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Determination of Area of Obscuration between Two Circles by Method of Integration

Gourav Vivek Kulkarni

B.E. (Mechanical), KLS Gogte Institute of Technology, Belagavi, Karnataka, India

ABSTRACT

Obscuration is the phenomenon of covering of a certain percentage of area of an object by another. It can be a governing variable in radiation heat transfer, fluid dynamics and so on. Although modern day analytical tools allow determination of area of obscuration, the non availability of handy formulae for theoretical calculations has lead to this study. In this study, the method of integration shall be used to determine the area of obscuration for various cases. Two circles of different radii shall be considered for this study. Various positions shall be considered and based on the combinations obtained, expressions for the area of obscuration shall be derived. This study aims at enumerating and exploring various cases so as to pave way towards a further study of general formulae to arrive at required expressions. Thus the theoretically obtained area can be used in various calculations as and when found of necessity.

Keywords: *Obscuration, area, integration, definite, modulus*

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A. Mathematical function

A mathematical function is defined as follows[3]

Let A and B be two non-empty sets. A function or mapping from f from A to B is a rule which associates every element of A with a unique element of B and is denoted by

f: A -> B

Thus every value of Set A has a correspondence with Set B by virtue of which, a relation can be developed and can be represented pictorially in the form of a curve. In order to establish measurement of various parameters of the curve, Cartesian coordinate system shall be used in this study.

Considering Cartesian coordinate system, there is a relation between the dependent variable y and the independent variable x. Thus x can take any values based on which, the value of y is obtained corresponding to the function.

B. Increments

For every value x, δx added to it is called as an increment to x. Thus $x+\delta x$ is an incremented value of the independent variable corresponding to which, the dependent variable can be mapped as $y+\delta y$. The increment is infinitesimally small and is thus considered to keep the overall value of the function unaffected in the neighbourhood of the point (x,y) mapped within the incremental limits. With these, one can define the derivative of a function.

C. Derivative of a function and its significance

By definition,

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{f(x+\delta x)-f(x)}{\delta x}$$

is the derivative of a function y=f(x)

D. Integral of a function

By definition,

If y is a function of x denoted by y=f(x),

Then f '(x) is the derivative of the function.

The integral of f '(x) is defined as $\int \mathbf{f}'(\mathbf{x}) = \mathbf{f}(\mathbf{x})$

E. Determination of Area under a curve by method of integration

An integral represents the sum of the ordinates of the curve corresponding to the relation established with the abscissa. Thus the sum such infinitesimal areas collectively represents the area under the curve.

The area under a required curve is evaluated with respect to a particular axis. In the context of this study, this shall be done with respect to the abscissa for all cases to maintain uniformity.

F. Formulation of expressions to determine area of obscuration between two circles

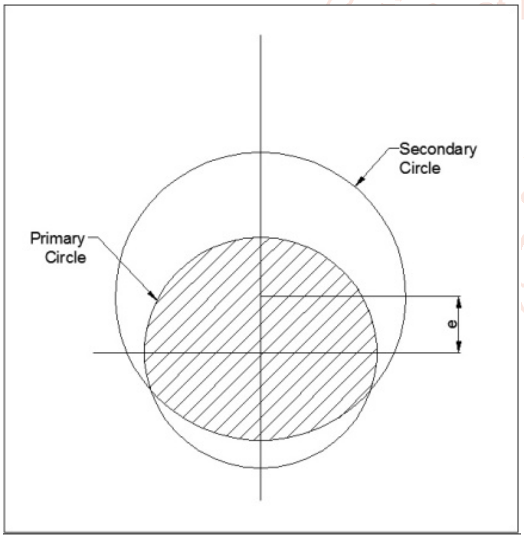
Thus by method of integration, the areas of obscuration between two circles shall be determined.

The methodology involves algebraic summation of individual areas with respect to the abscissa.

A primary circle with radius 'r' and centre at origin i.e. (0,0) and a secondary circle with radius 's' and centre at (0,e) shall be considered. Various cases shall be considered assuming the movement of the secondary circle along the ordinate. The primary circle shall therefore be fixed with its centre at the origin. The horizontal shadow on abscissa of point of intersection of the curves shall be represented by 'a' unless otherwise specified. Thus the limits of integration in most of the cases for this consideration shall be the origin i.e. (0,0) and (a,0).

Considering different combinations of radii of the primary and secondary circles, various positions and the area of obscuration shall be determined.

