International Journal of Trend in Scientific Research and Development (IJTSRD)
Volume 4 Issue 5, July-August 2020 Available Online: www.ijtsrd.com e-ISSN: 2456 – 6470

Some New Operators on Multi Intuitionistic Fuzzy Soft Matrix Theory

P. Rajarajeswari[1], J. Vanitha[2]

[1]Assistant Professor of Mathematics, Chikkanna Government Arts College, Tirupur, Tamil Nadu, India
[2]Assistant Professor of Mathematics, RVS College of Education, Sulur, Coimbatore, Tamil Nadu, India

ABSTRACT

In this paper, we have defined four new operators namely □, ◊, ⨁ , ⨂ on a new type of matrix namely Multi Intuitionistic Fuzzy soft Matrix were defined and some of their properties are studied. The concepts are illustrated with suitable numerical examples.

KEYWORDS: *Soft Set, Fuzzy Soft Set, Intuitionistic Fuzzy soft set, Multi-Fuzzy Soft Set*

How to cite this paper: P. Rajarajeswari | J. Vanitha "Some New Operators on Multi Intuitionistic Fuzzy Soft Matrix Theory" Published in International Journal of Trend in Scientific Research and Development (ijtsrd), ISSN: 2456-6470, Volume-4 | Issue-5, August 2020, pp.843-848, URL: www.ijtsrd.com/papers/ijtsrd33009.pdf

IJTSRD33009

1. INTRODUCTION

In real world problems we have uncertainties. Zadeh [11] in 1965, has introduced the concept namely Fuzzy sets to deal uncertainties which consists of degree of membership. Intuitionistic Fuzzy Sets are introduced by Atanasov[1,2] which are extension of Fuzzy Sets and consists of both membership value and non-membership value associated with every element. The concept Soft set theory have been introduced by Molodtsor [8] in 1999 and he also studied various properties of soft set. Representation of Soft sets in matrix form was given by Cagman et.al [5]. Maji et. Al. [7] have introduced the concept of Intuitionistic fuzzy soft set. Multi sets and Multi Fuzzy Sets were studied in [3,4] and [10]. Intuitionistic Multi fuzzy soft sets were introduced by Sujit Das and Samarjit Kar [9].

AMS Mathematics subject classification (2010): 08A72

2. PRELEMINARIES

In this section we have given some basic definitions and properties which are required for this paper.

Definition 2.1

Let X denotes a Universal set. Then the membership function μ_A by which a fuzzy set (FS) A is usually defined has the form μ_A: X → [0, 1], where [0, 1] denotes the interval of real numbers from 0 to 1 inclusive.

Definition 2.2

An Intuitionistic Fuzzy Set (IFS) A in E is defined as an object of the following form A= {(x, μ_A (x), ν_A (x))/ x ∈ E} where the functions, $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element x ∈ E respectively and for every x ∈ E: $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Definition 2.3

Let U be an initial Universe Set and E be the set of parameters. Let A⊆E. A pair (F,A) is called Fuzzy Soft Set over U where F is a mapping given by $F:A \to I^U$, where I^U denotes the collection of all fuzzy subsets of U. An fuzzy soft set is a parameterized family of fuzzy subsets of Universe U.

Definition 2.4

Let IP(U) denotes the set of all intuitionistic fuzzy set of U. A pair (F,A) is called a intuitionistic fuzzy soft set (IFSS) of over U, where F is a mapping given by $F:A \to IP(U)$. For any parameter e ∈A, F(e) is an intuitionistic fuzzy subset of U and is called Intuitionistic fuzzy value set of parameter e. Clearly F(e) can be written as an Intuitionistic fuzzy set such that F(e) = { x, $\mu_{F(e)}$ (x), $\nu_{F(e)}$ (x) ∣ x ∈U }. Here $\mu_{F(e)}$ (x) , $\nu_{F(e)}$ (x) are membership amd non-membership functions respectively and ∀x ∈U, $\mu_{F(e)}(x) + \nu_{F(e)}(x) \le 1$,

Definition 2.5

Let IMFS(U) denotes the set of all intuitionistic multi fuzzy set of U. A pair (F,A) is called a intuitionistic multi-fuzzy soft set (IMFSS) of dimension k over U, where F is a mapping given by $F:A \rightarrow IMFS^{K}(U)$. An intuitionistic multi fuzzy soft set is a mapping from parameters A to $IMFS^{K}(U)$. It is a parameterized family of intuitionistic multi fuzzy subsets of U. For e $\in$A, F(e) may be considered as the set of e- approximate elements of the intuitionistic multi fuzzy soft set(F,A).

3. Some New Operators on Multi Intuitionistic Fuzzy Soft Matrices

In this section, some new Operators on Multi Intuitionistic Fuzzy Soft Matrices are defined and based on these operators some properties are studied.

Definition 3.1

Let U = $\{u_1,u_2,\dots, u_m\}$ be the universal set and E = $\{e_1,e_2,\dots, e_n\}$ be the set of parameters. Let A$\subseteq$E and be a Multi Intuitionistic Fuzzy Soft Set on U. Then the matrix associated with this set namely Multi Intuitionistic Fuzzy Soft Matrix $\tilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n}$, i=1,2,…,m j=1,2,…,n

where,

$$a_{ij}{}^{(K)} = \begin{cases} \left(\mu_j^{(K)}(u_i), \nu_j^{(K)}(u_i)\right) \text{ (or) } \left(\mu_{\tilde{A}_{ij}}^{(K)}, \nu_{\tilde{A}_{ij}}^{(K)}\right) & \text{if } e_j \in A \\ \left(0^{(K)}, 1^{(K)}\right) & \text{if } e_j \notin A \end{cases}$$

Also $0 \le \mu_j^{(K)}(u_i) + \nu_j^{(K)}(u_i) \le 1$ and $\mu_j^{(K)}(u_i)$, $\nu_j^{(K)}(u_i)$ and $\mu_{\tilde{A}_{ij}}^{(K)}$, $\nu_{\tilde{A}_{ij}}^{(K)}$ represents the membership and non-membership of the Multi Intuitionistic Fuzzy Soft Set.

The set of all $m \times n$ Multi Intuitionistic Fuzzy Soft Matrices are denoted by $M^{(K)}$IVFSM.

