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On the Radio Number of Certain Classes of Circulant Graphs

Kins Yenoke

PG & Research, Department of Mathematics, Loyola College, Chennai, Tamil Nadu, India

ABSTRACT

Radio labelling problem is a special type of assignment problem which maximizes the number of channels in a specified bandwidth. A ***radio labelling*** of a connected graph $G = (V, E)$ is an injection $h: V(G) \rightarrow N$ such that $d(x, y) + |f(x) - f(y)| \geq 1 + d(G) \forall x, y \in V(G)$, where $d(G)$ is the diameter of the graph G. The ***radio number of h*** denoted $rn(h)$, is the maximum number assigned to any vertex of G. The ***radio number of G***, denoted $rn(G)$, is the minimum value of $rn(h)$ taken over all labelling's h of G. In this paper we have obtained the radio number certain classes of circulant graphs, namely $G\left(n; \left\{1,2 \ldots \left\lfloor\frac{n}{2}\right\rfloor - 1\right\}\right)$, $G\left(n; \left\{1, \frac{n}{2}\right\}\right)$, $G\left(n; \left\{1, \frac{n}{3}\right\}\right)$ and $G\left(n; \left\{1, \frac{n}{5}\right\}\right)$.

KEYWORDS: *Labelling, Radio labelling, Radio number, Circulant graphs*

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1. INTRODUCTION

In the modern world, all the communications are based on the wireless mode. In the end of the 20th century most of the communication were developed based on the wireless networks. These communications are working based on the specified allocation of the electromagnetic spectrum. One of the main applications is based on the low frequency long wavelength waves called radio waves. Among this, a fixed bandwidth ranges from 88.1 MHZ to 108 MHZ was allotted for the channel assignment of FM radio stations. 88.1 MHz and finished at 108MHz This real-life minimax problem motivated Chartrand et al. [6] in the year 2001 to introduced a new labelling called the radio labelling. He defined the formal graph theoretical definition for radio labelling problem as follows:

Let $diam(G)$ be the diameter of a connected graph G. An injection h from the vertex set of G to N such that $d(u, v) + |h(u) - h(v)| \geq 1 + diam(G))$ for every pair of vertices in G. The ***radio number of h***, denoted by $rn(h)$, is the maximum number assigned to any vertex of G. The ***radio number of G***, denoted by $rn(G)$, is the minimum value of $rn(h)$ taken over all labelling's h of G.

The radio number problem is NP-hard [9], even for graphs with diameter 2. In the past two decades plenty of research articles are published in this area and also developed new labelling problems based on the radio number.

2. An Overview of the Paper

For the past 20 years, several authors studied the radio labelling problem and its variations in various networks and graphs. Radio labelling problem is a particular case of radio K- Chromatic number [7]. In the recent years few new labelling's were introduced by different authors based on the k value, namely, radio mean labelling, radio multiplicative labelling, radial radio labelling etc. The radio number of square cycles was determined by Liu et.al [12]. Bharati et.al [2,3] obtained the bounds for the hexagonal mesh as $3n^2 - 3n + 2 + 12\sum_{i=0}^{n-2} i(i - n - 1) \leq rn(G) \leq n(3n^2 - 4n - 1) + 3$ and completely determined the radio number of graphs with small diameters. Kchikech et.al [10] studied the radio k-labelling of graphs. Fernandez et al. [8] computed the radio number for gear graph. Kins et. al. [11] investigated the radio number for mesh derived architectures and wheel extended graphs.

In this paper we have investigated the radio labelling of certain classes of circulant graphs.

3. Circulant Graphs

Circulant graphs have been used for several decades in the design of telecommunication networks because of their optimal fault-tolerance and routing capabilities [5]. For designing certain data alignment networks, the circulant graphs are being used. for complex memory systems [13]. Most of the earlier research concentrated on using the circulant graphs to build interconnection networks for distributed and parallel systems [2]. By using circulant graph we can adapt the performance of the network to user needs. It's a regular graph which includes standard such as the complete graph and the cycle.