F.A. Primary circle smaller than secondary circle with point of intersection of curves below abscissa



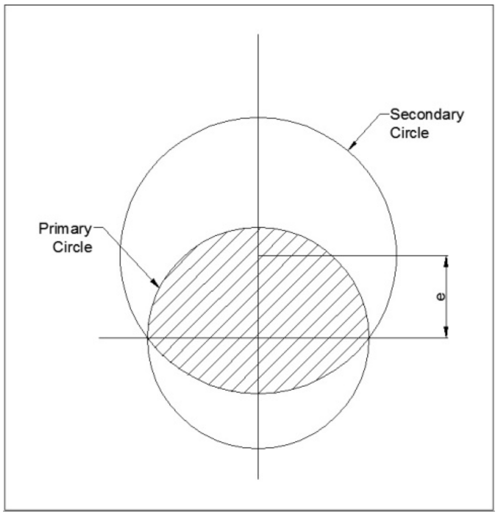
Mathematical expression for the shaded area in terms of the variables is as follows

$$\text{Shaded Area} = \frac{\pi r^2}{2} + 2\int_0^a \left(e + \sqrt{s^2 - x^2}\right) dx + 2\int_a^r \left(\sqrt{r^2 - x^2}\right) dx$$

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

$$A1 = \pi r^2 + 2ea + a\left(\sqrt{s^2 - a^2} - \sqrt{r^2 - a^2}\right) + s^2 \sin^{-1}\frac{a}{s} - r^2 \sin^{-1}\frac{a}{r} \quad \text{.....(1)}$$

F.B. Primary circle smaller than secondary circle with point of intersection of curves on abscissa



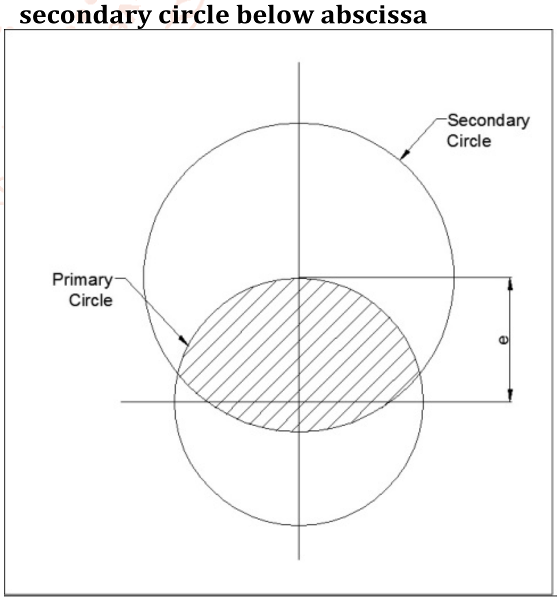
Mathematical expression for the shaded area in terms of the variables is as follows

$$\text{Shaded Area} = \frac{\pi r^2}{2} + 2\int_0^r \left(e + \sqrt{s^2 - x^2}\right) dx$$

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

$$A2 = \frac{\pi r^2}{2} + 2er + r\sqrt{s^2 - r^2} + s^2 \sin^{-1}\frac{r}{s} \quad \text{.....(2)}$$

F.C. Primary circle smaller than secondary circle with point of intersection of curves above abscissa and point on circumference of secondary circle below abscissa



Let the point of intersection of the curve indicating the secondary circle and the abscissa in the first quadrant be (b,0)

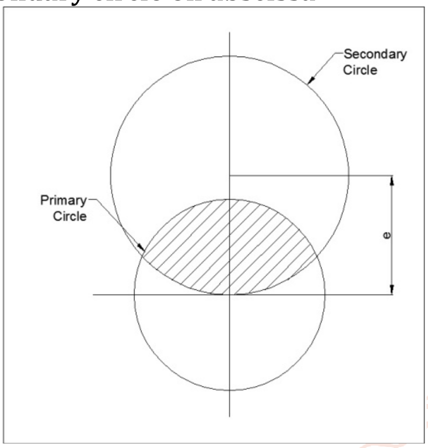
Mathematical expression for the shaded area in terms of the variables is as follows

$$\text{Shaded Area} = 2\int_0^b \left(e + \sqrt{s^2 - x^2}\right) dx + \frac{\pi r^2}{2} - 2\int_b^a \left(e + \sqrt{s^2 - x^2}\right) dx - 2\int_a^r \left(\sqrt{r^2 - x^2}\right) dx$$