Example 3.2

Suppose that U = $\{u_1,u_2,u_3\}$ is the universal set of students and E = $\{e_1,e_2\}$ is the set of parameters where e_1 = Academic performance and e_2 = Sports performance. Let A = E. Then the Multi Intuitionistic Fuzzy Soft Set (F,A), where F : n → $M^{(K)}$IFS (Multi Intuitionistic Fuzzy Sets on U) and is given by

F(A) = { F(e_1) = {(u_1, (0.8,0.1),(0.7,0.2))}, {(u_2, (0.5,0.3),(0.4,0.2))}, {(u_3, (0.6,0.2),(0.8,0.1))}} , F(e_2) = {(u_1, (0.7,0.2),(0.6,0.3))}, {(u_2, (0.4,0.2),(0.5,0.1))}, {(u_3, (0.7,0.1),(0.6,0.2))}}}

We can represent the above Multi Intuitionistic Fuzzy Soft Set in Matrix as follows.

$$\tilde{A}_{3\times 2}^{(K)} = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \end{array} \begin{array}{c} \begin{array}{cc} e_1 & e_2 \end{array} \\ \begin{pmatrix} ((0.8,0.1),(0.7,0.2)) & ((0.7,0.2),(0.6,0.3)) \\ ((0.5,0.3),(0.4,0.2)) & ((0.4,0.2),(0.5,0.1)) \\ ((0.6,0.2),(0.8,0.1)) & ((0.7,0.1),(0.6,0.2)) \end{pmatrix}_{3\times 2} \end{array}$$

Definition 3.3

Let $\tilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}$IFSM and $\tilde{B}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}$IFSM. Then $\tilde{A}^{(K)}$ is a Multi Intuitionistic Fuzzy Soft Sub Matrix of $\tilde{B}^{(K)}$, denoted by $\tilde{A}^{(K)} \subseteq \tilde{B}^{(K)}$ if $\mu_{\tilde{A}_{ij}}^{(K)} \le \mu_{\tilde{B}_{ij}}^{(K)}$ and $\nu_{\tilde{A}_{ij}}^{(K)} \ge \nu_{\tilde{B}_{ij}}^{(K)}$ for all i,j and K.

Definition 3.4

A Multi Intuitionistic Fuzzy Soft Matrix of order $m \times n$ with cardinality K is called Multi Intuitionistic Fuzzy Soft Null (Zero) Matrix if all of its elements are $\left(0^{(K)}, 1^{(K)}\right)$. It is denoted by $\tilde{\Phi}^{(K)}$.

Definition 3.5

A Multi Intuitionistic Fuzzy Soft Matrix of order $m \times n$ with cardinality K is called Multi Intuitionistic Fuzzy Soft Absolute Matrix if all of its elements are $\left(1^{(K)}, 0^{(K)}\right)$. It is denoted by $\tilde{I}^{(K)}$.

Definition 3.6

If $\tilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}$IFSM and $\tilde{B}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}$IFSM then Addition, Subtraction and Multiplication of two Multi Intuitionistic Fuzzy Soft Matrices $\tilde{A}^{(K)}$ and $\tilde{B}^{(K)}$ are defined as

$$\tilde{A}^{(K)} + \tilde{B}^{(K)} = \left[\left(\max\left(\mu_{\tilde{A}_{ij}}^{(K)}, \mu_{\tilde{B}_{ij}}^{(K)}\right), \min\left(\nu_{\tilde{A}_{ij}}^{(K)}, \nu_{\tilde{B}_{ij}}^{(K)}\right)\right)\right] \text{ for all i,j and K.}$$

$$\tilde{A}^{(K)} - \tilde{B}^{(K)} = \left[\left(\min\left(\mu_{\tilde{A}_{ij}}^{(K)}, \mu_{\tilde{B}_{ij}}^{(K)}\right), \max\left(\nu_{\tilde{A}_{ij}}^{(K)}, \nu_{\tilde{B}_{ij}}^{(K)}\right)\right)\right] \text{ for all i,j and K.}$$

$$\tilde{A}^{(K)} * \tilde{B}^{(K)} = [C_{il}]_{m\times p} = \left[\left(\max_{l}\min_{ij}\left(\mu_{\tilde{A}_{ij}}^{(K)}, \mu_{\tilde{B}_{jl}}^{(K)}\right), \min_{l}\max_{ij}\left(\nu_{\tilde{A}_{ij}}^{(K)}, \nu_{\tilde{B}_{jk}}^{(K)}\right)\right)\right].$$

Example 3.7

Consider $\tilde{A}_{2\times 2}{}^{(2)} = \begin{pmatrix} ((0.7,0.2)(0.8,0.1)) & ((0.6,0.2)(0.7,0.1)) \\ ((0.5,0.4)(0.4,0.5)) & ((0.2,0.6)(0.4,0.5)) \end{pmatrix}$ and

$\tilde{B}_{2\times 2}{}^{(2)} = \begin{pmatrix} ((0.6,0.2)(0.5,0.4)) & ((0.6,0.1)(0.7,0.2)) \\ ((0.7,0.1)(0.6,0.2)) & ((0.5,0.3)(0.2,0.7)) \end{pmatrix}$

are two Multi Intuitionistic Fuzzy Soft Matrices then

$$\tilde{A}^{(K)} + \tilde{B}^{(K)} = \begin{pmatrix} ((0.6,0.2)(0.5,0.4)) & ((0.6,0.1)(0.7,0.1)) \\ ((0.7,0.1)(0.6,0.2)) & ((0.5,0.3)(0.4,0.5)) \end{pmatrix}$$

$$\tilde{A}^{(K)} - \tilde{B}^{(K)} = \begin{pmatrix} ((0.6,0.2)(0.5,0.4)) & ((0.6,0.2)(0.7,0.2)) \\ ((0.5,0.4)(0.4,0.5)) & ((0.2,0.6)(0.2,0.7)) \end{pmatrix}$$

$$\tilde{A}^{(K)} * \tilde{B}^{(K)} = \begin{pmatrix} ((0.6,0.2)(0.6,0.2)) & ((0.6,0.2)(0.7,0.2)) \\ ((0.5,0.4)(0.4,0.5)) & ((0.5,0.4)(0.4,0.5)) \end{pmatrix}$$

Definition 3.8

Let $\tilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}$IFSM then $\widetilde{A^{T}}^{(K)}$ is the Multi Intuitionistic Fuzzy Soft Transpose Matrix of $\tilde{A}^{(K)}$ and is given by $\widetilde{A^{T}}^{(K)} = [a_{ji}{}^{(K)}]_{n\times m} \in M^{(K)}$IVFSM.