Definition 3.1: An undirected circulant graph denoted by $G(n;\pm\{1,2\ldots j\}), 1\le j\le \left\lfloor\frac{n}{2}\right\rfloor, n\ge 3$, is defined as a graph with vertex set $V=\{0,1\ldots n-1\}$ and the edge set $E=\{(i,j): |j-i|\equiv k(mod\ n), k\in\{1,2\ldots j\}$.

Remark 1: In this paper for our convenience we take the vertex set V as $\{v_1,v_2\ldots v_n\}$, which is named in clockwise order.

Remark 2: It is clear that, when $j=\frac{n}{2}$, the circulant graph $G\left(n;\pm\left\{1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor\right\}\right)$, become a complete graph.

Lemma 3.1: The diameter of the circulant graph $G\left(n;\left\{1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)$ is 2.

Proof: As we know that, the circulant graph $G\left(n;\left\{1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)$ is obtained from the circulant graph $G\left(n;\pm\left\{1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor\right\}\right)$ by the removal of an unique one edge from each vertex with maximum distance on the outer cycle. Since the diameter of $G\left(n;\pm\left\{1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor\right\}\right)$ is 1, it is obvious that the diameter of the circulant graph $G\left(n;\left\{1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)$ is 2.

***Theorem* 3.1:** The radio number of the circulant graph $G\left(n;\left\{1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)$ is given by $G\left(n;\left\{1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)=n$.

Proof: Let $V\left(G\left(n;\left\{1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)\right)=\{v_1,v_2\ldots v_n\}$. Define an injection $h:\{v_1,v_2\ldots v_n\}\to N$ as follows: $h(v_i)=2i-1, i=1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor$, $h\left(v_{\left\lfloor\frac{n}{2}\right\rfloor+i}\right)=2i, i=1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor$.

Next, we claim that the defined injection h is a valid radio labeling. Using Lemma 3.1, we must verify the radio labelling condition $d(x,y)+|h(x)-h(y)|\ge 3\ \forall\ x,y\in V\left(G\left(n;\left\{1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)\right)$.

Case 1: If $x=v_k$ and $y=v_m$, $1\le k\ne m\le\left\lfloor\frac{n}{2}\right\rfloor$, then $d(x,y)\ge 1, h(x)=2k-1$ and $h(y)=2m-1$. Hence $d(x,y)+|h(x)-h(y)|\ \ge 1+2(k-m)\ge 3$, since $k\ne m$.

Case 2: If $x=v_{\left\lfloor\frac{n}{2}\right\rfloor+k}$ and $y=v_{\left\lfloor\frac{n}{2}\right\rfloor+m}$, $1\le k\ne m\le\left\lfloor\frac{n}{2}\right\rfloor$, then $d(x,y)\ge 1, h(x)=2k$ and $h(y)=2m$. Hence $d(x,y)+|h(x)-h(y)|\ \ge 1+2(k-m)\ge 3$, since $k\ne m$.

Case 3: If $x=v_k$ and $y=v_{\left\lfloor\frac{n}{2}\right\rfloor+m}$, $1\le k\le\left\lfloor\frac{n}{2}\right\rfloor, 1\le m\le\left\lfloor\frac{n}{2}\right\rfloor$, then either $d(x,y)\ge 1$ and $|h(x)-h(y)|\ge 2$ or $d(x,y)=2\ |h(x)-h(y)|\ge 1$. In both the cases we have $d(x,y)+|h(x)-h(y)|\ge 3$.

Thus, $d(x,y)+|h(x)-h(y)|\ \ge 3$ for all $x,y\in V\left(G\left(n;\left\{1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)\right)$.

Therefore, h is a radio labelling and $rn(G)\le n$. Since the mapping is an injection, all the n vertices of $G\left(n;\left\{1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)$ received different radio labelling. Hence, we conclude that the radio number of $G\left(n;\left\{1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor-1\right\}\right)$ is exactly n.

Lemma 3.2: The diameter of $G\left(n;\left\{1,\frac{n}{3}\right\}\right)$ is $\left\lfloor\frac{n}{6}\right\rfloor+1$, whenever $n\equiv 0(mod\ 3)$.

