping, etc. Note that this cost requires ordering parts in large batches. Ordering spare parts in large batches lead-to a large inventory. The holding cost of keeping the inventory includes the cost of capital tied to inventory and because this money has been invested in inventory it cannot be used in other investment ways, this cost of capital is made up of lost return (referred to as opportunity cost). Other components of the cost of tenure include the cost of renting the storage space, the cost of insuring against loss of inventory due to fire, theft, or vandalism, taxes based on the value of the inventory, and the cost of the personnel who oversee and protect the inventory. Inventory costs are presented in the Table 2, Table 3, and Table 4

**Table 2 Holding cost for required items through holding periods**

	One period	Two periods	Three periods	Four periods	Five periods
No. 1: Oil Filter	29	58	87	116	145
No. 2: Spark Plug	20	40	60	80	100
No. 3: Air Filter	23	46	69	92	115
No. 4: Engine belt	28	56	84	112	140
No. 5 Fuel Filter	13	26	39	52	65
No. 6: Front brake pads	41	82	123	164	205
No. 7: Rear brake pads	45	90	135	180	225

**Table 3 Backorder Cost for required items according to backorder periods**

Items No.	One period	Two periods	Three periods	Four periods	Five periods
No. 1: Oil Filter	48	55	63	72	83
No. 2: Spark Plug	32	37	43	49	57
No. 3: Air Filter	38	43	50	57	66
No. 4: Engine belt	45	52	60	68	79
No. 5 Fuel Filter	22	25	29	33	38
No. 6: Front brake pads	67	77	88	101	116
No. 7: Rear brake pads	76	87	100	115	132

**Table 4 Ordering Cost**

Period No.	Ordering Cost (EGP)
1	64000
2	64000
3	64000
4	64000
5	64000
6	64000

**5.2. Formulation of the Inventory Practical Problem**

$$\begin{aligned}
 & \text{Min } \sum_{i=1}^6 \sum_{j=i+1}^6 \sum_{k=1}^7 h_{ijk} x_{ijk} + \sum_{i=2}^6 \sum_{j=1}^{i-1} \sum_{k=1}^7 b_{ijk} x_{ijk} + \sum_{i=1}^6 S_i y_i \\
 & \text{Subject to} \\
 & \sum_{j=1}^6 x_{ijk} \leq \tilde{Q}_k \cdot y_i \quad i = 1, 2, \dots, 6 \ \& \ k = 1, 2, \dots, 7 \\
 & \sum_{i=1}^6 x_{ijk} \leq \tilde{d}_{jk} \quad j = 1, 2, \dots, 6 \ \& \ k = 1, 2, \dots, 7 \\
 & y_i = 0, 1 \quad i=1, 2, \dots, 6 \\
 & x_{ijk} \geq 0, \text{ Integer}
 \end{aligned} \tag{7}$$

**Transform to Crisp Form**

$$\begin{aligned}
 & \text{Min } \sum_{i=1}^6 \sum_{j=i+1}^6 \sum_{k=1}^7 h_{ijk} x_{ijk} + \sum_{i=2}^6 \sum_{j=1}^{i-1} \sum_{k=1}^7 b_{ijk} x_{ijk} + \sum_{i=1}^6 S_i y_i \\
 & \text{Subject to} \\
 & Q_{kL} \cdot y_i \leq \sum_{j=1}^6 x_{ijk} \leq Q_{kU} \cdot y_i \quad i = 1, 2, \dots, 6 \ \& \ k = 1, 2, \dots, 7 \\
 & d_{jkL} \leq \sum_{i=1}^6 x_{ijk} \leq d_{jkU} \quad j = 1, 2, \dots, 6 \ \& \ k = 1, 2, \dots, 7 \\
 & y_i = 0, 1 \quad i=1, 2, \dots, 6 \\
 & x_{ijk} \geq 0, \text{ Integer}
 \end{aligned} \tag{8}$$

After making  $\alpha$ -cut the lower and upper values for the required quantities in every period will present in Table 5

**Table 5 The lower and Upper values for required quantities in every period at  $\alpha$ -cut =0.5**

Item	Period 1		Period 2		Period 3		Period 4		Period 5		Period 6	
	L	U	L	U	L	U	L	U	L	U	L	U
No. 1: Oil Filter	138	152	162	179	166	184	171	189	124	137	128	142
No. 2: Spark Plug	247	273	257	284	304	336	304	336	247	273	228	252
No. 3: Air Filter	67	74	67	74	76	84	76	84	67	74	57	63
No. 4: Engine belt	29	32	38	42	29	32	38	42	38	42	29	32
No. 5: Fuel Filter	162	179	190	210	190	210	200	221	171	189	152	168
No. 6: Front brake pads	409	452	485	536	499	551	523	578	428	473	380	420
No. 7: Rear brake pads	314	347	380	420	356	394	399	441	399	441	285	315

**5.3. Results & Discussions**

LINGO software is used for solving the problem to find the optimal inventory policy for six periods (i.e one year), which minimizes the inventory cost for spare parts of the regular maintenance for chosen brand car. **Table 5** introduces the optimal inventory policy, where the minimum total inventory cost through six periods (i.e. a year) is 344475 EGP plus purchase cost the Items