Definition 3.9

Let $\tilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}$IFSM where $a_{ij}{}^{(K)}$ = $(\mu_j^{(K)}(u_i), \nu_j^{(K)}(u_i))$. Then $\widetilde{A^{C}}^{(K)}$, the Multi Intuitionistic Fuzzy Soft Complement Matrix $\tilde{A}^{(K)}$ is defined as

$\widetilde{A^{C}}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} = (\nu_j^{(K)}(u_i), \mu_j^{(K)}(u_i))$, For all i,j and K.

Definition 3.10

Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ where $a_{ij}{}^{(K)} = (\mu_j^{(K)}(C_i), \nu_j^{(K)}(C_i))$. Then

I. $\Box\widetilde{A}^{(K)}$ is called a Multi Intuitionistic Fuzzy Soft Necessity Matrix of $\widetilde{A}^{(K)}$ and is defined as $\Box\widetilde{A}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n}$, where $b_{ij}{}^{(K)} = (\mu_j^{(K)}(C_i), 1-\nu_j^{(K)}(C_i))$ for all i,j and K.

II. $\Diamond\widetilde{A}^{(K)}$ is called a Multi Intuitionistic Fuzzy Soft Possibility Matrix of $\widetilde{A}^{(K)}$ and is defined as $\Diamond\widetilde{A}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n}$, where $b_{ij}{}^{(K)} = (1-\mu_j^{(K)}(C_i), \nu_j^{(K)}(C_i))$ for all i,j and K.

Example 3.11

Let $\widetilde{A}_{2\times 2}{}^{(2)}$ be a Multi Intuitionistic Fuzzy Soft Matrix given by

$$\widetilde{A}_{2\times 2}{}^{(2)} = \begin{pmatrix} ([0.6,0.3],[0.7,0.3]) & ([0.6,0.2],[0.5,0.3]) \\ ([0.5,0.4],[0.6,0.2]) & ([0.7,0.2],[0.8,0.1]) \end{pmatrix}_{2\times 2}$$

Then Multi Intuitionistic Fuzzy Soft Necessity Matrix of $\widetilde{A}^{(2)}$ is given by

$$\Box\widetilde{A}_{2\times 2}{}^{(2)} = \begin{pmatrix} ([0.6,0.4],[0.7,0.3]) & ([0.6,0.4],[0.5,0.5]) \\ ([0.5,0.5],[0.6,0.4]) & ([0.7,0.3],[0.8,0.2]) \end{pmatrix}_{2\times 2}$$

Also Multi Intuitionistic Fuzzy Soft Possibility Matrix of $\widetilde{A}^{(2)}$ is given by

$$\Diamond\widetilde{A}_{2\times 2}{}^{(2)} = \begin{pmatrix} ([0.7,0.3],[0.7,0.3]) & ([0.8,0.2],[0.7,0.3]) \\ ([0.6,0.4],[0.8,0.2]) & ([0.8,0.2],[0.9,0.1]) \end{pmatrix}_{2\times 2}$$

Proposition 3.12

Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ where $a_{ij}{}^{(K)} = (\mu_j^{(K)}(C_i), \nu_j^{(K)}(C_i))$. Then

I. $\left(\Box\widetilde{A^{C}}^{(K)}\right)^{C} = \Diamond\widetilde{A}^{(K)}$

II. $\left(\Diamond\,\widetilde{A^{C}}^{(K)}\right)^{C} = \Box\widetilde{A}^{(K)}$

III. $\Box\widetilde{A}^{(K)} \subseteq \widetilde{A}^{(K)} \subseteq \Diamond\widetilde{A}^{(K)}$

IV. $\Box(\Box\widetilde{A}^{(K)}) = \Box\widetilde{A}^{(K)}$

V. $\Diamond(\Diamond\widetilde{A}^{(K)}) = \Diamond\widetilde{A}^{(K)}$

VI. $\Box(\Diamond\widetilde{A}^{(K)}) = \Diamond\widetilde{A}^{(K)}$

VII. $\Diamond(\Box\widetilde{A}^{(K)}) = \Box\widetilde{A}^{(K)}$

VIII. $\Box^{n}(\widetilde{A}^{(K)}) = \Box(\Box(\Box(\ldots(A))) = \Box\widetilde{A}^{(K)}$ for all integer n > 0.

IX. $\Diamond^{n}(\widetilde{A}^{(K)}) = \Diamond(\Diamond(\Diamond(\ldots(A))) = \Diamond\widetilde{A}^{(K)}$ for all integer n > 0.

Proof

(i) $\widetilde{A^{C}}^{(K)} = (\nu_j^{(K)}(C_i), \mu_j^{(K)}(C_i))$ for all i,j and K.

Now, $\Box\widetilde{A^{C}}^{(K)} = (\nu_j^{(K)}(C_i), 1-\nu_j^{(K)}(C_i))$ for all i,j and K.

$\left(\Box\widetilde{A^{C}}^{(K)}\right)^{C} = (1-\nu_j^{(K)}(C_i), \nu_j^{(K)}(C_i))$ for all i,j and K.

$= \Diamond\,\widetilde{A}^{(K)}$

Similarly (ii) can be proved.

(iii). We have $0 \le \mu_j^{(K)}(C_i) + \nu_j^{(K)}(C_i) \le 1$ for all all i,j and K.

$[(\mu_j^{(K)}(C_i), 1-\mu_j^{(K)}(C_i))] \subseteq [(\mu_j^{(K)}(C_i), \nu_j^{(K)}(C_i))] \subseteq [(1-\nu_j^{(K)}(C_i), \nu_j^{(K)}(C_i))]$ for all all i,j and K.

Hence $\Box\widetilde{A}^{(K)} \subseteq \widetilde{A}^{(K)} \subseteq \Diamond\widetilde{A}^{(K)}$.

(iv). $\Box\widetilde{A}^{(K)} = [(\mu_j^{(K)}(C_i), 1-\mu_j^{(K)}(C_i))]$ for all all i,j and K.

$\Box\Box\widetilde{A}^{(K)} = [(\mu_j^{(K)}(C_i), 1-\mu_j^{(K)}(C_i))]$ for all all i,j and K.