Proof: As in the proof of Lemma 3.2, with a common difference of length $\frac{n}{3}-1$ in the outer cycle, we construct the graph $G\left(n;\left\{1,\frac{n}{3}\right\}\right)$ by joining the vertices v_1 to $v_{\frac{n}{3}}$, v_2 to $v_{\frac{n}{2}+1}$ .... Therefore, the diameter of $G\left(n;\left\{1,\frac{n}{3}\right\}\right)$ is $\left\lfloor\frac{n}{6}\right\rfloor+1$.

Theorem 3.2: The radio number of $G\left(n;\left\{1,\frac{n}{3}\right\}\right), n\equiv 0(mod3)$, satisfies $rn\left(G\left(n;\left\{1,\frac{n}{2}\right\}\right)\right)\le$

$$\begin{cases}\left(\frac{n}{6}+1\right)\left(\frac{n}{2}+1\right)+\left\lceil\frac{n}{12}\right\rceil+1, \text{if } n \text{ is even}\\ \left\lceil\frac{n}{6}\right\rceil\left(\frac{n-1}{2}\right)+, \text{if } n \text{ is odd.}\end{cases}$$

Proof: We partition the vertex set $V=\{v_1,v_2\ldots v_n\}$ into four disjoint set V_1, and V_2, where $V_1=\left\{v_1,v_2\ldots v_{\left\lceil\frac{n}{2}\right\rceil}\right\}, V_2=\left\{v_{\left\lceil\frac{n}{2}\right\rceil+1}, v_{v_{\left\lceil\frac{n}{2}\right\rceil+2}}\ldots v_n\right\}$. We discuss the proof for n even and odd case separately.

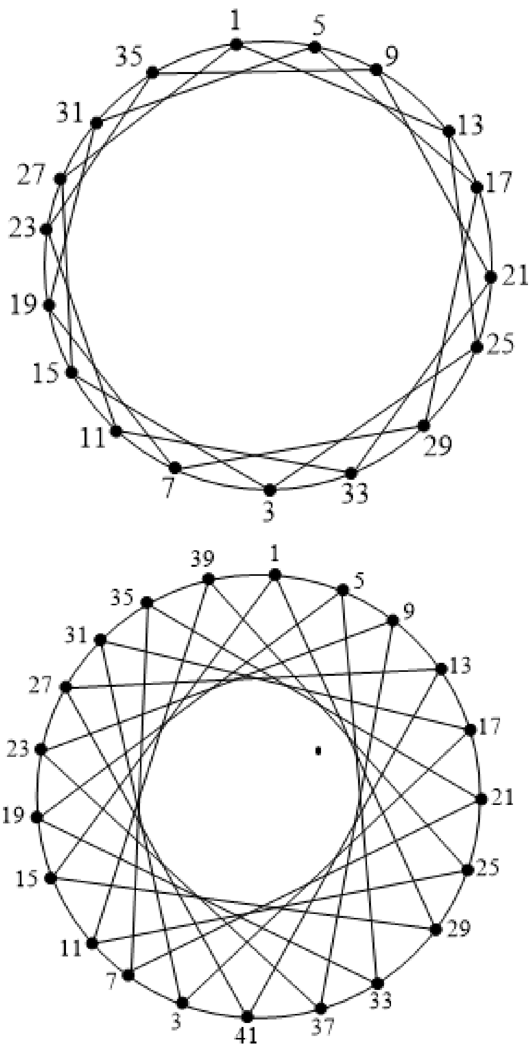

Figure 1: Radio labelling of circulant graphs $G(n;\{1,\frac{n}{3}\})$ with $n=18$ and 21.

Case 1: n is even

Define a mapping $h: V\left(G\left(n;\left\{1,\frac{n}{3}\right\}\right)\right) \to N$ as follows:

$h(v_i) = \left(\frac{n}{6}+1\right)(i-1)+1, i = 1,2 \ldots \frac{n}{2}$.

$h\left(v_{\frac{n}{2}+i}\right) = \left(\frac{n}{6}+1\right)(i-1)+\left\lceil\frac{n}{12}\right\rceil+1, i = 1,2 \ldots \frac{n}{2}$. See Figure 1.

