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Significance of Area of Obscuration and Derivation of General Expression by Method of Integration

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ABSTRACT

In a number of Engineering applications, there are certain governing variables that bear the potential to possess a major part of the control towards the success of the process. Some of these variables may be independent while others are dependent on each other is some or the other manner. In applications involving metered quantity of fluid flow, radiation heat transfer and so on, one of such governing variables is the flow rate. Flow rate may refer to that of any fluid or radiation or solids in certain cases. As flow rate depends on the area of opening that is available for flow, it is customary to either determine the area of opening or that of obscuration. In this paper, initially the physical significance of area of obscuration in applications related to fluid flow, radiation heat transfer, packaging applications and photo sensors shall be elaborated to establish a premise for the later derived general expression for determination of the area of obscuration for a combination of two circles with different radii, with one placed at any arbitrary point in space. This general expression shall be deemed to be applicable for provided certain necessary condition is met with respect the relation between the radii of the two circles and the distance between their centres. The derived general formula shall be verified towards the end to confirm the validity for any arbitrary case.

Keywords: *Obscuration, area, integration, definite, modulus*

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A. Formation area of obscuration

It is known that obscuration is the area of shadow between two curves. In the context of circles, it is the area bounded by the arcs of the two circles.

it is necessary that for the area of obscuration to form, the radii of the two circles considered and the distance between their centres is in a proportion without which, there with be no points of intersection of the circles. Since two circles of any radii intersect at only two points, it is not that cumbersome a task to determine the coordinates of the points of intersection. The area bounded in this region is common to both the circles and is defined as the area of obscuration. It can be better understood as an eclipsed area.

B. Physical significance of area of obscuration

Obscuration may occur in various conditions. However, in applications involving fluid flow, it can play a major role in the effectiveness of the process by governing the flow rate at a given flow velocity. In the following paragraphs, four such applications are considered wherein the area of obscuration plays a vital role.

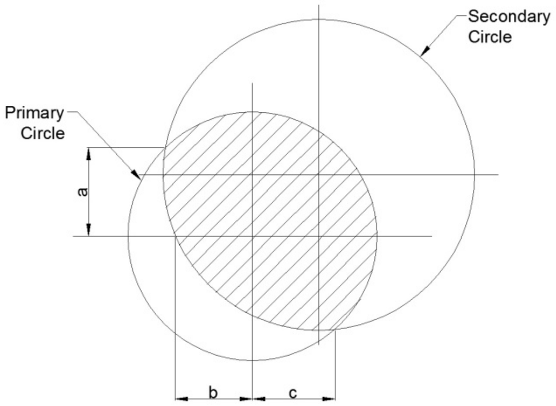
There are innumerable applications that involve flow through pipes. In these applications, in order to control the flow rate, industrial valves of various kinds are used which work on a common principle of allowing the fluid to flow by obscuring certain flow area. As the area of opening increases, the area obscuration diminishes. However for the success of the application, it is necessary that a metered quantity of fluid is allowed to flow. The area of opening can be related to the flow rate at given values of fluid velocity. Mathematically, the determination of the area of flow can aid to development of flow simulation software as well.

Another area of interest would be applications involving Radiation Heat Transfer. These may be related to the solar power plants, solar eclipses, Pyrometers and so on. In most of these cases, the key variable is the incoming radiation based on which the further process depends. According to the theory of thermal radiation, a surface emits or absorbs radiation according to its ability to do so which is termed as emissivity. However, in event this area emitting the radiation is obscures partially, there will be a difference in the outgoing radiation which would certainly lead to variations in the outputs. Such cases may be observed frequently in solar power plants in case of cloud cover and during the occurrence of solar eclipses. Pyrometers are devices that solely depend on the incoming radiation to generate faithful readings. Obscuration of incoming radiations can definitely lead to errors in the temperature outputs.

Packaging applications at times require a certain quantity of material to be filled into containers or bags within a certain time. This requires calculation of the area of opening required to facilitate the desired operation. Minor variations in this may lead to major errors in the material quantity, time to fill the container and so on which may have impact on the business.

In this digital era, day in and day out, photo sensors are encountered. These work on the basic principle of sensing the presence of light of required frequency or wavelength and carrying out further process based on the same. Like radiation heat transfer, these applications also depend on the magnitude of incoming radiation which is in turn dependent on the presence or absence of the area of obscuration.

C. Arrangement and necessary conditions for existence of area of obscuration



Consider two circles as shown in the figure. Let the primary circle be placed with its centre at the origin (0,0) with a radius 'r'. Let the secondary circle be placed at any arbitrary point in the space (h,k) with a radius 's'. In order to derive the equations in this case, let us consider that the point (h,k) is in the first quadrant.