$= \Box\widetilde{A}^{(K)}$

(v).Similarly we can prove $\Diamond(\Diamond\widetilde{A}^{(K)}) = \Diamond\widetilde{A}^{(K)}$

(vi). $\Box\,\Diamond\widetilde{A}^{(K)} = \Box([(1-\nu_j^{(K)}(C_i), \nu_j^{(K)}(C_i))])$ for all i,j and K.

$= [(1-\nu_j^{(K)}(C_i), \nu_j^{(K)}(C_i))]$ for all i,j and K.

$= \Diamond\widetilde{A}^{(K)}$

(vii). $\Diamond(\Box\widetilde{A}^{(K)}) = \Diamond([(\mu_j^{(K)}(C_i), 1-\mu_j^{(K)}(C_i))])$ for all i,j and K.

$= [(\mu_j^{(K)}(C_i), 1-\mu_j^{(K)}(C_i))]$ for all i,j and K.

$= \Box\widetilde{A}^{(K)}$

(viii). Proof follows from (iv)

(ix). Proof follows from (v)

Remark 3.13

Let $\widetilde{A}^{(K)}$ be a Multi Intuitionistic Fuzzy Soft Matrix. Then in general $\Box\,\Diamond\,\widetilde{A}^{(K)} \ne \Diamond\,\Box\widetilde{A}^{(K)}$.

Proposition 3.14

If $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ and $\widetilde{B}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ then

(i) $\Box(\widetilde{A}^{(K)} + \widetilde{B}^{(K)}) = \Box\widetilde{A}^{(K)} + \Box\widetilde{B}^{(K)}$

(ii) $\Diamond(\widetilde{A}^{(K)} + \widetilde{B}^{(K)}) = \Diamond\,\widetilde{A}^{(K)} + \Diamond\,\widetilde{B}^{(K)}$

(iii) $\Box(\widetilde{A^{C}}^{(K)}) = \left(\Diamond\widetilde{A}^{(K)}\right)^{C}$

(iv) $\Diamond(\widetilde{A^{C}}^{(K)}) = \left(\Box\widetilde{A}^{(K)}\right)^{C}$

Proof

(i). Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} = \left[(\mu_{\widetilde{A}_{ij}}^{(K)}, \nu_{\widetilde{A}_{ij}}^{(K)})\right] \in M^{(K)}IFSM$ and $\widetilde{B}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} = \left[(\mu_{\widetilde{B}_{ij}}^{(K)}, \nu_{\widetilde{B}_{ij}}^{(K)})\right] \in M^{(K)}IFSM$ then

$\widetilde{A}^{(K)} + \widetilde{B}^{(K)} = \left[\left(\max(\mu_{\widetilde{A}_{ij}}^{(K)}, \mu_{\widetilde{B}_{ij}}^{(K)}), \min(\nu_{\widetilde{A}_{ij}}^{(K)}, \nu_{\widetilde{B}_{ij}}^{(K)})\right)\right]$ for all i,j and K

$\Box(\widetilde{A}^{(K)} + \widetilde{B}^{(K)}) = \left[\left(\max(\mu_{\widetilde{A}_{ij}}^{(K)}, \mu_{\widetilde{B}_{ij}}^{(K)}), 1-\max(\mu_{\widetilde{A}_{ij}}^{(K)}, \mu_{\widetilde{B}_{ij}}^{(K)})\right)\right]$ for all i,j and K --------→(1)

$\Box\widetilde{A}^{(K)} = \left[\left(\mu_{\widetilde{A}_{ij}}^{(K)}, 1-\mu_{\widetilde{A}_{ij}}^{(K)}\right)\right]$ for all i,j and K

$\Box\widetilde{B}^{(K)} = \left[\left(\mu_{\widetilde{B}_{ij}}^{(K)}, 1-\mu_{\widetilde{B}_{ij}}^{(K)}\right)\right]$ for all i,j and K

$\Box\widetilde{A}^{(K)} + \Box\widetilde{B}^{(K)} = \left[\left(\max(\mu_{\widetilde{A}_{ij}}^{(K)}, \mu_{\widetilde{B}_{ij}}^{(K)}), \min\left(1-\mu_{\widetilde{A}_{ij}}^{(K)}, 1-\mu_{\widetilde{B}_{ij}}^{(K)}\right)\right)\right]$ for all i,j and K

$= \left[\left(\max(\mu_{\widetilde{A}_{ij}}^{(K)}, \mu_{\widetilde{B}_{ij}}^{(K)}), 1-\max(\mu_{\widetilde{A}_{ij}}^{(K)}, \mu_{\widetilde{B}_{ij}}^{(K)})\right)\right]$ for all i,j and K --------→(2)

From (1) and (2) $\Box(\widetilde{A}^{(K)} + \widetilde{B}^{(K)}) = \Box\widetilde{A}^{(K)} + \Box\widetilde{B}^{(K)}$.

Similarly (ii) can be proved.

(iii). Consider $\widetilde{A^{C}}^{(K)} = \left[(\nu^{(K)}_{\tilde{A}_{ij}}, \mu^{(K)}_{\tilde{A}_{ij}})\right]$ for all i,j and K

Now, $\square\widetilde{A^{C}}^{(K)} = \left[(\nu^{(K)}_{\tilde{A}_{ij}}, 1-\nu^{(K)}_{\tilde{A}_{ij}})\right]$ for all i,j and K ---→(3)

$\Diamond\tilde{A}^{(K)} = \left[(1-\nu^{(K)}_{\tilde{A}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}})\right]$ for all i,j and K

$\left(\Diamond\tilde{A}^{(K)}\right)^{C} = \left[(\nu^{(K)}_{\tilde{A}_{ij}}, 1-\nu^{(K)}_{\tilde{A}_{ij}})\right]$ for all i,j and K ---→(4)

From (3) and (4), $\square(\widetilde{A^{C}}^{(K)}) = \left(\Diamond\tilde{A}^{(K)}\right)^{C}$

Similarly, (iv) can be proved.

Definition 3.15

If $\tilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ and $\tilde{B}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ then the operators $\oplus$ and $\otimes$ are defined on these matrices as follows.