Case 1.1: Suppose $x = v_k$ and $y = v_m$, $1 \le k \ne m \le \frac{n}{2}$, then $d(x,y) \ge 1$. Also, $h(x) = \left(\frac{n}{6}+1\right)(k-1)+1$, and $h(y) = \left(\frac{n}{6}+1\right)(m-1)+1$,. Hence $d(x,y)+|h(x)-h(y)| \ge \left(\frac{n}{6}+1\right)(k-m) \ge 2+\left\lceil\frac{n}{6}\right\rceil$, since $k \ne m$.

Case 1.2: If $x = v_{\frac{n}{2}+k}$ and $y = v_{\frac{n}{2}+m}$, $1 \le k \ne m \le \frac{n}{2}$, then $|h(x)-h(y)| = \left|\left(\frac{n}{6}+1\right)(k-1)+\left\lceil\frac{n}{12}\right\rceil+1-\left(\left(\frac{n}{6}+1\right)(m-1)+\left\lceil\frac{n}{12}\right\rceil+1\right)\right| \ge \left(\frac{n}{6}+1\right)(k-m)$ and $d(x,y) \ge 1$. Hence $d(x,y)+|h(x)-h(y)| \ge \left\lceil\frac{n}{6}\right\rceil+2$, since $k \ne m$.

Case 1.3: If $x = x = v_k$ and $y = y = v_{\frac{n}{2}+m}$, $1 \le k, m \le \frac{n}{2}$ then $d(x,y) \ge 1$ and $|h(x)-h(y)| \ge \left|\left(\left(\frac{n}{6}+1\right)(k-1)+1\right)-\left((m-1)\left(\frac{n}{4}-1\right)+2\right)\right| \ge \left\lceil\frac{n}{6}\right\rceil+1$.

Therefore $d(x,y)+|h(x)-h(y)| \ge \left\lceil\frac{n}{6}\right\rceil+2$.

Thus, the radio labelling condition is true for the case when n is even.

Case 2: n is odd

Define an injection $h: V\left(G\left(n;\left\{1,\frac{n}{3}\right\}\right)\right) \to N$ as follows:

$h(v_i) = \left\lceil\frac{n}{6}\right\rceil(i-1)+1, i = 1,2 \ldots \frac{n+1}{2}$.

$h\left(v_{\frac{n}{2}+i}\right) = \left\lceil\frac{n}{6}\right\rceil(i-1)+\left\lceil\frac{n}{12}\right\rceil+1, i = 1,2 \ldots \frac{n-1}{2}$.

Proceeding as in previous case, we can show h is a radio labelling and that $rn\left(G\left(n;\left\{1,\frac{n}{2}\right\}\right)\right) \le \begin{cases}\left(\frac{n}{6}+1\right)\left(\frac{n}{2}+1\right)+\left\lceil\frac{n}{12}\right\rceil+1, \text{if } n \text{ is even}\\ \left\lceil\frac{n}{6}\right\rceil\left(\frac{n-1}{2}\right)+, \text{if } n \text{ is odd.}\end{cases}$.

Lemma 3.3: For $n \equiv 0(mod\ 4)$, the diameter of $G\left(n;\left\{1,\frac{n}{2}\right\}\right)$ is $\left\lceil\frac{n}{4}\right\rceil$.

Proof: Let $\{v_1, v_2 \ldots v_n\}$ be the vertices in the outer circle. We construct the graph $G\left(n;\left\{1,\frac{n}{2}\right\}\right)$ by joining the vertices v_1 to $v_{\frac{n}{2}}$, v_2 to $v_{\frac{n}{2}+1}$ … with a common difference of length $\frac{n}{2}-1$ till the process of joining gets over. Therefore, the maximum distance from a vertex to another vertex is at least half of $\frac{n}{2}-1$, which is equal to $\left\lceil\frac{n}{4}\right\rceil$, since $n \equiv 0(mod\ 4)$.

Theorem 3.3: The radio number of $G\left(n;\left\{1,\frac{n}{2}\right\}\right), n \equiv 0(mod\ 4), n > 16$, satisfies $rn\left(G\left(n;\left\{1,\frac{n}{2}\right\}\right)\right) \le \left(\frac{n}{2}-1\right)\left(\frac{n}{4}-1\right)+4$.