Equation of the Primary circle is as follows.

$$x^2 + y^2 = r^2 \quad .....(1)$$

$$\therefore \mathbf{x} = \sqrt{\mathbf{r}^2 - \mathbf{y}^2} \quad .....(2)$$

$$\therefore \mathbf{y} = \sqrt{\mathbf{r}^2 - \mathbf{x}^2} \quad .....(3)$$

Equation of the Secondary circle is as follows.

$$(x-h)^2 + (y-k)^2 = s^2 \quad .....(4)$$

$$\therefore \mathbf{x} = \mathbf{h} + \sqrt{\mathbf{s}^2 - (\mathbf{y}-\mathbf{k})^2} \quad .....(5)$$

$$\therefore \mathbf{y} = \mathbf{k} + \sqrt{\mathbf{s}^2 - (\mathbf{x}-\mathbf{h})^2} \quad .....(6)$$

These expressions shall be useful while determining the area of obscuration by method of integration.

Accordingly, the first quadrant of the primary circle is completely obscured while the second, third and fourth quadrants are partially obscured.

In the second quadrant, let the shadow of the point of intersection of the circles on the ordinate be 'a'.

Let the point of intersection of the secondary circle and the abscissa within the primary circle be at a distance 'b' from the origin.

In the fourth quadrant, let the shadow of the point of intersection of the circles on the abscissa be 'c'.

Thus having defined the various points of interest, the necessary and sufficient conditions to derive the equation are formulated as follows.

Necessary condition refers to a mandatory requirement without which, one cannot proceed further while sufficient conditions refer to various conditions that can facilitate the process of derivation.

Necessary condition for the existence of an area of obscuration is that the distance between the centres of the primary and the secondary circles shall be lesser than the sum of their radii.

$$\therefore \sqrt{\mathbf{h}^2 + \mathbf{k}^2} < r + s \quad .....(7)$$

Sufficient condition for the derivation is the knowledge of equations of both the curves with the help of which, the area can be determined mathematically.

D. Derivation of general expression for area of obscuration

In this process, let the shaded area be the sum of shaded areas in each quadrant.

$$\therefore \mathbf{A} = \mathbf{A1} + \mathbf{A2} + \mathbf{A3} + \mathbf{A4} \quad .....(8)$$

where the suffixes 1 to 4 represent the respective quadrants. Using the symbols as defined in the previous section, the general expression is derived as follows.

In the first quadrant, complete area of the primary circle is obscured. Mathematical expression for which is as follows.

$$\mathbf{A1} = \int_0^r \left(\sqrt{\mathbf{r}^2 + \mathbf{x}^2}\right) \mathbf{dx} \quad .....(9)$$

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

$$\mathbf{A1} = \frac{\pi \mathbf{r}^2}{4} \quad .....(10)$$

This is equal to a quarter of the area of the primary circle.

In the second quadrant, the shaded area can be evaluated by integrating the area between the curves and the ordinate. The limits of integration shall be first from the origin to the point 'a' and then from the point 'a' to the radius 'r'. The value of 'a' can be determined by solving the two equations of circles.

Mathematical expression is as follows.

$$\mathbf{A2} = \int_0^a \left(\mathbf{h} + \sqrt{\mathbf{s}^2 - (\mathbf{y}-\mathbf{k})^2}\right) \mathbf{dy} + \int_a^r \left(\sqrt{\mathbf{r}^2 - \mathbf{y}^2}\right) \mathbf{dy}$$

.....(11)

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

$$\mathbf{A2} = \frac{\pi \mathbf{r}^2}{4} + \mathbf{ha} + \left\{\frac{\mathbf{a}-\mathbf{k}}{2}\sqrt{\mathbf{s}^2 - (\mathbf{a}-\mathbf{k})^2} + \frac{\mathbf{k}}{2}\sqrt{\mathbf{s}^2 - \mathbf{k}^2} - \frac{\mathbf{a}}{2}\sqrt{\mathbf{r}^2 - \mathbf{a}^2}\right\} + \left\{\frac{\mathbf{s}^2}{2}\left(\sin^{-1}\frac{\mathbf{a}-\mathbf{k}}{\mathbf{s}} + \sin^{-1}\frac{\mathbf{k}}{\mathbf{s}}\right) - \frac{\mathbf{r}^2}{2}\sin^{-1}\frac{\mathbf{a}}{\mathbf{r}}\right\}$$

.....(12)

In the third quadrant, The curve indicating the secondary circle crosses the abscissa at a point b.