(i). $\tilde{A}^{(K)} \oplus \tilde{B}^{(K)} = \left[(\mu^{(K)}_{\tilde{A}_{ij}} + \mu^{(K)}_{\tilde{B}_{ij}} - \mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}},\ \nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}})\right]$ for all i,j and K

(ii). $\tilde{A}^{(K)} \otimes \tilde{B}^{(K)} = \left[(\mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}},\ \nu^{(K)}_{\tilde{A}_{ij}} + \nu^{(K)}_{\tilde{B}_{ij}} - \nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}})\right]$ for all i,j and K

$\tilde{A}^{(K)} \oplus \tilde{B}^{(K)} \in M^{(K)}IFSM$ and $\tilde{A}^{(K)} \otimes \tilde{B}^{(K)} \in M^{(K)}IFSM$. Here +, -, . represents usual addition, subtraction and multiplication.

Proposition 3.16

Let $\tilde{A}^{(K)}_{m\times n} \in M^{(K)}IFSM$ and $\tilde{B}^{(K)}_{m\times n} \in M^{(K)}IFSM$ then

(i) $\tilde{A}^{(K)} + \tilde{B}^{(K)} \subseteq \tilde{A}^{(K)} \oplus \tilde{B}^{(K)}$

(ii) $\tilde{A}^{(K)} \otimes \tilde{B}^{(K)} \subseteq \tilde{A}^{(K)} + \tilde{B}^{(K)}$

(iii) $\tilde{A}^{(K)} \otimes \tilde{B}^{(K)} \subseteq \tilde{A}^{(K)} + \tilde{B}^{(K)} \subseteq \tilde{A}^{(K)} \oplus \tilde{B}^{(K)}$

Proof

Let $\tilde{A}^{(K)}_{m\times n} = \left[(\mu^{(K)}_{\tilde{A}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}})\right] \in M^{(K)}IFSM$ and $\tilde{B}^{(K)}_{m\times n} = \left[(\mu^{(K)}_{\tilde{B}_{ij}}, \nu^{(K)}_{\tilde{B}_{ij}})\right] \in M^{(K)}IFSM$

Now, $\tilde{A}^{(K)} + \tilde{B}^{(K)} = \left[\left(\max(\mu^{(K)}_{\tilde{A}_{ij}}, \mu^{(K)}_{\tilde{B}_{ij}}), \min(\nu^{(K)}_{\tilde{A}_{ij}}, \nu^{(K)}_{\tilde{B}_{ij}})\right)\right]$ for all i,j and K

$\tilde{A}^{(K)} \oplus \tilde{B}^{(K)} = \left[(\mu^{(K)}_{\tilde{A}_{ij}} + \mu^{(K)}_{\tilde{B}_{ij}} - \mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}},\ \nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}})\right]$ for all i,j and K

Since $\max(\mu^{(K)}_{\tilde{A}_{ij}}, \mu^{(K)}_{\tilde{B}_{ij}}) \leq \mu^{(K)}_{\tilde{A}_{ij}} + \mu^{(K)}_{\tilde{B}_{ij}} - \mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}}$ and $\min(\nu^{(K)}_{\tilde{A}_{ij}}, \nu^{(K)}_{\tilde{B}_{ij}}) \geq \nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}}$

Hence $\tilde{A}^{(K)} + \tilde{B}^{(K)} \subseteq \tilde{A}^{(K)} \oplus \tilde{B}^{(K)}$

(ii). Now, $\tilde{A}^{(K)} \otimes \tilde{B}^{(K)} = \left[(\mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}},\ \nu^{(K)}_{\tilde{A}_{ij}} + \nu^{(K)}_{\tilde{B}_{ij}} - \nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}})\right]_{m\times n}$ for all i,j and K

Since max $(\mu^{(K)}_{\tilde{A}_{ij}}, \mu^{(K)}_{\tilde{B}_{ij}}) \geq \mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}}$ and min $(\nu^{(K)}_{\tilde{A}_{ij}}, \nu^{(K)}_{\tilde{B}_{ij}}) \leq \nu^{(K)}_{\tilde{A}_{ij}} + \nu^{(K)}_{\tilde{B}_{ij}} - \nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}}$

Therefore $\tilde{A}^{(K)} \otimes \tilde{B}^{(K)} \subseteq \tilde{A}^{(K)} + \tilde{B}^{(K)}$

(iii). From above results (i) and (ii) combining both we have

$\tilde{A}^{(K)} \otimes \tilde{B}^{(K)} \subseteq \tilde{A}^{(K)} + \tilde{B}^{(K)} \subseteq \tilde{A}^{(K)} \oplus \tilde{B}^{(K)}$

Corollary 3.17

Let $\tilde{A}^{(K)}_{m\times n} \in M^{(K)}IFSM$ then

(i) $\tilde{A}^{(K)} \subseteq \tilde{A}^{(K)} \oplus \tilde{A}^{(K)}$

(ii) $\tilde{A}^{(K)} \otimes \tilde{A}^{(K)} \subseteq \tilde{A}^{(K)}$

(iii) $\tilde{A}^{(K)} \otimes \tilde{A}^{(K)} \subseteq \tilde{A}^{(K)} \subseteq \tilde{A}^{(K)} \oplus \tilde{A}^{(K)}$

(iv) $\tilde{A}^{(K)} \otimes \tilde{A}^{(K)} \subseteq \tilde{A}^{(K)} \oplus \tilde{A}^{(K)}$

Proof is Obvious.

Proposition 3.18

Let $\tilde{A}^{(K)}, \tilde{B}^{(K)}$ and $\tilde{C}^{(K)} \in M^{(K)}IVFSM$ then

(i) $\tilde{A}^{(K)} \oplus \tilde{B}^{(K)} = \tilde{B}^{(K)} \oplus \tilde{A}^{(K)}$ (Commutative Law)

(ii) $(\tilde{A}^{(K)} \oplus \tilde{B}^{(K)}) \oplus \tilde{C}^{(K)} = \tilde{A}^{(K)} \oplus (\tilde{B}^{(K)} \oplus \tilde{C}^{(K)})$ (Associative Law)

(iii) $\tilde{A}^{(K)} \otimes \tilde{B}^{(K)} = \tilde{B}^{(K)} \otimes \tilde{A}^{(K)}$ (Commutative Law)

(iv) $(\tilde{A}^{(K)} \otimes \tilde{B}^{(K)}) \otimes \tilde{C}^{(K)} = \tilde{A}^{(K)} \otimes (\tilde{B}^{(K)} \otimes \tilde{C}^{(K)})$ (Associative Law)