Proof: We partition the vertex set $V = \{v_1, v_2 \ldots v_n\}$ into four disjoint set V_1, V_2, V_3 and V_4. Let $V_1 = \left\{v_1, v_2 \ldots v_{\frac{n}{4}}\right\}$, $V_2 = \left\{v_{\frac{n}{4}+1}, v_{\frac{n}{4}+2} \ldots v_{\frac{n}{2}}\right\}$, $V_3 = \left\{v_{\frac{n}{2}+1}, v_{\frac{n}{2}+2} \ldots v_{\frac{3n}{2}}\right\}$ and $V_3 = \left\{v_{3\frac{n}{2}+1}, v_{3\frac{n}{2}+2} \ldots v_n\right\}$.

Define a mapping $h: V\left(G\left(n;\left\{1,\frac{n}{2}\right\}\right)\right) \to N$ as follows:

$h(v_{2i-1}) = (i-1)\left(\frac{n}{4}-1\right), i = 1,2 \ldots \left\lceil\frac{n}{8}\right\rceil$.

$h(v_{2i}) = \left(\left\lceil\frac{n}{8}\right\rceil+(i-1)\right)\left(\frac{n}{4}-1\right)+1, i = 1,2 \ldots \left\lceil\frac{n}{8}\right\rceil$.

$h\left(v_{\frac{n}{4}+2i-1}\right) = (i-1)\left(\frac{n}{4}-1\right)+2, i = 1,2 \ldots \left\lceil\frac{n}{8}\right\rceil$.

$h\left(v_{\frac{n}{4}+2i}\right) = \left(\left\lceil\frac{n}{8}\right\rceil+(i-1)\right)\left(\frac{n}{4}-1\right)+2, i = 1,2 \ldots \left\lceil\frac{n}{8}\right\rceil$.

$h\left(v_{n-\left(2\left\lceil\frac{n}{8}\right\rceil+2i-3\right)}\right) = \left(\frac{n}{4}+i-1\right)\left(\frac{n}{4}-1\right)\right)+4, i = 1,2 \ldots \left\lceil\frac{n}{8}\right\rceil$

$h\left(v_{n-\left(2\left\lceil\frac{n}{8}\right\rceil+2i-2\right)}\right) = \left(\left\lceil\frac{n}{8}\right\rceil+i+\frac{n}{4}-1\right)\left(\frac{n}{4}-1\right)\right)+4, i = 1,2 \ldots \left\lceil\frac{n}{8}\right\rceil$.

$h\left(v_{n-2(i-1)}\right) = \left(\frac{n}{4}+(i-1)\right)\left(\frac{n}{4}-1\right)+3, i = 1,2 \ldots \left\lceil\frac{n}{8}\right\rceil$.

$h\left(v_{n-(2i-1)}\right) = \left(\left\lceil\frac{n}{8}\right\rceil+\frac{n}{4}+(i-1)\right)\left(\frac{n}{4}-1\right)+3, i = 1,2 \ldots \left\lceil\frac{n}{8}\right\rceil$. See Figure 2.

Next, we verify that $d(x,y)+|h(x)-h(y)| \ge 1+\left\lceil\frac{n}{4}\right\rceil$ for all $x, y \in V\left(G\left(n;\left\{1,\frac{n}{2}\right\}\right)\right)$

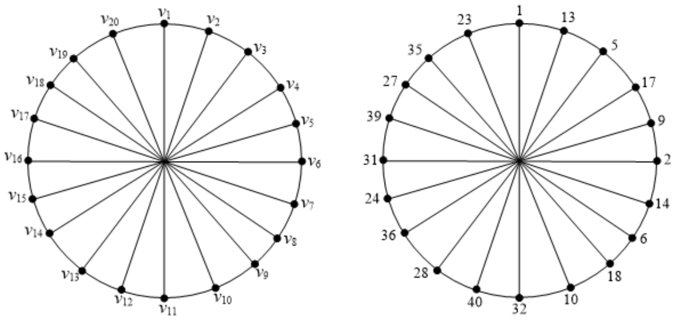

Figure 2: A circulant graph graph $G(20, \{1, 10\})$ and its radio labelling

Case 1: Suppose $x = v_{2k-1}$ and $y = v_{2m-1m}$, $1 \le k \ne m \le \left\lceil\frac{n}{8}\right\rceil$, then the distance between them is at least 1. Also, $h(x) = (k-1)\left(\frac{n}{4}-1\right)$ and $h(y) = (m-1)\left(\frac{n}{4}-1\right)$.

