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Mathematical Model for Modified Population Growth

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ABSTRACT
Generally, the people in urban areas have better living in standard cities, particularly, the big cities, have their own attraction. The urban areas have better job opportunities, transportation, educational, entertainment, medical facilities and scope for industrial growth. On the other side the rural areas are devoid of most of the facilities. They depend mostly on agriculture. Their return from agricultural product is inadequate; people have to look towards urban cities for alternative job opportunities. Industrial areas and cities, too, attract labour force and job seekers. Hence there is out migration from rural to urban areas. In the present study, the solution of population growth model equations have been obtained by the discussion with the marriage rate as a quadratic function time, and discussion with the death rate of unmarried people and married couple under this occurrence. The total population has been represented by the final solution, who express the all developing countries like India marriage go on increasing every year which produce the tremendous growth of the population. It is an effort to represents the population growth model.

KEYWORDS: *Mathematical-model; Overcrowding; Population-growth*

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1. INTRODUCTION

The urban areas have better job opportunities, transportation, educational, entertainment, medical facilities and scope for industrial growth, In general, the people in urban areas have better living standard cities, particularly, the big cities, have their own attraction. On the other side the rural areas and devoid of most of the facilities **(Spear, 1974; Sarin, 1993).**

They depend most on agriculture. Their return from agricultural product is inadequate people have to look towards urban cities for alternative job opportunities **(Saradamani, 1995).** Industrial areas and cities, too, attract labour force and job seekers. Hence there is out migration from to urban areas **(Dholakia and Dholakia, 1978).** This in-migration causes additional overcrowding in the cities and imposes strain on civic facilities there **(Fawcett, et. al., 1984).** It is rightly said that rural poverty and urban misery are the two faces of the same coin.

A migration carries a culture of its community and of the area of place of birth to the receiving community, which may have cultural, ethnical of language diversities **(Keeling and Rohani, 2007).** This may out the migrant in tension **(Yue, et. al., 2010).** Both the migrants communicates try to adjust themselves with each other, the process of may be slow **(Hugo, 2000).** If the number is large then in may modify the culture the receiving society **(Anderson, and May, 1979).** Thus migration is also an instrument of the culture and social interaction **(Terrink, 1995; Grundmann and Hellriegel, 2006).**

The Kendall's Model has represented in other way and get the result. **Kendall (1949)** extended the problem to two sexes and suggested following Model:

$\frac{dM}{dt}$ + af = (a+u) N - K (M,F)

$\frac{df}{dt}$ + af = (a+u) N - K (M, F)

And written as

$\frac{d}{dt}$ (M - F) = - a (M -F)

And$\frac{dN}{dt}$+ 2aN = K (M, F)

Where M,F and N denote the number of unmarried men, unmarried woman and married couples respectively at time, **a** and**u** denotes death rate and birth respectively. Emphasizing the need of study of migration aspect of human population **Bogue (1969),** has pointed out "If the problem of human were not so critical at the present time t is almost certain that human migration and the plight of migration (especially in the development countries) would be listed as a top priority problem for research and action **(Sibly, et. al. 2005; Schmolke, et. al., 2010)** .

2. PROPOSED MODEL FOR POPULATION GROWTH:

Let K (M,F) be the marriage rate per unit time where M, F and N denote the number of unmarried men, unmarried couples respectively at a time t, **Mishra (1985)** proposed the modified Kendall`s population model (1) as given by the following equations.

$\frac{dM}{dt}$ + $a_1M = (a_3+u_1) N - K (M,F)$ (2.1)

$\frac{dM}{dt}$ + $a_2M = (a_3+u_2) N - K (M,F)$ (2.2)

And

$\frac{dN}{dt}$ + $2a_3N = K (M,F)$ (2.3)

Where a_1, a_2, a_3 denote the death rate per head per unit time of unmarried women and married couple respectively, u_1 and u_2 denote the sale birth rate and female birth rate per married couple pr unit time, we get Kendall's model when $a_1=a_2=a_3=a$ and $u_1=u_2=u$. It is observed that in all the developing countries like India, marriage go on increasing every year which the tremendous growth of the population. **Mishra (1988)** has supposed the marriage rate k as an exponential and linear function of time and obtained the solution. Female are more vital factor to increase the population so we obtain the general solution of these population growth model equations considering the marriage rate depending linearly on the number of unmarried female only.

3. SOLUTION OF POPULATION GROWTH MODEL EQUATION:

Let us assume that the marriage rate K (M, F) depends only on the F as a linear function given by

$K (F) = \alpha_1 F + \alpha_0$ (3.1)

Where α_1 and α_0 are non-negative constants. The population growth model equations are expressed with the help of (3.1) as:

$\frac{dM}{dt}$ + $a_1M = (a_3+u_1) N - (\alpha_1 F + \alpha_0)$ (3.2)

$\frac{dF}{dt}$ + $(a_2 + \alpha_1) F = (a_3+u_2) u - \alpha_0$ (3.3)

And

$\frac{dN}{dt}$ + $2a_3N = \alpha_1 F + \alpha_0$ (3.3)

Differentiating (3.3) with respect to t and eliminating $\frac{dN}{dt}$ and N from equation (3.4) and (3.3) we get:

$\frac{d^2F}{dt}$ + $(a_2 + \alpha_1 + 2a_3)$ $\frac{dF}{dt}$ + $\{ \alpha_1 (a_3-u_2) - 2a_2a_3\} = \alpha_0 (u_2 - a_3)$ (3.5)

Whose particular integral is?

