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Application of Arithmetic Progression in Equilateral Triangle as Mathematics Brain Teaser

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ABSTRACT

In a classroom situation generally homogeneous teaching goes for heterogeneous groups of students. Mathematically gifted students always require some differentiated instruction from those of other students because they demonstrate an uneven pattern of mathematical understanding and development. Though they are endowed with high computation skills, problem-solving strategies, inferential thinking skills, or deductive reasoning and are often able to discern answers with unusual speed and accuracy even smart enough in finding the inter-relationships among and between ideas, topics, and concepts without the intervention of formal instruction specifically geared to that particular content, yet several factors never allow them to nourish their creativity as teachers are overloaded, non friendly mathematics curriculum, sometimes lack of parental support. To address this issue a mathematical brain teaser typically designed to stimulate cognitive processes and involves lateral logical thinking of those gifted students in order to solve the teaser for amusement even. Actually in a specially designed equilateral triangle (Several equal number of intra parallel lines from each side of triangle with same distance between the parallels) produces many triangles, parallelograms, trapeziums, rhombus, and regular hexagons. Gifted students may count those Geometrical figures faster than the others and those answers can be justified by using formulas developed by me. Those formulas are developed with the help of simple concept of arithmetic progression.

Example: (Only for one geometrical figure parallelogram)
Total number of parallelograms inside largest triangle = $6[1(n-2)^2+2(n-4)^2+3(n-6)^2+.....]$
n *is number of parallel lines from each side of triangle (Negative number within the first bracket is neglected)*

It would help to develop and improve cognitive functions via logical reasoning of those gifted students.

KEYWORDS: *Gifted student, Arithmetic progression, Mathematical brain teaser*

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1. INTRODUCTION

Several gifted students and prodigies are not provided healthy instructions as per their mental heights by their mathematics teacher. They are completely different from others in three key areas viz. pace of learning, depth of comprehension, and the high interest they are endowed with. It is a grave problem in our system of education. There is a myth that gifted students and prodigies do not required special attention as they are well versant with harder problems of mathematics and is easy to learn by them. On the contrary they need dictate curriculum that is more than a mile wide and an inch deep, and faster that would push them into new realms of understanding than what is delivered to other students. Due to intuitive understanding of mathematical function and process, often they skip over steps and are able to explain how they arrive at the correct answer to the problem. It is a mathematical brain teaser typically designed to stimulate cognitive processes and involves lateral logical thinking of those students in order to solve the teaser for amusement even. It would help to develop and improve cognitive functions via logical reasoning of those prodigies and gifted students.

2. OBJECTIVES OF THE STUDY

A. To promote critical thinking and reasoning abilities in new and novel ideas of mathematic.
B. To protect and enrich the tremendous interest in mathematics before being snuffed out early by unhealthy influence of material and non-material elements.
C. To develop and expand thinking skills.
D. To utilize differentiated strategies for learning mathematics.

3. BODY OF THE WORK

Inside this largest equilateral triangle there may be several smaller triangles, rhombus, parallelograms, trapeziums and regular hexagons. Students have to count it traditionally first and the result may be confirmed with the help of formulas. To obtain those formulas the simple concept of AP was applied. Where n= Number of parallel lines from any base. And for this left side figure n= 3 (This can be extend).

Size of the equilateral triangle is flexible. Number of parallel lines from any base may be infinite, i.e. **n = {1, 2, 3, 4 …∞}.**

§ For triangle (△Equilateral)