Hence $d(x,y)+|h(x)-h(y)| \ge 1+\frac{n}{4}(k-m) \ge 1+\left\lceil\frac{n}{4}\right\rceil$, since $k \ne m$.

Case 2: If $x = v_{2k}$ and $y = v_{2m}$, $1 \le k \ne m \le \left\lfloor \frac{n}{8} \right\rfloor$, then $h(x) = \left(\left\lfloor \frac{n}{8} \right\rfloor + (k-1)\right)\left(\frac{n}{4} - 1\right)$ and $h(y) = \left(\left\lfloor \frac{n}{8} \right\rfloor + (m-1)\right)\left(\frac{n}{4} - 1\right)$ and $d(x,y) \ge 1$. Hence $d(x,y) + |h(x) - h(y)| \ge \frac{n}{4}(k-m) \ge 1 + \left\lfloor \frac{n}{4} \right\rfloor$, since $k \ne m$.

Case 3: If $x = v_{\frac{n}{4}+2k-1}$ and $y = v_{\frac{n}{4}+2m-1}$, $1 \le k \ne m \le \left\lfloor \frac{n}{8} \right\rfloor$, then $d(x,y) \ge 1$ and $|h(x) - h(y)| \ge \left|(k-1)\left(\frac{n}{4} - 1\right) + 2 - \left((m-1)\left(\frac{n}{4} - 1\right) + 2\right)\right| \ge \left\lfloor \frac{n}{4} \right\rfloor$, since $k \ne m$.

Therefore $d(x,y) + |h(x) - h(y)| \ge 1 + \left\lfloor \frac{n}{4} \right\rfloor$.

Case 4: If $x = v_{\frac{n}{4}+2m}$ and $y = v_{\frac{n}{4}+2k}$, $1 \le k \ne m \le \left\lfloor \frac{n}{8} \right\rfloor$, then $d(x,y) \ge 1$ and the modulus difference of $h(x)$ and $h(y)$ is altleast $\left\lfloor \frac{n}{4} \right\rfloor$.

Therefore $d(x,y) + |h(x) - h(y)| \ge 1 + \left\lfloor \frac{n}{4} \right\rfloor$.

$$|h(x) - h(y)| \ge \left|(k-1)\left(\frac{n}{4} - 1\right) + 2 - \left((m-1)\left(\frac{n}{4} - 1\right) + 2\right)\right| \ge \left\lfloor \frac{n}{4} \right\rfloor, \text{since } k \ne m.$$

Therefore $d(x,y) + |h(x) - h(y)| \ge 1 + \left\lfloor \frac{n}{4} \right\rfloor$.

Case 5: Suppose x and y are of the form $v_{n-(2\left\lfloor \frac{n}{8} \right\rfloor+2k-3}$ and $v_{n-(2\left\lfloor \frac{n}{8} \right\rfloor+2m-3}$, $1 \le k \ne m \le \left\lfloor \frac{n}{8} \right\rfloor$, then the distance between them is at least 1 and $|h(x) - h(y)| \ge \left|\left(\left(\frac{n}{4} + k - 1\right)\left(\frac{n}{4} - 1\right) + 4\right) - \left(\frac{n}{4} + m - 1\right)\left(\frac{n}{4} - 1\right) + 4\right| \ge \left|\left(\frac{n^2}{16} + k\frac{n}{4}\right) - \left(\frac{n^2}{16} + m\frac{n}{4}\right)\right|$.

Hence $d(x,y) + |h(x) - h(y)| \ge 1 + \left\lfloor \frac{n}{4} \right\rfloor$, since $k \ne m$.

Case 6: If $x = v_{2k-1}$ and $y = v_{\frac{n}{4}+2k}$, $1 \le k \le \left\lfloor \frac{n}{8} \right\rfloor$, $1 \le m \le \left\lfloor \frac{n}{8} \right\rfloor$, then $d(x,y) \ge 3$ and $h(x) = (k-1)\left(\frac{n}{4} - 1\right)$, $h(y) = \left(\left\lfloor \frac{n}{8} \right\rfloor + (k-1)\right)\left(\frac{n}{4} - 1\right) + 2$.

