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Vertex Odd Power Mean Labeling of Graphs

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ABSTRACT

We define Odd Power Mean labeling for the graph G(V, E) with p vertices and q edges, if it is feasible to label the vertices $x\epsilon V$ with different labelings$f(x)$ from 1, 3, 5, ..., $2q-1$ in such a way that when each edge $e = uv$ is labeled with

$$f(e=uv) = \left\lceil \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rceil$$

$$(\text{or}) f(e=uv) = \left\lfloor \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rfloor$$

and the edge labelings are distinct. The graph which admits the vertex odd Power Mean labeling is called vertex odd Power Mean graph.

KEYWORD: *Power mean labeling, vertex odd power mean labeling*

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1. INTRODUCTION

The graphs, with p vertices and q edges, considered here are simple, finite and undirected graphs. For a detailed survey of graph labeling we refer to Gallian [1]. For other standard terminology and notations we follow the book by Harary [2]. In [3]Somasundaram and Ponraj introduced Mean labeling for some standard graphs in 2003. Somasundaram et al.[4] introduced the concept of Geometric Mean labeling of graphs in the year 2011 and studied further for more graphs. Sandhya and Somasundaram [5] introduced Harmonic Mean labeling of graphs in 2012 and then Sandhya et al. studied the technique in detail. In 2020 S. S. Sandhya, S. Somasundaram and S.Anusa introduced the concept of Root Square Mean labeling of graphs in [6].

P. Mercy and S. Somasundaram defined Power Mean labeling and investigated for some standard graphs [7].Subsequently in 2020, S. Sreeji and S. S. Sandhya introduced another labeling called the Power 3 Mean Labeling of Graphs in [8].

After studying the above works we have interested toextend Vertex Odd Power Mean Labeling for some Graphs.

In this paper we define Vertex Odd Power Mean labeling and investigate the same for some graphs.

2. DEFINITION AND RESULTS

Here we present the basic definitions and some results.

Definition 2.1. A graph G = (V, E) with p vertices and q edges is said to be vertex odd power mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from 1, 3, … ,$2q$ - 1 in such a way that when each edge$e = uv$is labeled with

$$f(e=uv) = \left\lceil \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rceil (\text{or}) f(e=uv) = \left\lfloor \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rfloor$$

and the edge labelings are distinct. The graph which admits the vertex odd power mean labeling, is called vertex odd power mean graph.

Definition 2.1: Circuit C_n: A circuit is a path which ends at the vertex it begins. Hence a loop is an circuit of length one.

Definition 2.2: Star graph S_n: Star graph is a special type of graph in which n-1 vertices have degree 1 and a single vertex have degree n – 1. This looks like that n – 1 vertices are connected to a single central vertex. A star graph with total n – vertex is termed as S_n.

Definition 2.3: Connected graph $C_n + 2P_2$ is a graph having the circuit $C_n$and path P_2which is attached in 2 vertices of C_n opposite to each other.

Theorem 2.1: The circuit $C_n$is vertex odd power meangraph.

Proof: We have the circuit$Cn$of length n.

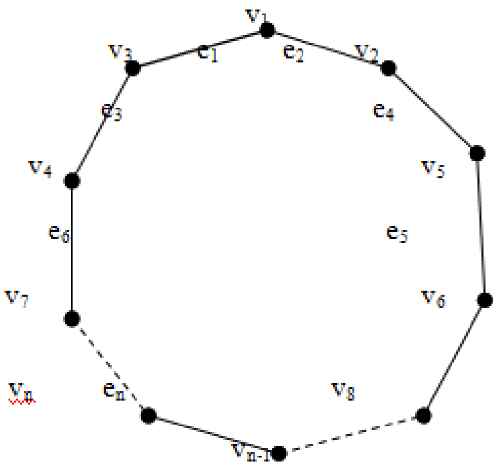

Figure 1: Odd Power Mean labeling of the Cycle C_n

Define a function f: $(Cn) \to \{1,3,\ldots,2q - 1\}$ by
$f(x) = v_i = 2i$-1 ; $1 \le i \le n$.
The edges are labeled according to the definition 1. The edge labelings are distinct. Hence $f$is vertex odd power mean labeling. Hence the circuit $Cn$is vertex odd power mean graph.

Example 2.1: A vertex odd power mean labeling of C_6 is given in Figure 2.

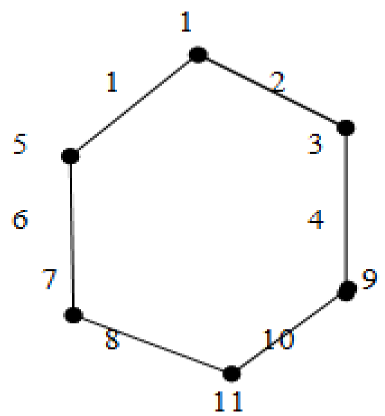

Figure 2: Vertex odd power mean labeling of the Cycle C_6

Theorem 2.2: The star graph S_n is vertex odd power mean graph.