Proof

Let $\tilde{A}^{(K)}_{m\times n} = [a_{ij}{}^{(K)}]_{m\times n} = \left[(\mu^{(K)}_{\tilde{A}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}})\right]_{m\times n} \in M^{(K)}IFSM$,

$\tilde{B}^{(K)}_{m\times n} = [b_{ij}{}^{(K)}]_{m\times n} = \left[(\mu^{(K)}_{\tilde{B}_{ij}}, \nu^{(K)}_{\tilde{B}_{ij}})\right]_{m\times n} \in M^{(K)}IFSM$ and

$\tilde{C}^{(K)}_{m\times n} = [c_{ij}{}^{(K)}]_{m\times n} = \left[(\mu^{(K)}_{\tilde{C}_{ij}}, \nu^{(K)}_{\tilde{C}_{ij}})\right]_{m\times n} \in M^{(K)}IFSM$ and

Now,

$\tilde{A}^{(K)} \oplus \tilde{B}^{(K)} = \left[(\mu^{(K)}_{\tilde{A}_{ij}} + \mu^{(K)}_{\tilde{B}_{ij}} - \mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}},\ \nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}})\right]$

$= \left[(\mu^{(K)}_{\tilde{B}_{ij}} + \mu^{(K)}_{\tilde{A}_{ij}} - \mu^{(K)}_{\tilde{B}_{ij}}.\mu^{(K)}_{\tilde{A}_{ij}},\ \nu^{(K)}_{\tilde{B}_{ij}}.\nu^{(K)}_{\tilde{A}_{ij}})\right]$ for all i,j and K

$= \tilde{B}^{(K)} \oplus \tilde{A}^{(K)}$

Similarly (ii) can be proved.

(ii). For all i,j and K. $\tilde{A}^{(K)} \otimes \tilde{B}^{(K)} = \left[(\mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}},\ \nu^{(K)}_{\tilde{A}_{ij}} + \nu^{(K)}_{\tilde{B}_{ij}} - \nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}})\right]$

$= \left[(\mu^{(K)}_{\tilde{B}_{ij}}.\mu^{(K)}_{\tilde{A}_{ij}},\ \nu^{(K)}_{\tilde{B}_{ij}} + \nu^{(K)}_{\tilde{A}_{ij}} - \nu^{(K)}_{\tilde{B}_{ij}}.\nu^{(K)}_{\tilde{A}_{ij}})\right]$

$= \tilde{B}^{(K)} \otimes \tilde{A}^{(K)}$

Similarly (iv) can be proved.

Proposition 3.19

Let $\tilde{A}^{(K)}_{m\times n} = [a_{ij}{}^{(K)}]_{m\times n} = \left[(\mu^{(K)}_{\tilde{A}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}})\right]_{m\times n} \in M^{(K)}IFSM$ and

$\tilde{B}^{(K)}_{m\times n} = [b_{ij}{}^{(K)}]_{m\times n} = \left[(\mu^{(K)}_{\tilde{B}_{ij}}, \nu^{(K)}_{\tilde{B}_{ij}})\right]_{m\times n} \in M^{(K)}IFSM$. If $\tilde{A}^{(K)}$ and $\tilde{B}^{(K)}$ are Symmetric, then $\tilde{A}^{(K)} \oplus \tilde{B}^{(K)}$ and $\tilde{A}^{(K)} \otimes \tilde{B}^{(K)}$ are also Symmetric.

Proof

Let $\tilde{A}^{(K)}_{m\times n} = [a_{ij}{}^{(K)}]_{m\times n} = \left[(\mu^{(K)}_{\tilde{A}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}})\right]_{m\times n} \in M^{(K)}IFSM$ and

$\tilde{B}^{(K)}_{m\times n} = [b_{ij}{}^{(K)}]_{m\times n} = \left[(\mu^{(K)}_{\tilde{B}_{ij}}, \nu^{(K)}_{\tilde{B}_{ij}})\right]_{m\times n} \in M^{(K)}IFSM$.

Then we have $0 \leq \mu^{(K)}_{\tilde{A}_{ij}} + \nu^{(K)}_{\tilde{A}_{ij}} \leq 1$ and $0 \leq \mu^{(K)}_{\tilde{B}_{ij}} + \nu^{(K)}_{\tilde{B}_{ij}} \leq 1$ for all i,j and K.

Given that $\tilde{A}^{(K)}$ and $\tilde{B}^{(K)}$ are symmetric. We have $\widetilde{A^{T}}^{(K)} = \tilde{A}^{(K)}$ and $\widetilde{B^{T}}^{(K)} = \tilde{B}^{(K)}$.

Therefore, $\mu^{(K)}_{\tilde{A}_{ji}} = \mu^{(K)}_{\tilde{A}_{ij}}$ and $\nu^{(K)}_{\tilde{A}_{ji}} = \nu^{(K)}_{\tilde{A}_{ij}}$.

Also $\mu^{(K)}_{\tilde{B}_{ji}} = \mu^{(K)}_{\tilde{B}_{ij}}$ and $\nu^{(K)}_{\tilde{B}_{ji}} = \nu^{(K)}_{\tilde{B}_{ij}}$.

For all i,j and K, $\left(\tilde{A}^{(K)} \oplus \tilde{B}^{(K)}\right)^T = \left[(\mu^{(K)}_{\tilde{A}_{ij}} + \mu^{(K)}_{\tilde{B}_{ij}} - \mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}})\right]^T$

$= \left[(\mu^{(K)}_{\tilde{A}_{ji}} + \mu^{(K)}_{\tilde{B}_{ji}} - \mu^{(K)}_{\tilde{A}_{ji}}.\nu^{(K)}_{\tilde{B}_{ji}}, \nu^{(K)}_{\tilde{A}_{ji}}.\nu^{(K)}_{\tilde{B}_{ji}})\right]$

$= \left[(\mu^{(K)}_{\tilde{A}_{ij}} + \mu^{(K)}_{\tilde{B}_{ij}} - \mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}})\right]$

$= \tilde{A}^{(K)} \oplus \tilde{B}^{(K)}$

Therefore $\tilde{A}^{(K)} \oplus \tilde{B}^{(K)}$ is Symmetric.

Similarly we can prove $\tilde{A}^{(K)} \otimes \tilde{B}^{(K)}$ is Symmetric.