$P = \frac{(u_2-a_3)\alpha 0}{a_1 (a_3-u_2) + 2 a_2 a_3}$ (3.6)

The solution of (3.5) is of the form

$F = A_1 e^{\xi 1t} + A_2 e^{\xi 2t} + P$ (3.7)

Where A1 and A2 are arbitrary constant ξ_1 and ξ_2 are given

$\xi_1 = \frac{(\alpha_1+a_2+2a_3)+\sqrt{(\alpha_1+a_2-2a_3)^2+4\alpha_1 (a_3+u_2)}}{2}$ (3.8a)

And

$\xi_2 = \frac{-(\alpha_1+a_2+2a_3)+\sqrt{(\alpha_1+a_2-2a_3)^2+4\alpha_1 (a_3+u_2)}}{2}$ (3.8b)

Again differentiating (3.4) and using the equation (3.2) we get:

$\frac{d^2N}{dt2}$ + $(a_2 + \alpha_1 + 2a_3)$ $\frac{dN}{dt}$ + $\{ \alpha_1 (a_3-u_2) + 2a_1a_3\}$ N = $\alpha_0 a_2$ (3.9)

Where the particular solution is:

$Q = \frac{\alpha_0 a_2}{\alpha 1(a3-u2) + 2a_1a_3}$ (3.10)

The complete of (3.9) is of the form

$N = B_1 e^{\xi 1t} + B_2 e^{\xi 2t} + Q$ (3.11)

Where B_1 and B_2 are arbitrary constants $\xi 1$ and $\xi 2$ are given by (3.8a) and (3.8b) .

Substituting the value of F and N in (3.2) and using (3.4) we get K in the form of:

$M = \frac{\{B_1+(a_3+u_1)-a_1A_1\}e^{\xi_1 t}}{a_1+\xi_1} + \frac{\{B_2+(a_3+u_1)-a_1A_2\}e^{\xi_2 t}}{a_1+\xi_2} + Ce^{-a1t} + \frac{R}{a_1}$ (3.12)

Where

$R = \frac{(a_2-u_1)a_2\alpha 0}{a_1 (a_3-u_2)+2 a_2 a_3} - \frac{\alpha_1(u_2-a_3)\alpha_0}{a_1 (a_3-u_2)+2 a_2 a_3} - \alpha_0$

4. TOTAL POPULATION:

If we consider the difference of Kand F and put:

D = K - F

Then we get from equation (3.7) and (3.12)

$D = \frac{\{B_1+(a_3+u_1)-A(a_1+\xi_1+\alpha_1)\}e^{\xi_1 t}}{(a_1+\xi_1)} + \frac{\{B_1+(a_3+u_1)-A1\,(a_1+\xi_2+\alpha_1)\}e^{\xi_2 t}}{(a_1+\xi_2)} + Ce^{a1t} + \frac{R}{a_1} - P$ (4.1)

If we put (i) M = SN

(ii) F = TN

Then is consequence of (3.2) and (3.3) and using (3.4) we obtain:

$N\frac{dS}{dt} + S\{\alpha_1 (A_1e^{\xi 1t} + A_2e^{\xi 2t} + P) - (2a_{3-}a_1) N+\alpha_0\}$

$= (a_3 + u_1) N - \alpha_1 (A_1e^{\xi 1t} + A_2e^{\xi 2t}+P) - \alpha_0$ (4.2)

$N\frac{dS}{dt} + T\{\alpha_1 (A_1e^{\xi 1t} + A_2e^{\xi 2t} + P) - (2a_{3-}a_{2-}a1) N+\alpha_0\}$

$= (a_3 + u_2) N - \alpha_0$ (4.3)

Where P is given by the equation (3.6).

In view of the equation (3.7), (3.11) and (3.12) the total population is express Total

Population = M + F + 2 N

Therefore we obtain from the equation (3.7), (3.11) and (3.12) that Total population

$= [A_1 (a_2 + \xi_1 - \alpha_1) - B_1 (a_2 + u_1 + 2 (a_1 + \xi_1))]e^{\xi 1t} +$

$[A_2 (a_1 + \xi_2 - \alpha_1) - B_2 (a_3 + u_1 + 2 (a_1 + \xi_2))]e^{\xi 2t}$

$+ P + 2 Q + \frac{R}{a_1} + Ce^{-a1t}$ (4.4)

We prove the following theorems from equation (3.7), (3.11) and (3.12) .

Theorem (3.1): In the stationary stage of the population model (2.8) the population of unmarried males tendsto:

$$\frac{1}{a_1}\left[\frac{(a_2-u_1)a_2\alpha 0}{\alpha_1(a_3-u_2)+2a_2a_3} - \frac{\alpha_1\alpha 0(u_2-a_3)\alpha_0}{\alpha_1(a_3-u_2)+2a_2a_3} - \alpha_0\right]$$

Population is

$$\frac{(u_2-a_3)}{a_2}$$

Theorem (3.5): In the ultimate stationary population stage of the population model, the total population will be equal to

$$\frac{\alpha_0}{a_1[\alpha_1(a_3-u_2)+2a_2a_3]}\left[(a_3-\alpha_1)(u_2-a_3)+a_2(2a_1+a_3+u_1)\right] - \frac{\alpha_0}{a}$$

5. RESULTS AND DISCUSSION:

The solution of population growth model equations have been obtained by the above discussion with the marriage rate as a quadratic function time, and discuss the death rate of unmarried people and married couple under this occurrence. The total population has been represented by the final solution, who express the all developing countries like India marriage go on increasing every year which produce the tremendous growth of the population. It is an effort to represents the population growth model.

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