Therefore $d(x,y) + |h(x) - h(y)| \ge 3 + \left|\left(\left\lfloor \frac{n}{8} \right\rfloor + (m-1)\right)\left(\frac{n}{4} - 1\right) + 2 - \left((k-1)\left(\frac{n}{4} - 1\right)\right)\right| \ge 3 + \left\lfloor \frac{n}{8} \right\rfloor\left(\frac{n}{4} - 1\right) > \left\lfloor \frac{n}{4} \right\rfloor + 1$.

Case 7: Suppose $x = v_{n-(2\left\lfloor \frac{n}{8} \right\rfloor+2k-2)}$ and $v_{n-(2m-1)}$, $1 \le k, m \le \left\lfloor \frac{n}{8} \right\rfloor$, then $h(x) = \left(\left\lfloor \frac{n}{8} \right\rfloor + m + \frac{n}{4} - 1\right)\left(\frac{n}{4} - 1\right) + 4$ and $h(y) = \left(\left\lfloor \frac{n}{8} \right\rfloor + \frac{n}{4} + (m-1)\right)\left(\frac{n}{4} - 1\right) + 3$.

Also $d(x,y) \ge 2$. Hence, we get$(x,y) + |h(x) - h(y)| \ge 2 + \frac{n}{4} + 1 > \left\lfloor \frac{n}{4} \right\rfloor + 1$.

Similarly, we can prove the rest of the cases. Thus, h is a valid radio labelling and satisfies $rn\left(G\left(n; \left\{1, \frac{n}{2}\right\}\right)\right) \le \left(\frac{n}{2} - 1\right)\left(\frac{n}{4} - 1\right) + 4$.

Lemma 3.4: If $n \equiv 0 (mod\ 10)$, then the diameter of $G\left(n; \left\{1, \frac{n}{5}\right\}\right)$ is $\frac{n}{10} + 2$.

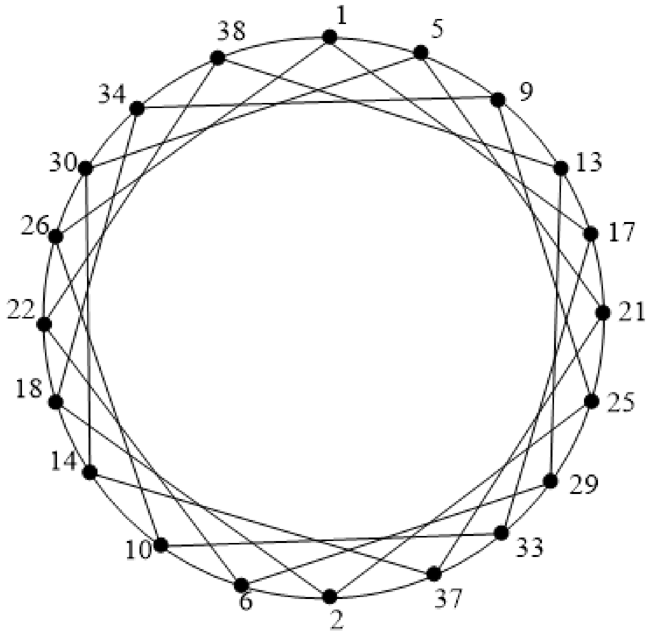

Figure 3: Radio labelling of a circulant graph $G(n; \{1, \frac{n}{5}\})$ with $n = 20$.

Proof: As the proof is similar to Lemma 3.2, we omit the proof.

Theorem 3.4: The radio number of $G\left(n; \left\{1, \frac{n}{5}\right\}\right)$, $n \equiv 0 (mod\ 10)$, satisfies $rn\left(G\left(n; \left\{1, \frac{n}{2}\right\}\right)\right) \le \left\{\frac{n}{20}(n + 18) + \left\lfloor \frac{n}{20} \right\rfloor\right.$.

Proof: We omit the proof. Figure 3, illustrates the proof of the theorem

4. Conclusion

In this article we have obtained the radio number of certain classes of circulant graphs. Further the work is extended to other extensions of channel assignment problems such as radial radio number, antipodal radio mean labelling etc.

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