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

$$A3 = (4eb - 2ea) + \left(2b\sqrt{s^2 - b^2} + a\left(\sqrt{r^2 - a^2} - \sqrt{s^2 - a^2}\right)\right) + \left(2s^2 \sin^{-1}\frac{b}{s} + r^2 \sin^{-1}\frac{a}{r} - s^2 \sin^{-1}\frac{a}{s}\right)$$
.....(3)

F.D Primary circle smaller than secondary circle with point of intersection of curves above abscissa and point on circumference of secondary circle on abscissa



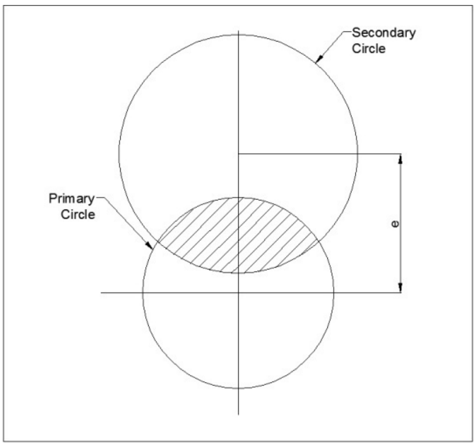
Mathematical expression for the shaded area in terms of the variables is as follows

$$\textbf{Shaded Area} = \frac{\pi r^2}{2} - 2\int_0^a \left(e + \sqrt{s^2 - x^2}\right) dx - 2\int_a^r \left(\sqrt{r^2 - x^2}\right) dx$$

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

$$A4 = -2ea + a\left(\sqrt{r^2 - a^2} - \sqrt{s^2 - a^2}\right) + r^2 \sin^{-1}\frac{a}{r} - s^2 \sin^{-1}\frac{a}{s} \quad .....(4)$$

F.E. Primary circle smaller than secondary circle with point of intersection of curves above abscissa



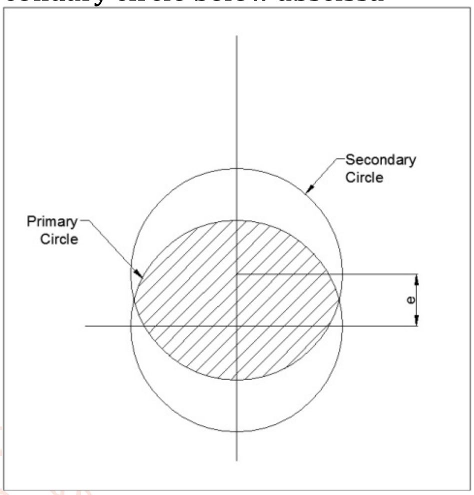
Mathematical expression for the shaded area in terms of the variables is as follows

$$\textbf{Shaded Area} = \frac{\pi r^2}{2} - 2\int_0^a \left(e + \sqrt{s^2 - x^2}\right) dx - 2\int_a^r \left(\sqrt{r^2 - x^2}\right) dx$$

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

$$A5 = -2ea + a\left(\sqrt{r^2 - a^2} - \sqrt{s^2 - a^2}\right) + r^2 \sin^{-1}\frac{a}{r} - s^2 \sin^{-1}\frac{a}{s} \quad .....(5)$$

F.F. Primary circle of same radius as secondary circle with point of intersection of curves above abscissa and point on circumference of secondary circle below abscissa



Let the point of intersection of the curve indicating the secondary circle and the abscissa in the first quadrant be (c,0)

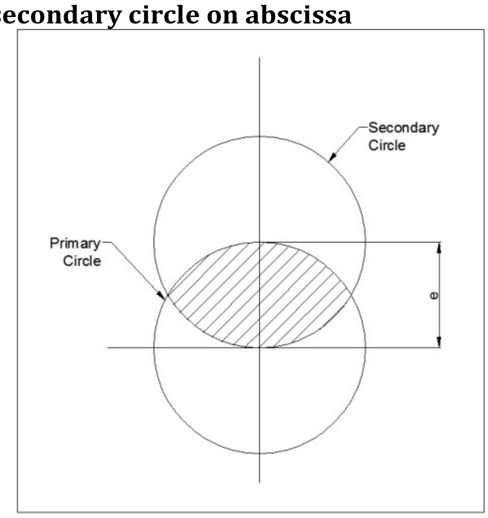
Mathematical expression for the shaded area in terms of the variables is as follows

$$\textbf{Shaded Area} = 2\int_0^c \left(e + \sqrt{r^2 - x^2}\right) dx + \frac{\pi r^2}{2} - 2\int_b^a \left(e + \sqrt{r^2 - x^2}\right) dx - 2\int_a^r \left(\sqrt{r^2 - x^2}\right) dx$$