In order to find coordinates of the point 'b', put y=0 in (4). On simplification, it is found that,

$$\mathbf{b = h + s^2 - k^2} \qquad .....(13)$$

This value can be used in the actual determination when the equations of the circles are known.

Mathematical expression is as follows

$$\mathbf{A3 = \int_0^b (k + \sqrt{s^2 - (x-h)^2})\,dx} \qquad .....(14)$$

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

$$\mathbf{A3 = hb + \left\{\frac{b-h}{2}\sqrt{s^2-(b-h)^2} + \frac{h}{2}\sqrt{s^2-h^2}\right\} + \left\{\frac{s^2}{2}\left(\sin^{-1}\frac{b-h}{s} + \sin^{-1}\frac{h}{s}\right)\right\}}$$

.....(15)

As a general observation, Equation (15) is similar to Equation (12) considering similar variables sans the primary circle radius term as it does not come into the picture.

In the fourth quadrant, the shaded area can be evaluated by integrating the area between the curves and the abscissa. The limits of integration shall be first from the origin to the point 'c' and then from the point 'c' to the radius 'r'. The value of 'c' can be determined by solving the two equations of circles.

Mathematical expression is as follows.

$$\mathbf{A4 = \int_0^c (k + \sqrt{s^2-(x-h)^2})\,dx + \int_c^r (\sqrt{r^2-x^2})\,dx}$$

.....(16)

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

$$\mathbf{A4 = \frac{\pi r^2}{4} + kc + \left\{\frac{c-h}{2}\sqrt{s^2-(c-h)^2} + \frac{h}{2}\sqrt{s^2-h^2} - \frac{c}{2}\sqrt{r^2-c^2}\right\} + \left\{\frac{s^2}{2}\left(\sin^{-1}\frac{c-h}{s} + \sin^{-1}\frac{h}{s}\right) - \frac{r^2}{2}\sin^{-1}\frac{c}{r}\right\}} \qquad ....(17)$$

As a general observation, Equation (17) is also similar to Equation (12) considering similar variables.

The sum of areas obtained till this point shall give the total shaded area according to Equation (8). The simplified form of the general expression for area of obscuration by method of integration is as follows.

$$\mathbf{A = \frac{3\pi r^2}{4} + (ha + kb + kc) + \left(\frac{a-k}{2}\sqrt{s^2-(a-k)^2} + \frac{b-h}{2}\sqrt{s^2-(b-h)^2} + \frac{c-h}{2}\sqrt{s^2-(c-h)^2}\right) + \left(\frac{k}{2}\sqrt{s^2-k^2} + h\sqrt{s^2-h^2}\right) + \left(-\frac{a}{2}\sqrt{r^2-a^2} - \frac{c}{2}\sqrt{r^2-c^2}\right) + \left(\frac{s^2}{2}\left(\sin^{-1}\frac{a-k}{s} + \sin^{-1}\frac{k}{s} + \sin^{-1}\frac{b-h}{s} + 2\sin^{-1}\frac{h}{s}\sin^{-1}\frac{c-h}{s}\right)\right) - \left(\frac{r^2}{2}\left(\sin^{-1}\frac{a}{r} + \sin^{-1}\frac{c}{r}\right)\right)} \qquad .....(18)$$

Is the required expression considering all the intercepts and areas enclosed by the curves.

E. Verification of derived formula

Every general formula needs to be verified. This validates the formula and makes it capable of taking any values for the variables therein for a given family of curves in event the values of constants are not known.

Thus for verification, if in Equation (18),

h=0, k=0, a=b=c=s=r, The secondary circle reduces to the primary circle. In that event, the area as evaluated by the derived formula should be equal to that of the complete primary circle.

on substituting the conditional values in Equation (18), following is obtained.

$$\mathbf{A = \frac{3\pi r^2}{4} + (0) + (0) + (0) + (0) + \left(\frac{r^2}{2}\left(\frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2}\right)\right) - \left(\frac{r^2}{2}\left(\frac{\pi}{2} + \frac{\pi}{2}\right)\right)}$$

On simplification,

$$\therefore \mathbf{A = \pi r^2}$$

which validates the derived formula and thus claims that the formula can be used for determination of area of obscuration between two circles with one fixed at the origin and the other with centre at any arbitrary point in space for the arrangement shown in the figure.

Conclusion

Thus it can be concluded that by the method of integration the area of obscuration has been found out for a combination of two circles, one with centre at the origin and the other with centre at an arbitrary point. The necessary condition for existence has also been mentioned in addition to the physical significance of area of obscuration for a few applications. The formula has been verified and validated using an arbitrary condition. Any other case shall be undertaken for evaluation in a similar manner as with the verification, the methodology is validated as well.

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