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Impact of Teachers' Identification of Written Mathematical Points on Students' Learning

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ABSTRACT

I examined the relationship between teachers' identification of mathematical points in written lessons and students' mathematical learning opportunities. Lessons in teachers' guides and classroom instruction were analyzed for written mathematical points and those articulated by teachers during instruction. Teachers who appropriately identified written mathematical points together with suggested curricular resources to realize them had a positive impact on students' mathematical learning opportunities. Positive impact was influenced by the teacher's ability to appropriately identify the role of available curricular resources in supporting the achievements of written mathematical points, recognize relationships between suggested activities and curricular resources toward written mathematical points, and develop a productive mathematical storyline.

***KEYWORDS**: Elementary School Education, Curriculum, Assessment*

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Quality teaching begins with teachers clearly identifying what students need to learn from their teacher's guide and then designing activities or tasks to support them to reach the intended learning goal. Many research studies (e.g., Morris et al., 2009) have investigated teachers' abilities to clearly identify learning goals from students' written work. For example, Morris et al. (2009) investigated the ability of preservice teachers to identify subconcepts in mathematical ideas students are to learn and found that they could accurately identify at least one subconcept of a learning goal but not all when correct student work was presented to them. This result suggests that unpacking learning goals from resources available to teachers or additional resources they may want to include in their lessons might be a challenging skill to develop.

Hiebert et al. (2007) emphasized the need for teachers to clearly state what students should learn from a lesson. According to Hiebert et al., learning goals help teachers determine whether or not students have arrived at the intended learning. Hiebert et al. further argued that breaking general learning goals into sub-goals provides better guidance to examine the link between teaching and learning. This implies that teachers should carefully evaluate and interpret written lessons together with suggested resources to identify what mathematics students ought to learn. After identification of the lesson goals, teachers look forward to planning moves that will enable them have student learning reach these goals.

Sleep (2009) unpacked ways teachers steer lessons toward mathematical points. Sleep defined mathematical point (MP) as "the mathematical learning goal for an activity as well as the connection between an activity and its goal" (p. 13). Sleep's definition focused on the learning goals of an activity and describes the work involved in steering the lesson towards these goals. Van Zoest et al. (2016) defined MP with respect to student thinking in-the-moment during whole class discussion. According to them, "an MP is a mathematical statement of what could be gained from considering a particular instance of student thinking" (p. 323). From these, we see that Sleep (2009) focused on the activities students are to engage in while Van Zoest et al. (2016) focused on student thinking in-the-moment. Despite this interest in MPs, however, little is known about teachers' ability to identify MPs embedded in suggested curricular resources (CRs)—"valuable support provided to teachers within each lesson in the teacher's guide" (Atanga, 2014, p. 3). By mathematical point (MP), I mean important mathematical ideas students ought to learn for each lesson as communicated in a teacher's guide, which may be different from, or the same as, lesson goals. My definition of MP is similar to those of Sleep (2009) and Van Zoest et al. (2016) in that the emphasis is on the mathematical idea students are to learn, but different in that while Sleep looks at activities students are to engage with and Van Zoest et al. (2016) looks at student thinking, I focus on CRs embedded in teachers' guides.

Specifically, this study examined teachers' ability to identify stated MPs for written lessons in teachers' guides and the impact on the mathematics students ought to learn. I also investigated whether the MPs teachers seem to pursue during instruction as they make use of suggested CRs are the same or different from those communicated in the teacher's guide. This study focused on the impact of teachers' identification of MPs embedded in suggested CRs on the mathematical content students have the opportunity to learn. As such, I asked the research question, *How does teachers' identification of written MPs impact students' mathematical learning?*

Theoretical Perspectives

Sleep (2012) defined the work of steering instruction as involving three mutually dependent actions: "(a) articulating the mathematical point, (b) orienting the instructional activity, and (c) steering the instruction" (p. 937). She described the first two as "mathematical purposing" (p. 938) to involve stating learning goals for students. Morris et al. (2009) found that some preservice teachers either correctly (exactly) or partially identified learning subgoals when presented with student work. Steering instruction towards identified learning goals involves teacher moves deployed during planning or enactment of lessons to support students in achieving the mathematics of the lesson.

