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Using Cauchy Inequality to Find Function Extremums

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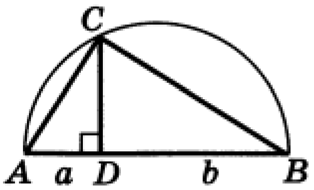
ABSTRACT

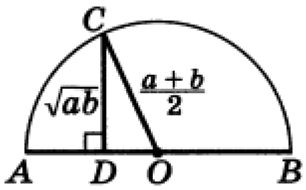
The article explains the relationships between the mean values using triangles. Here are some ways to determine the extreme values of some functions using the Cauchy inequality, which represents the relationship between arithmetic mean and geometric mean.

KEYWORDS: *function extremums, averages, Cauchy inequality*

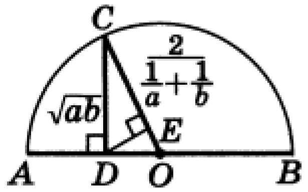
Consider sections with lengths a and$b$. Their average arithmetic is $\frac{a+b}{2}$, their average geometry is $\sqrt{ab}$, and their average harmonic is $\frac{2}{\frac{1}{a}+\frac{1}{b}}$. We can compare these values in the following ways (Figure 1).



1-rasm



2-rasm



3-rasm

From the similarity of triangles ADC and *CDB* $\frac{AD}{CD}=\frac{CD}{BD}\Rightarrow CD=\sqrt{ab}$

It can be seen from Figure 2 that it is $\frac{a+b}{2}\geq\sqrt{ab}$. (The arithmetic mean of a and b is equal to the mean geometry at $a = b$.) From the similarity of triangles CDE and COD $\frac{CE}{CD}=\frac{CD}{CO}$; $\Rightarrow CE=\frac{ab}{\frac{a+b}{2}}$ or $E=\frac{2}{\frac{1}{a}+\frac{1}{b}}$; $CE < CD$ because $\frac{2}{\frac{1}{a}+\frac{1}{b}}\leq\sqrt{ab}$ (fig. 3).

The three mean values are equal to $a = b$.

Now let's talk about the average values application in solving problems to find the largest values of the function.

1-example. At what value of x does the $f(x) = x(1-x)$ $(0 < x < 1)$ function take the largest value?

Solution: According to $\frac{a+b}{2}\geq\sqrt{ab}$ inequalities

$$x(1-x)\leq\left(\frac{x+1-x}{2}\right)^2=\frac{1}{4}$$

$$x(1-x)=\frac{1}{4}\Leftrightarrow x=\frac{1}{2}$$

Answer: $\max f(x)=f\left(\frac{1}{2}\right)=\frac{1}{4}$

2-example. At what value of x does the function $g(x) = x(x-1)$ $(0 < x < 1)$ take the smallest value?

Solution. If $c < d$ *then* $-c > -d$.

$$x(x-1)=-x(1-x)\geq-\left(\frac{x+1-x}{2}\right)^2=-\frac{1}{4}$$

$$x(x-1)=-\frac{1}{4}\Leftrightarrow x=\frac{1}{2}$$

Answer: $\min g(x)=g\left(\frac{1}{2}\right)=-\frac{1}{4}$

3-example. At what value of x does the function $h(x)=4x+\frac{9}{x}, x > 0$ take the smallest value?

Solution. According to $a+b\geq2\sqrt{ab}$

$$4x+\frac{9}{x}\geq2\sqrt{\frac{9}{x}\cdot4x}=12$$

$$4x+\frac{9}{x}=12\Leftrightarrow x=\frac{3}{2}$$

Answer: $\min h(x)=h\left(\frac{3}{2}\right)=12$

4-example. At what value of x does $t(x)=\frac{2}{4x+\frac{9}{x}}$, the function $x > 0$ take the largest value?

Solution: $\frac{2}{4x+\frac{9}{x}}\leq\sqrt{\frac{1}{4x}\cdot\frac{x}{9}}=\frac{1}{6}$

$$\frac{2}{4x+\frac{9}{x}}=\frac{1}{6}\Leftrightarrow x=\frac{3}{2}$$

Answer: $\max t(x)=t\left(\frac{3}{2}\right)=\frac{1}{6}$

5-example. At what values of $y(x)=\frac{x^2+4x+53}{x+2}$, the function $x > -2$ takes the smallest value.

Solution: According to $a+b\geq2\sqrt{ab}$

$$\frac{x^2+4x+53}{x+2}=x+2+\frac{49}{x+2}\geq2\sqrt{(x+2)\cdot\frac{49}{x+2}}=14$$

$$\frac{x^2+4x+53}{x+2}=14\Leftrightarrow x=5$$

Answer: $\min y(x)=y(5)=14$

6-example. Prove that a rectangle with equal sides has a square with the smallest perimeter.

Let one side of the rectangle be the one length, then the other side of the given surface $S = a^2 (a > 0)$ is $\frac{a^2}{x}$.

Proof: Let one side of the rectangle be the one length, then the other side of the given surface $S = a^2 (a > 0)$ is $\frac{a^2}{x}$.

This is the rectangle perimeter

$$P(x) = 2(x + \frac{a^2}{x})$$

It's clear,

$$2\left(x + \frac{a^2}{x}\right) \geq 2 \cdot 2\sqrt{x \cdot \frac{a^2}{x}} = 4a.$$

$$2\left(x + \frac{a^2}{x}\right) = 4a \Leftrightarrow x^2 - 2ax + a^2 = 0 \Leftrightarrow x = a$$

$\min P(x) = 4a$

if $\frac{x^2}{x} = a$ $x = a$. In that case, the rectangles with equal sides have the smallest perimeter.

7-example. If $A = \dfrac{x^2+3}{\sqrt{x^2+2}}$, $B = 2$, then $A > B$.

Proof. $A = \dfrac{x^2+3}{\sqrt{x^2+2}} = \sqrt{x^2+2} + \dfrac{1}{\sqrt{x^2+2}} > 2$.

So $A > B$. Proved.

References

[1] Sivashinsky I.Kh. Inequalities in tasks. M: Science,1967

[2] Zaitsev V. V., Ryzhkov V. V., Skanavi M. I. Elementary mathematics. M: Science,1976