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An Exponential Observer Design for a Class of Chaotic Systems with Exponential Nonlinearity

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ABSTRACT

In this paper, a class of generalized chaotic systems with exponential nonlinearity is studied and the state observation problem of such systems is explored. Using differential inequality with time domain analysis, a practical state observer for such generalized chaotic systems is constructed to ensure the global exponential stability of the resulting error system. Besides, the guaranteed exponential decay rate can be correctly estimated. Finally, several numerical simulations are given to demonstrate the validity, effectiveness, and correctness of the obtained result.

KEYWORDS: *Generalized chaotic systems with exponential nonlinearity, state observer, exponential decay rate, Ten-ring chaotic system*

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1. INTRODUCTION

In the past few years, various types of chaotic systems have been widely studied; see, for example, [1-4] and the references therein. The investigation of chaotic systems not only allows us to understand the chaotic characteristics, but we can use the research results in various chaos applications; such as image processing and chaotic secure communication. Because the state variables of a chaotic system are highly sensitive to initial values and the output signal is unpredictable, the state variables of a chaotic system are always more difficult to estimate than those of a non-chaotic system.

Due to the excessive number of state variables or the lack of measurement equipment, the state variables of real physical systems are often difficult to estimate; see, for example, [5-10]. For chaotic systems, designing a suitable state observer has always been one of the goals pursued by researchers engaged in nonlinear systems.

In this paper, the state observer for a class of generalized chaotic systems with exponential nonlinearity is explored and studied. Based on the differential inequality and time-domain approach, a state observer of such generalized chaotic systems will be developed to guarantee the global exponential stability of the resulting error system. In addition, the guaranteed exponential decay rate can be accurately estimated. Finally, some numerical examples will be provided to illustrate the effectiveness of the obtained results.

2. PROBLEM FORMULATION AND MAIN RESULTS

In this paper, we consider the following generalized chaotic system with exponential nonlinearity

$$\dot{x}_1(t) = a_1 x_1(t) + a_2 x_2(t) \tag{1a}$$

$$\dot{x}_2(t) = f_1(x_1(t), x_2(t), x_3(t)) \tag{1b}$$

$$\dot{x}_3(t) = -a_3 x_3(t) + e^{x_1^2(t)}, \tag{1c}$$

$$y(t) = 2x_1(t) + x_2(t), \tag{1d}$$

where $x(t) := [x_1(t) \quad x_2(t) \quad x_3(t)]^T \in \Re^3$ is the state vector, $y(t) \in \Re$ is the system output, f_1 is a smooth function, and a_1, a_2, a_3 are the parameters of the system (1), with $a_3 > 0$ and $2a_2 > a_1$. In addition, we assume that the signal of $x_1(t)$ is bounded.

Remark 1: It is emphasized that the famous Ten-ring chaotic system [4] is a special case of the system (1).

It is a well-known fact that since states are not always available for direct measurement, states must be estimated. The objective of this paper is to search a suitable state observer for the nonlinear system (1) such that the global exponential stability of the resulting error systems can be guaranteed. In what follows, $\|x\|$ denotes the Euclidean norm of the column vector *x* and $|a|$ denotes the absolute value of a real number *a*.

Before presenting the main result, let us introduce a definition which will be used in the main theorem.

Definition 1. The system (1) is exponentially state reconstructible if there exist a state estimator $E\,\dot{z}(t)=h(z(t),y(t))$ and positive numbers k and α such that

$$\|e(t)\|:=\|x(t)-z(t)\|\le k\exp(-\alpha t),\quad \forall\, t\ge 0,$$

where $z(t)$ expresses the reconstructed state of the system (1). In this case, the positive number α is called the exponential decay rate.

Now we present the main result.

Theorem 1. The system (1) is exponentially state reconstructible. Besides, a suitable state observer is given by

$$\dot{z}_1(t)=(a_1-2a_2)z_1(t)+a_2y(t), \qquad (2a)$$

$$z_2(t)=y(t)-2z_1(t), \qquad (2b)$$

$$\dot{z}_3(t)=-a_3z_3(t)+e^{z_1^2(t)}, \qquad (2c)$$

with the guaranteed exponential decay rate

$$\alpha:=\min\left\{2a_2-a_1,\frac{a_3}{2}\right\}.$$

Proof. Define $M\ge|x_1(t)|$, from (1), (2) with

$$e_i(t):=x_i(t)-z_i(t),\quad \forall\, i\in\{1,2,3\}, \qquad (3)$$

it can be readily obtained that

$$\begin{aligned}\dot{e}_1(t)&=\dot{x}_1(t)-\dot{z}_1(t)\\&=a_1x_1(t)+a_2x_2(t)-(a_1-2a_2)z_1(t)-a_2y(t)\\&=a_1x_1(t)+a_2[y(t)-2x_1(t)]\\&\quad-(a_1-2a_2)z_1(t)-a_2y(t)\\&=-(2a_2-a_1)[x_1(t)-z_1(t)]\\&=-(2a_2-a_1)e_1(t),\quad \forall\, t\ge 0.\end{aligned}$$