Sleep (2012) identified such moves and elaborated on them. The moves shown in Figure 1 are relevant for this study as they have direct impact on reaching the MPs. These moves provide a framework for the analysis in my study.

Work of Steering Instruction	Teacher Moves
1. Making sure students are doing the mathematical work	a) Asking questions that engage students in mathematical reasoning b) Getting students into the work without doing it for them c) Distributing the mathematical talk and the kinds of mathematical talk
2. Developing and maintaining a mathematical storyline	a) Developing a coherent within-lesson storyline by making mathematical connections across a lesson's activities b) Progressing the mathematical storyline by engaging with new ideas/practices or engaging with ideas/practices in new (more challenging) ways c) Developing an across-lesson mathematical storyline by looking for mathematical coherence across students' prior and future work d) Conveying the mathematical storyline to students by framing, narrating, and summarizing the mathematical work
3. Opening up and emphasizing key mathematical ideas	a) Using intentional redundancy b) Pointing out the use of a focal concept or skill c) Providing definitions d) Spending more time on key ideas e) Using a combination of teacher and student talk
4. Keeping a focus on meaning	a) Deploying representations in ways that highlight intended meaning b) Explicitly connecting the activity to the intended mathematics

Figure1: Work of Steering Instruction and Corresponding Teacher Moves (Sleep, 2012)

Method

Curriculum programs and teacher participants: Data were gathered from teachers in grades 3-5 using two different curriculum programs: *Investigations in Number, Data, and Space* (*Investigations*) and *Scott Foresman Addison Wesley-Mathematics* (*SFAW-Mathematics*), both published in 2008. The former is an NSF-funded program, while the latter was commercially developed. Six teachers participated in this study; Lisa, Maria, and Jennifer used *Investigations*, while Caroline, Dan, and John used *SFAW-Mathematics*. These teachers have teaching experience ranging from 8 to 11 years, from Head Start to grade 8. They have also taught a variety of subjects for the different grades and have been exposed to both NSF-funded and commercially developed CMs.

Data sources: Data used in this study included classroom observations and post-observation teacher interviews. Each teacher was observed teaching three consecutive lessons in spring 2012 and all enacted lessons were videotaped and transcribed. Each of these six teachers was interviewed to determine whether or not they identified MPs of the written lessons embedded in recommended CRs in their teacher's guide.

Data analysis: I determined written or implied MPs by identifying important mathematical ideas students ought to learn for each written lesson based on the suggested activities in each teacher's guide. Suggested CRs used in written lessons were also analyzed to identify what MPs, written or implied, they are intended to foster. As classroom videos and transcripts were analyzed, issues about teachers' identification of MPs in suggested CRs emerged. I then observed each classroom video to identify the MPs each teacher articulated to students. I compared the MPs articulated by teachers to those written or implied in the teacher's guide using the codes (1) *exactly* (*when the articulated MP is exactly the same as that written or implied*), (2) *differently* (*when the articulated MP is exactly different as that written or implied*), (3) *partially* (*when part of the articulated MP is partially the same as that written or implied and part is different*). This enabled me to determine whether MPs articulated by teachers were similar or different from those written in the teacher's guide or implied by the researcher. In addition, from the classroom videos, I coded teacher moves using Figure 1. This enabled me to determine which MPs are pursued in the lesson to determine which MPs students are effectively exposed to and the kind of learning each teacher likely promoted, by the opportunities available to students. I compared what students effectively learned to intended learning to determine whether teachers' identification of MPs impacted student learning fully positively (*when students actually learned what was intended for them by the opportunities to learn available and are able to demonstrate that with accurate execution of assigned task*), partially positively (*when students partially learned what was intended for them and the other part not encountered and are only partially able to execute assigned task*), or fully negatively (*when students actually did not encounter or learn what was intended for them and are not able to execute assigned task*). Also, I looked at all lessons by teacher to determine the overall impact on student learning and categorize each teacher based on the MPs they articulated as measured by the mathematical content students had the opportunity to learn. Lastly, I searched for patterns across all teachers in each category to describe possible reasons for such impact on student learning.