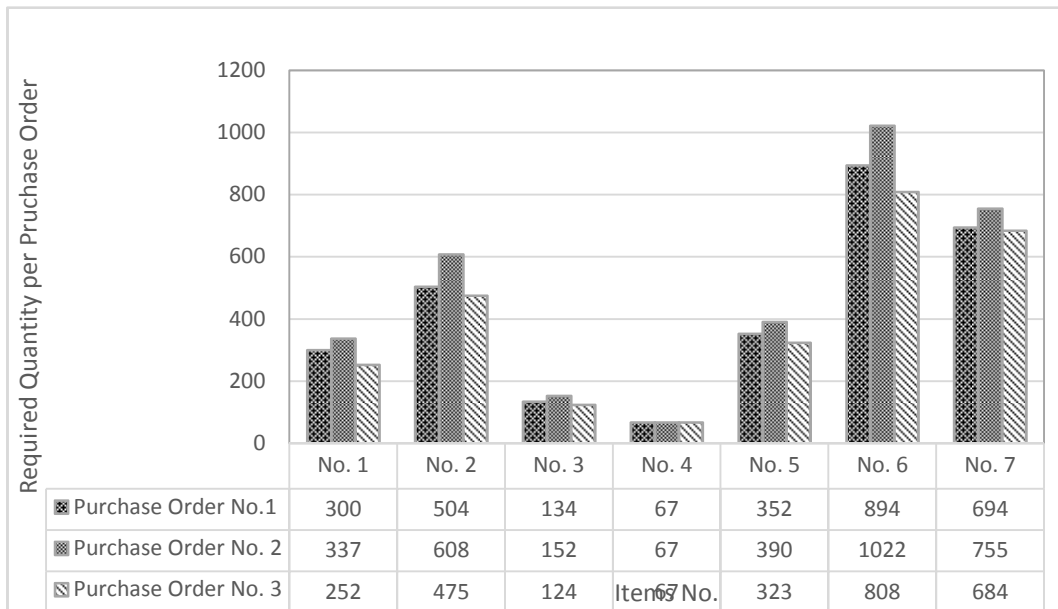
**Table 5 Optimal Inventory Policy by Using LINGO Software**

Global optimal solution found.	
Objective value:	344475.0
Objective bound:	344475.0
Infeasibilities:	0.000000
Extended solver steps:	0
Total solver iterations:	737
Elapsed runtime seconds:	0.15
Objective value:	344475.0
Model Class:	MILP
Total variables:	258
Nonlinear variables:	0
Integer variables:	6
Total constraints:	91
Nonlinear constraints:	0
Total nonzeros:	978
Nonlinear nonzeros:	0

Variable	Value	Reduced Cost
X121	162.0000	0.000000
X341	171.0000	0.000000
X561	128.0000	0.000000
X122	257.0000	0.000000
X342	304.0000	0.000000
X562	228.0000	0.000000
X123	67.00000	0.000000
X343	76.00000	0.000000
X563	57.00000	0.000000
X124	38.00000	0.000000
X344	38.00000	0.000000
X564	29.00000	0.000000
X125	190.0000	0.000000
X345	200.0000	0.000000
X365	152.0000	0.000000
X126	485.0000	0.000000
X346	523.0000	0.000000
X566	380.0000	0.000000
X127	380.0000	0.000000
X347	399.0000	0.000000
X567	285.0000	0.000000
Y1	1.000000	64000.00
Y3	1.000000	64000.00
Y5	1.000000	64000.00
X111	138.0000	0.000000
X112	247.0000	0.000000
X113	67.00000	0.000000
X114	29.00000	0.000000
X115	162.0000	0.000000
X116	409.0000	0.000000
X117	314.0000	0.000000
X331	166.0000	0.000000
X332	304.0000	0.000000
X333	76.00000	0.000000
X334	29.00000	0.000000
X335	190.0000	0.000000
X336	499.0000	0.000000
X337	356.0000	0.000000
X551	124.0000	0.000000
X552	247.0000	0.000000
X553	67.00000	0.000000
X554	38.00000	0.000000
X555	171.0000	0.000000
X556	428.0000	0.000000
X557	399.0000	0.000000

We can summarize the optimal solution as the following as shown in Fig.7: No. of orders through a year are Three orders; the first order occurs at first period to cover the required quantities for the first and the second planned periods. While the second purchase order occurs at the third period to cover the required quantities for the third and fourth planned periods. On the other hands, the third purchase order happens at the fifth period to cover the required quantities for fifth and sixth planned periods. by total inventory cost equal to 344475 EGP. When comparing with the results of the proposed model by the current strategy that costs 384000 EGP for the inventory cost (i.e. six order multiple with ordering cost to avoid the holding and backorder cost), the new proposed strategy will save about 10.3 % from the total inventory cost.





**Fig 7 Order quantities for all Required Spare Parts According to Purchase Order No.**

**6. Conclusion**

In this paper, we present a proposed model for fuzzy inventory model to minimization the total inventory cost that represents in holding cost, setup cost, and backorder cost and apply the proposed model on a service station the find the optimal policy to replenishments the orders to minimization the total inventory cost. The current situation in service station makes purchase order every two month after applied the proposed model we can make purchase order every four month. This policy saves a money about 39525 EGP (i.e. about 10.3%) from the inventory cost per year. In the finally, we can be generalized the proposed model to can save a lot of money in many companies after adaptive the model on a company condition.

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