Total number of triangles formed by adjacent $1^1, 2^2, 3^2, 4^2, 5^2$,… triangle(s) shaping a triangle	
n=10	(1+2+…+10)+**2**(1+2+…+9)+(1+2+…+8)+**2**(1+2+…+7)+(1+2+…+6)+**2**(1+2+…+5)+….+**2**(1)
n=9	(1+2+…+9)+**2**(1+2+…+8)+ (1+2+…+7)+**2**(1+2+…+6)+ (1+2+…+5)+ **2**(1+2+3+4)**….**+1
n=8	(1+2+…+8)+**2**(1+2+…+7)+(1+2+…+6)+**2**(1+2+…+5)+(1+2+3+4)+….+**2**(1)
n=7	(1+2+…+7)+**2**(1+2+…+6)+ (1+2+…+5)+**2**(1+2+3+4)+(1+2+3)+….+1
n=6	(1+2+…+6)+**2**(1+2+…+5)+(1+2+3+4)+**2**(1+2+3)+….+**2**(1)
n=5	(1+2+…+5)+**2**(1+2+3+4)+(1+2+3)+**2**(1+2)+….+1
n=4	(1+2+3+4)+**2**(1+2+3)+(1+2)+**2**(1)
n=3	(1+2+3)+**2**(1+2)+ 1
n=2	(1+2)+**2**(1)

Number of terms in that sequence = n

1. **Total number of triangles inside the largest triangle =** $\frac{2n^3+5n^2+2n}{8}$ **(When "n" is even).**

2. **Total number of triangles inside the largest triangle =** $\frac{2n^3+5n^2+2n-1}{8}$ **(When "n" is odd).**

§ For rhombus (▱ considered only whose four sides are congruent)

Total number of rhombus formed by adjacent $2.1^1, 2.2^2, 2.3^2$, … triangle(s) shaping rhombus	
n=10	3{(1+2+3+….+8+9)+(1+2+3+…+6+7)+(1+2+3+4+5)+(1+2+3)+1}
n=9	3{(1+2+3+…+7+8)+(1+2+…+5+6)+(1+2+3+4)+(1+2)}
n=8	3{(1+2+3+…+6+7)+(1+2+3+4+5)+(1+2+3)+1}
n=7	3{(1+2+3+4+5+6)+(1+2+3+4)+(1+2)}
n=6	3{(1+2+3+4+5)+(1+2+3)+1}
n=5	3{(1+2+3+4)+(1+2)}
n=4	3{(1+2+3)+1}
n=3	3(1+2)
n=2	3(1)

Number of terms in that sequence = (n/2) when 'n' is even OR {(n-1)/2 when 'n' is odd

3. **Total number of rhombus inside the largest triangle =** $\frac{n(2n^2+3n-2)}{8}$ **(When "n" is even)**

4. **Total number of rhombus inside the largest triangle =** $\frac{(n-1)(2n^2+5n+3)}{8}$ **(When "n" is odd)**

§ For parallelogram (▱ considered only whose adjacent sides are not congruent)

For example the number of parallel lines from any base is taken here 7 (i.e. n=7). Here 'R' would be considered as rhombus to form the parallelogram.

While counting (parallelogram) form bottom to top from both left to right & right to left side					
Between two // lines (Left to right and right to left)	2(1+2+3+4+5) Under 2R	2(1+2+3+4) under 3R	2(1+2+3) under 4R	2(1+2) 5R	2(1) 6R
One line gap between two // lines (Left to right and right to left)	xx	xx	2(1+2+3) under 6R	2(1+2) 8R	2(1) 10R
Two lines gap between two // lines (Left to right and right to left)	xx	xx	xx	xx	2(1) 12R
Total number of parallelograms counting from bottom to top	= 2(1+2+3+4+5) + 2(1+2+3+4) + 4(1+2+3) + 4(1+2) + 6(1)				

Hence total number of parallelograms (counted from three different sides of the triangle) = 3 {2(1+2+3+4+5) + 2(1+2+3+4) + 4(1+2+3) + 4(1+2) + 6(1)}.

(For 7 parallel lines there would be at least one parallelogram containing maximum 12 smaller sized rhombus)
In general,

5. **Total number of parallelogram inside largest triangle = $6[1(n-2)^2+2(n-4)^2+3(n-6)^2+.....]$ (Negative number within the first bracket is neglected)**

§ For trapezium

For example the number of parallel lines from any base is taken here 7 (i.e. n=7). Here 'T' would be considered as adjacent triangles to form the trapezium.