Results

Because the written or implied MPs I identified and those articulated by the teacher are highly related to the possible impact on student learning, I present the results together in order to illustrate their connectedness. Lisa's and Maria's identification of written MPs was classified as negatively impacting student learning, while Caroline's, Dan's, Jennifer's, and John's identification of written MPs was classified as positively impacting student learning. The difference between these two impacts on student learning can be attributed to the MPs teachers articulated and the way teachers in the different categories emphasized key mathematical ideas, developed meaning, and developed and maintained a mathematical storyline—"following a deliberate progression and making connections among mathematical ideas toward the mathematical points over a course of lessons" (Atanga, 2014, p. 154). Teachers' identification of written MPs is explained below with examples from Maria's and John's lessons to illustrate possible impact on student learning.

Negative Impact on Student Learning

The MPs in the CRs for the three lessons Maria taught are "(1) using the inverse relationship between multiplication and division to solve problems, (2) identifying characteristics of these problems, and (3) write multiplication and division story problems" (Wittenberg et al., 2008, Grade 3, Unit 5, pp. 122-136). The MPs Maria articulated to the students and pursued for the three lessons were "to identify key words to determine whether a problem is multiplication or division, to solve problems, and then to write story problems." The MPs identified and stated by Maria are similar to those written in the curriculum in that students ought to solve problems and write their own story problems. Maria's articulated MPs are different from those written in the teacher's guide in two ways. First, Maria did not specify the suggested methods in the teacher's guide students ought to learn to solve the problems, while written MPs indicated solution strategies students should learn. Second, Maria introduced the identification of "key words" to provide a clue for students to determine whether assigned problems use multiplication or division. Maria hoped that these "key words" would support students in writing their own story problems.

Investigations provides a set of six problems (three pairs) for students to solve. Each pair of problems uses the same numbers, one of them being a multiplication problem and the other division. The curriculum particularly suggests that teachers highlight problems 2 and 3 for discussion with students to achieve the written MPs mentioned above. After this discussion, it is expected that students notice the other pairs of problems, use the inverse relationship between these operations to solve them, and subsequently write their own division and multiplication story problems.

During enactment, Maria led a classroom discussion of each of the six problems. She began by reading each problem and consistently asked, "What's my key word on this problem?" Maria underlined the key word and asked the students what notation could be used, and she wrote either × or ÷ beside the problem as appropriately determined. Afterwards, Maria asked students for the number sentence, which she wrote when correctly provided. In addition, Maria always asked, "How do I solve this one?" and together with students, a correct solution was provided.

As Maria led students through the solution of the six problems one after another, she focused students on problems 2 and 3 as recommended and orchestrated the following interaction.

Maria:…look at question number 2 and question number 3...Do you notice anything special about question number 2 and question number 3? …What do you notice?

Student: The top one's 5 and the bottom one's 20 because 20 divided by 4 is 5 and then 5 times 4 is 20.

Maria: Good. Adding and subtracting are exact opposites, right? So are multiplication and division so these have the same set of numbers in them it's just that this one is the inverse or opposite of the one right above it. Kind of cool. So if you solve this one and you solve the same numbers you automatically know the answer without having to even solve them. They're part of the same…?

Student: Fact family.

Maria: Fact family, absolutely.

In this interaction, Maria ended up using problems 2 and 3 to get students to see those numbers as members of a "fact family," an MP not intended for this lesson, which neither highlighted attributes of multiplication and division problems nor the potential of using the inverse relationship between these operations to solve the problems. Although the idea of inverse relationships surfaced in the above excerpt, Maria did not pursue it beyond making a comparison with addition and subtraction. Maria also did not push her students to see how the inverse relationship between multiplication and division could be used to solve problems 2 and 3. Figure 2 shows a suggested representation in the teacher's guide, which Maria did not use.