Remark 3.20

Let $\tilde{A}^{(K)}_{m\times n} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}$IFSM and $\tilde{B}^{(K)}_{m\times n} = [b_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}$IFSM. Then

(i) $\left(\tilde{A}^{(K)} \oplus \tilde{B}^{(K)}\right)^T \neq \widetilde{A^T}^{(K)} \otimes \widetilde{B^T}^{(K)}$

(ii) $\left(\tilde{A}^{(K)} \otimes \tilde{B}^{(K)}\right)^T \neq \widetilde{A^T}^{(K)} \oplus \widetilde{B^T}^{(K)}$

Proposition 3.21

Let $\tilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} = \left[(\mu^{(K)}_{\tilde{A}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}})\right]_{m\times n} \in M^{(K)}$IFSM,

$\tilde{B}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} = \left[\left(\mu^{(K)}_{\tilde{B}_{ij}}, \nu^{(K)}_{\tilde{B}_{ij}}\right)\right]_{m\times n} \in M^{(K)}$IFSM and

$\tilde{C}^{(K)} = [c_{ij}{}^{(K)}]_{m\times n} = \left[\left(\mu^{(K)}_{\tilde{C}_{ij}}, \nu^{(K)}_{\tilde{C}_{ij}}\right)\right]_{m\times n} \in M^{(K)}$IFSM,

(i) $\left(\tilde{A}^{(K)} \oplus \tilde{B}^{(K)}\right)^T = \left(\tilde{B}^{(K)} \oplus \tilde{A}^{(K)}\right)^T$ and $\left(\tilde{A}^{(K)} \oplus \tilde{B}^{(K)}\right)^T = \widetilde{A^T}^{(K)} \oplus \widetilde{B^T}^{(K)}$

(ii) $\left(\tilde{A}^{(K)} \otimes \tilde{B}^{(K)}\right)^T = \tilde{B}^{(K)} \otimes \tilde{A}^{(K)}$ and $\left(\tilde{A}^{(K)} \otimes \tilde{B}^{(K)}\right)^T = \widetilde{A^T}^{(K)} \otimes \widetilde{B^T}^{(K)}$

(iii) If $\tilde{A}^{(K)} \subseteq \tilde{B}^{(K)}$ then $\tilde{A}^{(K)} \oplus \tilde{C}^{(K)} \subseteq \tilde{B}^{(K)} \oplus \tilde{C}^{(K)}$ and $\tilde{A}^{(K)} \otimes \tilde{C}^{(K)} \subseteq \tilde{B}^{(K)} \otimes \tilde{C}^{(K)}$

Proof

$\left(\tilde{A}^{(K)} \oplus \tilde{B}^{(K)}\right)^T = \left(\tilde{B}^{(K)} \oplus \tilde{A}^{(K)}\right)^T$

For all i,j and K, $\left(\tilde{A}^{(K)} \oplus \tilde{B}^{(K)}\right)^T = \left[(\mu^{(K)}_{\tilde{A}_{ij}} + \mu^{(K)}_{\tilde{B}_{ij}} - \mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}})\right]^T$

$= \left[(\mu^{(K)}_{\tilde{A}_{ji}} + \mu^{(K)}_{\tilde{B}_{ji}} - \mu^{(K)}_{\tilde{A}_{ji}}.\nu^{(K)}_{\tilde{B}_{ji}}, \nu^{(K)}_{\tilde{A}_{ji}}.\nu^{(K)}_{\tilde{B}_{ji}})\right]$ -----→(1)

$\widetilde{A^T}^{(K)} = \left[(\mu^{(K)}_{\tilde{A}_{ji}}, \nu^{(K)}_{\tilde{A}_{ji}})\right]$, $\widetilde{B^T}^{(K)} = \left[(\mu^{(K)}_{\tilde{B}_{ji}}, \nu^{(K)}_{\tilde{B}_{ji}})\right]$

Now, $\widetilde{A^T}^{(K)} \oplus \widetilde{B^T}^{(K)} = \left[(\mu^{(K)}_{\tilde{A}_{ji}} + \mu^{(K)}_{\tilde{B}_{ji}} - \mu^{(K)}_{\tilde{A}_{ji}}.\nu^{(K)}_{\tilde{B}_{ji}}, \nu^{(K)}_{\tilde{A}_{ji}}.\nu^{(K)}_{\tilde{B}_{ji}})\right]$ -----→(2)

From (1) and (2), $\left(\tilde{A}^{(K)} \oplus \tilde{B}^{(K)}\right)^T = \widetilde{A^T}^{(K)} \oplus \widetilde{B^T}^{(K)}$.

Similarly we can prove (ii).

(iii). Let $\tilde{A}^{(K)} \subseteq \tilde{B}^{(K)}$. Therefore for all i,j and K we have

$\mu^{(K)}_{\tilde{A}_{ij}} \leq \mu^{(K)}_{\tilde{B}_{ij}}$ and $\nu^{(K)}_{\tilde{A}_{ij}} \leq \nu^{(K)}_{\tilde{B}_{ij}}$

$\Rightarrow \mu^{(K)}_{\tilde{A}_{ji}} + \mu^{(K)}_{\tilde{C}_{ij}} - \mu^{(K)}_{\tilde{A}_{ji}}.\mu^{(K)}_{\tilde{C}_{ij}} \leq \mu^{(K)}_{\tilde{B}_{ij}} + \mu^{(K)}_{\tilde{C}_{ij}} - \mu^{(K)}_{\tilde{B}_{ij}}.\mu^{(K)}_{\tilde{C}_{ij}}$ ----→(3)

and $\nu^{(K)}_{\tilde{B}_{ij}}.\nu^{(K)}_{\tilde{C}_{ij}} \geq \nu^{(K)}_{\tilde{B}_{ij}}.\nu^{(K)}_{\tilde{C}_{ij}}$.

Now, $\tilde{A}^{(K)} \oplus \tilde{C}^{(K)} = \left[(\mu^{(K)}_{\tilde{A}_{ij}} + \mu^{(K)}_{\tilde{C}_{ij}} - \mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{C}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{C}_{ij}})\right]$

Also, $\tilde{B}^{(K)} \oplus \tilde{C}^{(K)} = \left[(\mu^{(K)}_{\tilde{B}_{ij}} + \mu^{(K)}_{\tilde{C}_{ij}} - \mu^{(K)}_{\tilde{B}_{ij}}.\mu^{(K)}_{\tilde{C}_{ij}}, \nu^{(K)}_{\tilde{B}_{ij}}.\nu^{(K)}_{\tilde{C}_{ij}})\right]$

From (3) we have, $\tilde{A}^{(K)} \oplus \tilde{C}^{(K)} \subseteq \tilde{B}^{(K)} \oplus \tilde{C}^{(K)}$.