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

$$A6 = (2ec - 2ea) + \left(2c\sqrt{r^2 - c^2}\right) + \left(2r^2 \sin^{-1}\frac{c}{r}\right)$$
.....(6)

F.G. Primary circle of same radius as secondary circle with point of intersection of curves above abscissa and point on circumference of secondary circle on abscissa



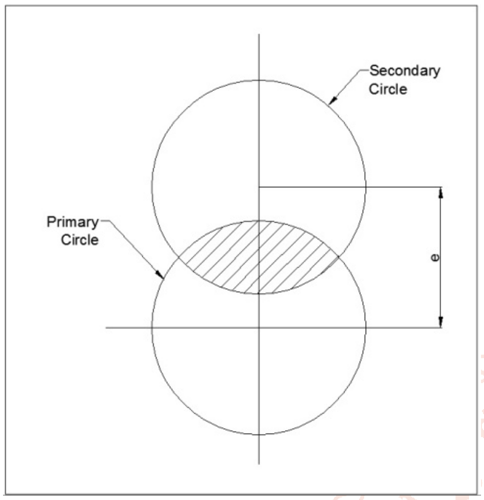
Mathematical expression for the shaded area in terms of the variables is as follows

$$\textbf{Shaded Area} = \frac{\pi r^2}{2} - 2\int_0^a \left(e + \sqrt{r^2 - x^2}\right) dx - 2\int_a^r \left(\sqrt{r^2 - x^2}\right) dx$$

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

A7 $= 2ea$ .....(7)

F.H. Primary circle of same radius as secondary circle with point of intersection of curves above abscissa



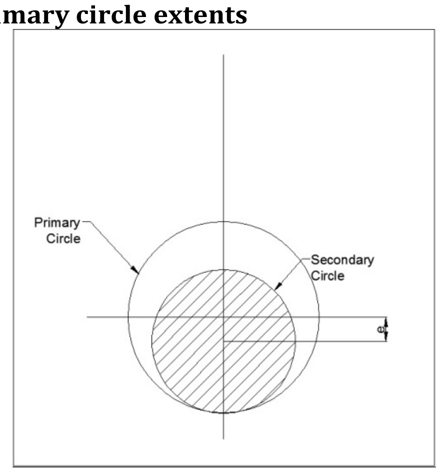
Mathematical expression for the shaded area in terms of the variables is as follows

$$\textbf{Shaded Area} = \frac{\pi r^2}{2} - 2\int_0^a \left(e + \sqrt{r^2 - x^2}\right) dx - 2\int_a^r \left(\sqrt{r^2 - x^2}\right) dx$$

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

A8 $= 2ea$ .....(8)

F.I. Primary circle larger than secondary circle with secondary circle completely within primary circle extents



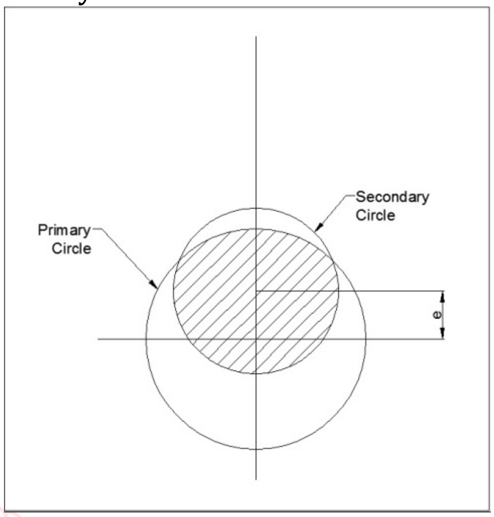
Without loss of accuracy, the secondary circle can be assumed to be placed with centre at the origin. Mathematical expression for the shaded area in terms of the variables is as follows

$$\textbf{Shaded Area} = 4\int_0^s \left(\sqrt{s^2 - x^2}\right) dx$$

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

A9 $= \pi s^2$ .....(9)

F.J. Primary circle larger than secondary circle with major arc of secondary circle within the primary circle



Let the point of intersection of the curve indicating the secondary circle and the abscissa in the first quadrant be (b,0)

Mathematical expression for the shaded area in terms of the variables is as follows