Number of Groups	Number in Each group	Product	Equation
?	4 muffins	20	20 ÷ 4 = or ___ × 4 = 20
5 packs	4 yogurt cups	?	5 × 4 = 20

Figure 2 Visual to Illustrate Inverse Relationship Between Multiplication and Division and their Attributes (Wittenberg et al., 2008, Grade 3, Unit 5, p. 124)

This suggested representation is to support teachers in accomplishing the written MPs, but when asked during the post-observation interview about what this representation communicated, Maria said, "I don't always use that table…so we talk about the differences in those notations, rather than relying so much on this chart. I don't know that I feel like the chart aids a whole lot." According to Maria, the chart is basically focused on differentiating the notations for both operations and hence is not particularly helpful.

After solving all six problems, Maria and her students identified and created a list of key words list for each operation; those for multiplication problems included *in all, altogether, how many, total* and those for division problems included *how many equally, share equally, how many groups, how many in each group, divide, put in each*. Following this summary of key words for each operation, Maria asked students to create their own multiplication and division story problems as required by the CM. After three days of teaching, students in Maria's class neither wrote correct multiplication and division story problems nor used the inverse relationship between multiplication and division to solve problems. Hence, it can be concluded that Maria's articulated MPs had a negative impact on students' opportunities for mathematical learning.

Positive Impact on Student Learning

Written MPs for a lesson John taught are "(1) a plane figure has two dimensions: length and width; (2) a solid figure has three dimensions: length, width, and height; (3) there is a unique relationship between solid figures and flat shapes; (4) definitions of mathematical terms" (Charles et al., 2008, Gr. 4, Vol. 3, p. 434). John articulated what students ought to learn during enactment as "today we're going to relate two different types of figures together. What we call plane figures and what we call solid figures." John's MP is the same as the third MP in the teacher's guide, but he did not explicitly articulate to students the first two and the fourth MPs.

In the lesson John taught, *SFAW-Mathematics* suggests that teachers distribute copies of the net of a cube to students, cut out the net, and construct a cube. The teacher's guide for this lesson suggests teachers introduce and illustrate the terms *face, edge,* and *vertex* to students and ask questions to determine the number of each. During enactment, John led students through the construction of the net of a cube, using graph paper. He asked questions such as, "The lines that you have on your graph paper are all making what type of shape?" and students answered, "Squares." John added and illustrated that, "A square is an example of a plane figure. Meaning it's flat. It's one surface. It has basically what we call two-dimensions. It has length and it has width, now the square."

In addition, John asked, "How many squares make up this shape [the net of a cube]?" This question focused students on the constructed net and students could see that there are six squares, to answer correctly. John continued to direct students on what has to be done to create a cube from the net. Students followed John's guidance and experienced the transformation from a net to a cube. John explained, "Now, you have six squares that made up the cube. So we have turned six plane figures, in other words flat figures, into a solid figure that has now three dimensions. We have length, width and height," pointing at each dimension to concretize it. Two things about John's actions are noteworthy here. First, a relationship between plane and solid figures was established using a cube. Second, John established a one-to-one correspondence between a solid figure and its dimensions. Therefore, John used suggested representations and guidance provided in the teacher's guide to develop and maintain a storyline from the net of a cube to the cube together with its dimensions and established a relationship between plane and solid figures.

John used the constructed cube to define the other terms—a *face*, an *edge*, and a *vertex*—students were to learn. He held the cube and said,

Squares. So it's 6, the 6 faces of your cube are all squares. So a flat...so in flat surfaced figures, which is what we're going to be talking about today for the most part, flat surface is a face. Your cubes have six faces those six faces are all squares. Yes? John called the six squares of the cube faces. He emphasized that because this solid figure is formed from a plane figure, the faces must be flat. Furthermore, John defined an edge and a vertex as below:

Ok, so an edge, look at the next highlighted part, it says: An edge is a line segment where two faces meet. Everyone hold up your cube. Run your finger along an edge. Very good, that is an edge. Notice where two faces come together is an edge. Any place where you folded them and those faces came together you created an edge. The last one is a vertex, a vertex is where three or more edges meet, the plural is vertices. So, point on your cube to a vertex.