In the following Proposition, we prove that the Complement based on the operators ⨁ and ⨂ follows De-Morgan's Laws.

Proposition 3.22

Let $\tilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} = \left[(\mu^{(K)}_{\tilde{A}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}})\right]_{m\times n} \in M^{(K)}$IFSM and

$\tilde{B}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} = \left[\left(\mu^{(K)}_{\tilde{B}_{ij}}, \nu^{(K)}_{\tilde{B}_{ij}}\right)\right]_{m\times n} \in M^{(K)}$IFSM. Then

(i) $\left(\tilde{A}^{(K)} \oplus \tilde{B}^{(K)}\right)^C = \widetilde{A^C}^{(K)} \otimes \widetilde{B^C}^{(K)}$ (ii) $\left(\tilde{A}^{(K)} \otimes \tilde{B}^{(K)}\right)^C = \widetilde{A^C}^{(K)} \oplus \widetilde{B^C}^{(K)}$

(iii) $\left(\tilde{A}^{(K)} \oplus \tilde{B}^{(K)}\right)^C \subseteq \widetilde{A^C}^{(K)} \oplus \widetilde{B^C}^{(K)}$ (iv) $\widetilde{A^C}^{(K)} \otimes \widetilde{B^C}^{(K)} \subseteq \left(\tilde{A}^{(K)} \otimes \tilde{B}^{(K)}\right)^C$

Proof

For all i,j and K, $\left(\tilde{A}^{(K)} \oplus \tilde{B}^{(K)}\right)^C = \left[(\mu^{(K)}_{\tilde{A}_{ij}} + \mu^{(K)}_{\tilde{B}_{ij}} - \mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}})\right]^C$

$= \left[(\nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}}, \mu^{(K)}_{\tilde{A}_{ij}} + \mu^{(K)}_{\tilde{B}_{ij}} - \mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}})\right]$ ----→(1)

For all i,j and K, $\widetilde{A^C}^{(K)} = \left[(\nu^{(K)}_{\tilde{A}_{ij}}, \mu^{(K)}_{\tilde{A}_{ij}})\right]$ and $\widetilde{B^C}^{(K)} = \left[\left(\nu^{(K)}_{\tilde{B}_{ij}}, \mu^{(K)}_{\tilde{B}_{ij}}\right)\right]$

$\widetilde{A^C}^{(K)} \otimes \widetilde{B^C}^{(K)} = \left[(\nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}}, \mu^{(K)}_{\tilde{A}_{ij}} + \mu^{(K)}_{\tilde{B}_{ij}} - \mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}})\right]$ ----→(2)

From (1) and (2), $\left(\tilde{A}^{(K)} \oplus \tilde{B}^{(K)}\right)^C = \widetilde{A^C}^{(K)} \otimes \widetilde{B^C}^{(K)}$

Similarly we can prove (ii).

Now, $\widetilde{A^C}^{(K)} \oplus \widetilde{B^C}^{(K)} = \left[\nu^{(K)}_{\tilde{A}_{ij}} + \nu^{(K)}_{\tilde{B}_{ij}} - \nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}}, \mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}}\right]$ ----→(3)

Since, $\nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}} \leq \nu^{(K)}_{\tilde{A}_{ij}} + \nu^{(K)}_{\tilde{B}_{ij}} - \nu^{(K)}_{\tilde{A}_{ij}}.\nu^{(K)}_{\tilde{B}_{ij}}$ and $\mu^{(K)}_{\tilde{A}_{ij}} + \mu^{(K)}_{\tilde{B}_{ij}} - \mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}} \geq \mu^{(K)}_{\tilde{A}_{ij}}.\mu^{(K)}_{\tilde{B}_{ij}}$

From (1) and (3) we have, $\left(\tilde{A}^{(K)} \oplus \tilde{B}^{(K)}\right)^C \subseteq \widetilde{A^C}^{(K)} \oplus \widetilde{B^C}^{(K)}$

In a Similar way result (iv) can be proved.

Conclusion:

In this paper, we have defined some new operators namely □, ◊ on a new type of matrix namely Multi Intuitionistic Fuzzy soft Matrix were defined and some of their properties are studied. Also we have defined another two new operators namely ⨁ , ⨂ on Multi Intuitionistic Fuzzy soft Matrix and we have studied some of their properties. The concepts are illustrated with suitable numerical examples.

Reference

[1] Atanassov, K., Intuitionistc fuzzy sets, Fuzzy Sets and Systems, 20(1) (1986), 87-96.

[2] Atanassov, K., Intuitionistic fuzzy sets, Physica-Verlag, Heidelberg, Newyork, 1999.

[3] Baruah H. K., Towards Forming a Field of Fuzzy Sets. International Journal of Energy, Information and Communications, Vol.2, Issue 1(2011) 16-20.

[4] Baruah H. K., The Theory of Fuzzy Sets. Beliefs and Realities. International Journal of Energy, Information and Communications, Vol.2, Issue 2(2011) 1-22.

[5] *Cağman*, N. *Enginoğlu*, *S*., Soft matrix theory and its decision making, Computers and Mathematics with Applications, 59(2010), 3308-3314.

[6] Gorzalczany, M. B., A method of inference in approximate reasoning based on interval valued fuzzy sets, Fuzzy sets and systems 21(1987), 1-7.

[7] Maji, P. K., Biswas, R. and Roy, A. R., Fuzzy Soft sets, Journal of Fuzzy Mathematics, 9(3)(2001), 677-691.

[8] Molodtsov, D., Soft set theory-first result, Computers and Mathematics with Applications, 37(1999), 19-31.

[9] Sujit Das and Samarjit Kar Intuitionistic Multi Fuzzy Soft Set and its Application in Decision Making International Conference on Pattern Recognition and Machine Intelligence (2013), LNCS 8251, 587-592.

[10] Yager R. R., On the theory of bags (Multi Sets). International Journal of General System, 13(1986) 23-37.

[11] Zadeh, L. A., Fuzzy sets, Information and Control, 8(1965), 338-353.