Proof: The star graph Sis having one vertex in common. There are n vertices and (n-1) edges.

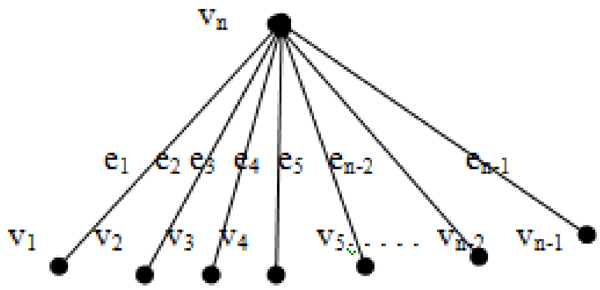

Figure 3: Odd Power Mean labeling of the star S_n

Define a function f: $(Cn) \to \{1,3,\ldots,2q - 1\}$ by
$f(x) = v_i = 2i$-1 ; $1 \le i \le n$.

Hence the edge labeling will be $f(e = uv) = e_1 = 1$, and $f(e_i) = 2i$, $i = 2, 3, 4, \ldots, n-1$

The edges are labeled according to the definition 1. The edge labelings are distinct. Hence $f$is vertex odd power mean labeling. Hence the circuit $Cn$is vertex odd Power mean graph.

Example 2.2:A vertex odd power mean labeling of S_6 is given in Figure 4.

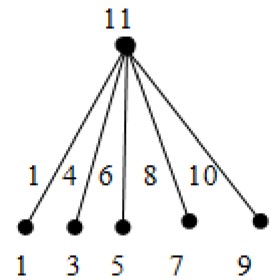

Figure 4: Vertex odd power mean labeling of the Star S_n

Theorem 2.3: The connected graph $C_n + 2P_2$ is vertex odd power mean graph.

Proof: The connected graph$C_n + 2P_2$ has n + 2vertices and edges in common. There are n vertices and n edges.

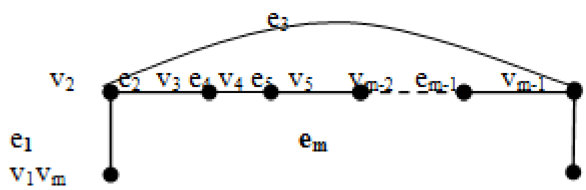

Figure 5: Vertex Odd Power Mean labeling of the graph $C_n + 2P_2$

Define a function $f: V(C_n + 2P_2) \rightarrow \{1,3,\ldots,2q - 1\}$ by
$f(x) = v_i = 2i-1$; $1 \leq i \leq n$.

The edges are labeled according to the definition 1. The edge labelings are distinct. Hence $f$is vertex odd power mean labeling. Hence the circuit $C_n + 2P_2$ is vertex odd Power mean graph.

Example 2.3:A vertex odd power mean labeling of the graph $C_5 + 2P_2$is given in Figure 6.

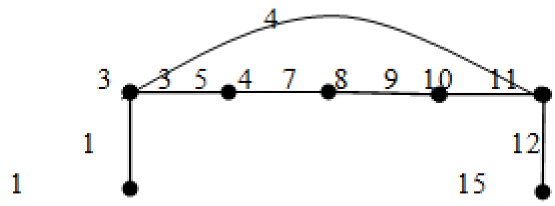

Figure 1.14: Vertex Odd Power Mean labeling of the graph $P_m + C_5$

Example 1.8
The connected graph $P_7 + C_5$is a vertex odd power mean graph.

It has 7 vertices and 7 edges. The graph is labeled as per figure 5.7 and is given below:

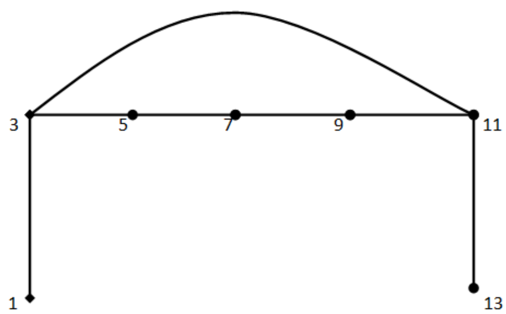

Figure 6: Vertex Odd Power Mean labeling of the graph $C_5 + 2P_2$

Conclusion

The study of labeled graph is important due to its diversified applications. It is very interesting to investigate graphs which admit vertex odd power mean labeling. In this paper, we proved for vertex odd power mean labeling for the graphs Cycle, Star, the connected graph $C_5 + 2P_2$and the suitable examples are also provided. We further investigate similar results for other graphs.

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