John accurately defined a face, an edge, and a vertex, mapping the terms to a surface, line, and point, respectively, to illustrate what they represent. Also, John asked for the number of faces, edges, and vertices of the constructed cube. As students provided the correct number, John counted the distinct faces, edges, and vertices to concretely justify students' responses. Students in John's class proceeded with assigned problems from the text with minor difficulties, suggesting that his actions had provided them the opportunity to learn the mathematics.

When John was asked during the post-observation interview about the mathematical significance of the activity from the net of a cube to constructing a cube, he said,

Well, what it's showing us is the faces of a cube are made up of six squares, and it, and it teaches us that, it teaches students that the faces of solid figures are plane figures, …knowing what the definitions are, knowing that, what a face is, what a vertex is, what an edge is…

This revealed that John had an understanding of the mathematical ideas of the lesson students ought to learn. Hence, I concluded that John's identification of MPs likely has a positive impact on students' mathematical learning.

Discussion/Implications

The findings indicate that a big responsibility in teaching is to accurately identify MPs for the lesson as well as how suggested CRs support their realizations. This study revealed three interdependent aspects of teaching that teachers need to attend to with care in order to expose their students to intended mathematical concepts they ought to learn.

First, teachers must appropriately identify MPs that suggested problems are intended to communicate. Understanding the rich mathematical concepts embedded in suggested problems together with solution strategies is a key in supporting students in reaching the intended learning target of the lesson. Second, teachers should be able to identify written MPs that suggested representations are designed to foster. Kilpatrick, Swafford, and Findell (2001) found that use of representations have significant positive influence on student understanding of the mathematics they ought to learn. Maria failed to make use of Figure 2 that could have helped her communicate to her students the attributes of multiplication and division problems as well as the use of inverse relationship between the operations to solve problems. In contrast, John used the representations available in the teacher's guide to communicate and establish a relationship between plane and solid figures. Third, teachers should identify relationships among CRs toward written MPs of the lesson. Understanding and identifying these relationships require deep knowledge of the representations (Castro Superfine, Canty, & Marshall, 2009) and knowledge about how to translate between the different representations while preserving the structural information presented in each of them (Novick, 2004). Maria seemed not to understand the information conveyed by problems 2 and 3 and Figure 2, making it hard for her to make meaningful connections between them. This resulted in her omitting the use of Figure 2 and attempting to use the suggested problems in isolation, and opportunities to learn important mathematical ideas and solution strategies were missed. In contrast, John seemed to understand each representation suggested in the teacher's guide and translated between the net of a cube and the constructed cube, calling the faces of the cube squares and preserving their structural information.

These aspects of teaching extend our understanding of "mathematical purposing" (Sleep, 2012, p. 938). Identifying written MPs of the lesson and those embedded in CRs, identifying and establishing relationships among CRs toward written MPs, and mapping out and developing a productive mathematical storyline from one MP to another provide us with additional fine-grain details of the work of mathematical purposing in classrooms. Mathematics educators might include into their methods and content courses for preservice teachers activities such as identifying MPs of written lessons and CRs, and discussing relationships among CRs in achieving the stated learning goals. This might help improve teachers' mathematical knowledge for teaching and can ultimately add value to teacher training programs. In addition, results of this study has potential of being used by educators to develop teachers' specialized content knowledge (SCK). Morris et al. (2009) argued that an aspect of SCK is focused on what type of representations teachers might use to effectively communicate a particular mathematical idea to students. So, focusing on teachers' ability to identify MPs in suggested CRs might support the development of needed SCK and hence teachers' subject matter knowledge, because the former is a subset of the latter (see Ball et al.'s 2008 model).

Although this study suggested important skills teachers need in order to promote student learning, the absence of student data to substantiate further the benefits of identifying MPs and developing a "productive" storyline and the small number of teachers and lessons involved limit its wide applicability. Therefore, further studies involving student data and a greater number of lessons and teachers over an extended period of time are needed to investigate what it means for teachers to identify written MPs and develop a potentially productive mathematical storyline toward them.

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