It results that

$$e_1(t)=e_1(0)\exp[-(2a_2-a_1)t],\ \forall\, t\ge 0. \qquad (4)$$

Moreover, form (1)-(4), we have

$$\begin{aligned}e_2(t)&=x_2(t)-z_2(t)\\&=[y(t)-2x_1(t)]-[y(t)-2z_1(t)]\\&=-2[x_1(t)-z_1(t)]\\&=-2e_1(t)\\&=-2e_1(0)\exp[-(2a_2-a_1)t],\ \forall\, t\ge 0. \qquad (5)\end{aligned}$$

Define $M\ge|x_1(t)|$ and form (1)-(5), it yields

$$\begin{aligned}\dot{e}_3(t)&=\dot{x}_3(t)-\dot{z}_3(t)\\&=-a_3x_3(t)+e^{x_1^2(t)}+a_3z_3(t)-e^{z_1^2(t)}\\&=-a_3e_3(t)+e^{x_1^2(t)}-e^{z_1^2(t)}\end{aligned}$$

$$\Rightarrow\quad 2e_3(t)\dot{e}_3(t)=-a_3e_3^2(t)+2e_3(t)\left[e^{x_1^2(t)}-e^{z_1^2(t)}\right]$$

This implies that

$$\begin{aligned}\frac{d\left[e_3^2(t)\right]}{dt}&=-2a_3e_3^2(t)+2e_3(t)\left[e^{x_1^2(t)}-e^{z_1^2(t)}\right]\\&\le-2a_3e_3^2(t)+a_3e_3^2(t)\\&\quad+\frac{1}{a_3}\left[e^{x_1^2(t)}-e^{z_1^2(t)}\right]^2\\&=-a_3e_3^2(t)+\frac{1}{a_3}\left[e^{x_1^2(t)}-e^{z_1^2(t)}\right]^2\end{aligned}$$

$$\Rightarrow\quad \frac{d\left[e^{a_3t}e_3^2(t)\right]}{dt}\le\frac{e^{a_3t}}{a_3}\left[e^{x_1^2(t)}-e^{z_1^2(t)}\right]$$

$$\begin{aligned}\Rightarrow\quad & e^{a_3t}e_3^2(t)-e_3^2(0)\\&\le\int_0^t\frac{e^{a_3t}}{a_3}\left[e^{x_1^2(t)}-e^{z_1^2(t)}\right]dt\\&\le\frac{1}{a_3}\int_0^t e^{M^2}\left[x_1^2(t)-z_1^2(t)\right]dt\\&\le\frac{e^{M^2}}{a_3}\cdot(2M+|e_1(0)|)\int_0^t|e_1(0)|e^{-(2a_2-a_1)t}\,dt\\&=\frac{e^{M^2}}{a_3}\cdot(2M+|e_1(0)|)\cdot\frac{|e_1(0)|}{2a_2-a_1}\\&\quad\cdot\left[1-e^{-(2a_2-a_1)t}\right]\end{aligned}$$

$$\begin{aligned}\Rightarrow\quad & e_3^2(t)\le e^{-a_3t}e_3^2(0)\\&\quad+\frac{e^{M^2}}{a_3}\cdot(2M+|e_1(0)|)\cdot\frac{|e_1(0)|}{2a_2-a_1}\\&\quad\cdot\left[e^{-a_3t}-e^{-(2a_2-a_1+a_3)t}\right]\\&\le\left[\frac{e^{M^2}}{a_3}\cdot(2M+|e_1(0)|)\cdot\frac{|e_1(0)|}{2a_2-a_1}\right.\\&\quad\left.+e_3^2(0)\right]e^{-a_3t}\end{aligned}$$

$$\begin{aligned}\Rightarrow\quad |e_3(t)|&\le\left[\frac{e^{M^2}}{a_3}\cdot(2M+|e_1(0)|)\cdot\frac{|e_1(0)|}{2a_2-a_1}\right.\\&\quad\left.+e_3^2(0)\right]^{\frac{1}{2}}\cdot e^{\frac{-a_3}{2}t},\ \forall\, t\ge 0. \qquad (6)\end{aligned}$$