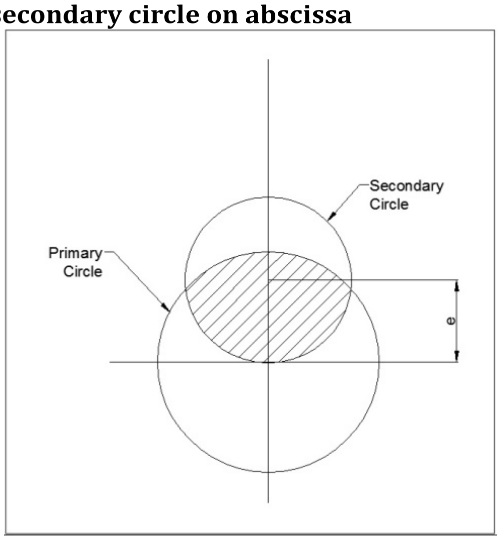
$$\textbf{Shaded Area} = 2\int_0^b \left(e + \sqrt{s^2 - x^2}\right) dx + \frac{\pi r^2}{2} - 2\int_b^a \left(e + \sqrt{s^2 - x^2}\right) dx - 2\int_a^r \left(\sqrt{r^2 - x^2}\right) dx$$

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

$$\mathbf{A10} = (4eb - 2ea) + \left(2b\sqrt{s^2 - b^2} + a\left(\sqrt{r^2 - a^2} - \sqrt{s^2 - a^2}\right)\right) + \left(2s^2 \sin^{-1}\frac{b}{s} + r^2 \sin^{-1}\frac{a}{r} - s^2 \sin^{-1}\frac{a}{s}\right)$$

.....(10)

F.K. Primary circle larger than secondary circle with point of intersection of curves above abscissa and point on circumference of secondary circle on abscissa



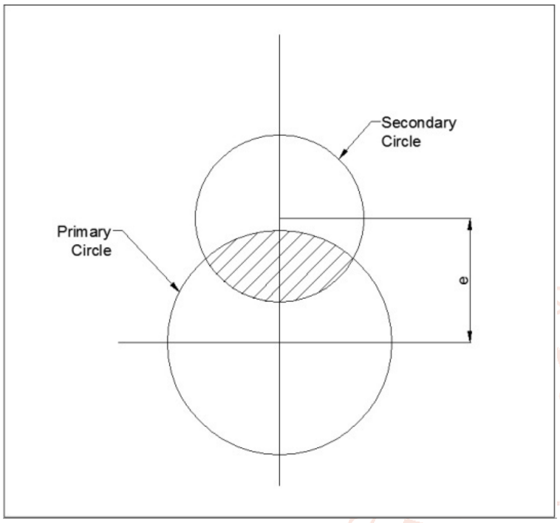
Mathematical expression for the shaded area in terms of the variables is as follows

$$\textbf{Shaded Area} = \frac{\pi r^2}{2} - 2\int_0^a \left(e + \sqrt{s^2 - x^2}\right) dx - 2\int_a^r \left(\sqrt{r^2 - x^2}\right) dx$$

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

$$A11 = -2ea + a\left(\sqrt{r^2 - a^2} - \sqrt{s^2 - a^2}\right) + r^2 \sin^{-1}\frac{a}{r} - s^2 \sin^{-1}\frac{a}{s} \quad .....(11)$$

F.L. Primary circle larger than secondary circle with point of intersection of curves above abscissa



Mathematical expression for the shaded area in terms of the variables is as follows

$$\textbf{Shaded Area} = \frac{\pi r^2}{2} - 2\int_0^a \left(e + \sqrt{s^2 - x^2}\right) dx - 2\int_a^r \left(\sqrt{r^2 - x^2}\right) dx$$

On evaluating the integral, applying the limits and on further simplification, the following expression is obtained which indicates the shaded area.

$$A12 = -2ea + a\left(\sqrt{r^2 - a^2} - \sqrt{s^2 - a^2}\right) + r^2 \sin^{-1}\frac{a}{r} - s^2 \sin^{-1}\frac{a}{s} \quad .....(12)$$

Conclusion

Thus it can be concluded that by the method of integration the area of obscuration has been found out for a combination of two circles considering three cases of different radii conditions. These may be handy with knowledge of the equations of the circles to determine various parameters like fluid flow rate, rate of radiation emitted from an obscured surface, time taken to empty a tank filled with a fluid and so on. The analysis can be further extended to generalize a formula which shall encompass any location and magnitude of radii of the pair of circles.

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