While counting (Trapezium) from bottom to top						
Between two // lines	(1+2+3+4+5+6) under 3T	(1+2+3+4+5) under 5T	(1+2+3+4) under 7T	(1+2+3) under 9T	(1+2) U 11T	(1) U 13T
One line gap between two // lines	xx	(1+2+3+4+5) under 8T	(1+2+3+4) under 12T	(1+2+3) under 16T	(1+2) U 20T	(1) U 24T
Two lines gap between two // lines	xx	xx	(1+2+3+4) under 15T	(1+2+3) under 21T	(1+2) U 27T	(1) U 33T
Three lines gap between two // lines	xx	xx	xx	(1+2+3) under 24T	(1+2) U 32T	(1) U 40T
Four lines gap between two // lines	xx	xx	xx	xx	(1+2) U 35T	(1) U 45T
Five lines gap between two // lines	xx	xx	xx	xx	xx	(1) U 48T
Total number of trapezium from bottom to top	= (1+2+3+4+5+6) + 2(1+2+3+4+5) + 3(1+2+3+4) + 4(1+2+3) +5(1+2) + 6(1)					

Hence total number of trapezium (counted from three different sides of the triangle) = 3{(1+2+3+4+5+6) + 2(1+2+3+4+5) + 3(1+2+3+4) + 4(1+2+3) +5(1+2) + 6(1)}
(For 7 parallel lines there would be at least one trapezium containing maximum 48 smallest sized triangle)

In general,

6. **Total number of trapezium inside largest triangle = $\frac{3n(2n^2-n-2)}{8}$ (When "n" is even)**

7. **Total number of trapezium inside largest triangle = $\frac{3(n-1)(2n^2+n-1)}{8}$ (When "n" is odd)**

§ For regular hexagon

For example the number of parallel lines from any base is taken here 11 (i.e. n=11). Here 'T' would be considered as adjacent triangles to form the regular hexagon.

While counting (regular hexagon) with →	$6x1^1$ smallest triangles	$6x2^2$ smallest triangles	$6x3^2$ smallest triangles
One line gap between two // lines	(1+2+.....+9)	xx	xx
Three lines gap between two // lines	xx	(1+2+.....+6)	xx
Five lines gap between two // lines	xx	xx	(1+2+3)
Total number of regular hexagon	= (1+2+....+8+9) + (1+2+....+5+6) + (1+2+3)		

Hence total number of regular hexagon = {(1+2+..... +8+9) + (1+2+...+6) + (1+2+3)}

(For 11 parallel lines there would be at least six regular hexagon containing maximum $6 \times 3^2 = 54$ smallest sized triangles)

In general,

8. **Total number of regular hexagon inside the largest triangle = ½ [(n-1)(n-2)+(n-4)(n-5)+(n- 7)(n-8)+(n-10)(n-11)+(n-13)(n-14)+…………………] (When "n" is a natural number) (Negative number within the first bracket is neglected)**
9. **Total number of regular hexagon inside largest triangle = $\frac{n(n2-3)}{8}$ (When 'n' is multiple of 3)**

4. DISCUSSION

All the geometrical two dimensional figures (i.e. equilateral triangles, rhombus whose four sides are congruent, parallelogram whose adjacent sides are not same, trapezium, and regular hexagon) formed by parallel lines kept in equal distance inside the largest equilateral triangle, can be counted traditionally as well as by using the formulas. Those formulas are deduced from the simple application of the AP's (Arithmetical Progression) formulas.

5. CONCLUSION

In secondary level mathematics class generally homogeneous instructional practices are disseminated to heterogeneous groups of students, their individual characteristics, needs, abilities, and interest are personal and unique. Mathematically gifted, prodigy, and savant are always endowed with remarkable spectacular talent and always reflect high computation skills, problem-solving strategies, inferential thinking skills, or deductive reasoning and are often able to discern answers with unusual speed and accuracy. Essentially an effective basic curriculum for those students who are mathematically gifted, prodigy, and savant are to be developed to meet their needs. But several factors never allow them to nourish their creativity as teachers are overloaded, non friendly mathematics curriculum, sometimes lack of parental support. In order to eradicate these problems a small effort is made to fulfill the demands of that group under the title 'Application of arithmetic progression in equilateral triangle as mathematics brain teaser', which can be used according to their pace, place and time.

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