Consequently, by (4)-(6), we conclude that

$$\|e(t)\|=\sqrt{e_1^2(t)+e_2^2(t)+e_3^2(t)}\le k\cdot\exp(\alpha t),\quad \forall\, t\ge 0,$$

with $k=5e_1^2(0)+e_3^2(0)\dfrac{e^{M^2}}{a_3}\cdot(2M+|e_1(0)|)\cdot\dfrac{|e_1(0)|}{2a_2-a_1}$. This completes the proof. □

3. NUMERICAL SIMULATIONS

Example 1: Consider the following Ten-ring chaotic system [4]:

$$\dot{x}_1(t)=20x_1(t)+20x_2(t), \qquad (7a)$$

$$\dot{x}_2(t)=0.5x_1(t)-28x_2(t)-x_1(t)x_3(t), \qquad (7b)$$

$$\dot{x}_3(t)=-2x_3(t)+e^{x_1^2(t)}, \qquad (7c)$$

$$y(t)=2x_1(t)+x_2(t), \qquad (7d)$$

Comparison of (7) with (1), one has

$a_1 = 20, a_2 = 20, a_3 = 2$, and

$$f_1(x_1(t), x_2(t), x_3(t)) = 0.5x_1(t) - 28x_2(t) - x_1(t)x_3(t).$$

By Theorem 1, we conclude that the system (7) is exponentially state reconstructible by the state observer

$$\dot{z}_1(t) = -20z_1(t) + 20y(t), \quad (8a)$$

$$z_2(t) = y(t) - 2z_1(t), \quad (8b)$$

$$\dot{z}_3(t) = -2z_3(t) + e^{z_1^2(t)}, \quad (8c)$$

with the guaranteed exponential decay rate $\alpha := 1$. The typical state trajectories of the systems (7) and (8) are depicted in Figure 1 and Figure 2, respectively. Besides, the time response of error states between the systems (7) and (8) is shown in Figure 3.

4. CONCLUSION

In this paper, a class of generalized chaotic system with exponential nonlinearity has been studied and the state observation problem of such systems has been explored. Using differential inequality with time domain analysis, a practical state observer for such generalized chaotic systems has been built to ensure the global exponential stability of the resulting error system. Moreover, the guaranteed exponential decay rate can be correctly calculated. Finally, several numerical simulations have been offered to demonstrate the validity and effectiveness of the obtained result.

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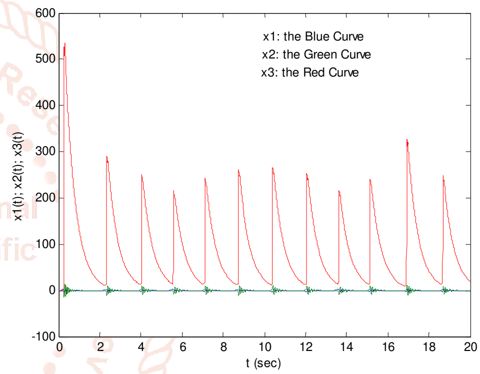

Figure 1: Typical state trajectories of the system (7).

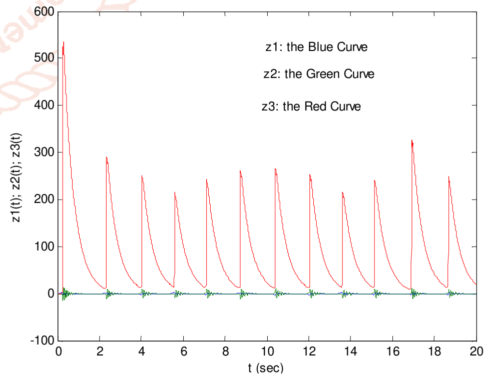

Figure 2: Typical state trajectories of the system (8).

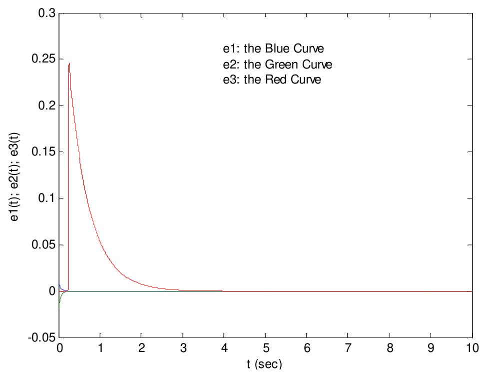

Figure 3: The time response of error states.