

A Mathematical Analysis of Blood Flow through Artery with Mild Stenosis

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ABSTRACT

Presented herein are the studies of blood flow through artery with mild stenosis. The parameter specified μ and R . It has been observed that the increases the viscosity increase the load capacity. Again it has been observed that increases the value of R decreases the load capacity.

KEYWORDS: artery, blood flow, biological fluid, load capacity, human skeletal

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INTRODUCTION

The term stenosis is the narrowing of the artery due the development of arteriosclerotic plaques. The lumen of the artery, blood flow is obstructed. The obstruction may damage the interval cells of the wall and may lead to further growth of the stenosis. Then there is a coupling between the growth of the stenosis and the flow of blood in the artery affect each other. The stenosis growth usually passes. There is no separation of flow and there is no back flow.

The development of stenosis in an artery can have serious consequences. It may lead to increased resistance to flow, with possible serve reduction in blood flow and increased danger of complex occlusion. Abnormal cellular growth in the vicinity of the stenosis, which increases the intensity of the stenosis and damage tissues leading to post-stenosis dilatation.

Blood has been taken homogeneous which obeys the Newton's law of friction. But this law hold goods if the flow occurs in tubes in which internal diameter is large compared with the size of the red cells. Most of the biological fluid are in fact non-Newtonian. In a study of the non-Newtonian behaviour of blood. It is observed that the non-Newtonian fluid flow involves many new features.

However for most purposes blood can be treated theoretically as an ordinary with an appropriate effective viscosity coefficient.

Physiologist concerned the blood circulation problem of wave propagation in a system of tube. The problem too has been studied by various authors theoretically as well experimentally^{1,2,3} Some investigator have studied the problem. Biomechanics has attracted mathematicians, engineering's, and scientists to study the functioning behavior of human skeletal system.

Various mathematician's considered the problem of lubrication of approaching porous surfaces in reference to human joints^{9,10}. Having discussed the structural properties of the arterial, we now turn our attention to the transport medium. Physiologically, amongst other function blood serve as transportation of respiratory gases, nutrients, waste products of metabolism etc. Blood is a suspension of particles in an aqueous solution, it is not a homogeneous fluid and has some unusual properties.

Blood flow problems are more complicated than the fluid flow problems. The properties arise from fact that the vessels walls are formed of different substances such as

elastic, collagen and smooth muscles with entirely different properties.

In this research paper we considered the steady flow of a Newtonian fluid in a stenosis.

Formulation of the problem

The detail some of the model outputs will be performed. This model is of relevance studies in particularly in the real world. In this model considered the study flow of a Newtonian fluid in a stenosis.

The Navier stokes equation;

$$-\frac{\partial p}{\partial r} = 0 \quad (1)$$

and

$$0 = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} \right) \quad (2)$$

Equation (2) can be written as

$$0 = -\frac{\partial p}{\partial z} + \frac{\mu}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) \quad (3)$$

The no-slip condition on the stenosis, Surface gives

$$\begin{aligned} V &= 0 \text{ at } r = R \\ V &= 0 \text{ at } r = R_0 \end{aligned} \quad (4)$$

And

$$\frac{\partial V}{\partial r} = 0 \text{ at } r = 0 \quad (5)$$

Result and Discussion

The present paper proposes a more realistic model for explaining the blood flow in a stenosis. The load capacity depends on various values of parameter μ and R . It has been observed that the increases the viscosity increase the load capacity. Again it has been observed that increases the value of R decrease the load capacity.

Solution of the problem

Differentiating equation (3) partially w.r.to z , we get

$$-\frac{\partial^2 p}{\partial z^2} = 0 \quad (6)$$

Integrating we get

$$-\frac{\partial p}{\partial z} = \Delta p \quad (7)$$

From equation (3) and (7), we get

$$\frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) = \frac{\Delta p r}{\mu} \quad (8)$$

Integrating equation (8) and applying the boundary condition, we get

$$V = \frac{\Delta p}{4\mu} (r^2 - R^2) \quad (9)$$

The volume flow rate is

$$\begin{aligned} Q &= \int_0^R V 2\pi r dr \\ &= \frac{\pi \Delta p}{2\mu} \int_0^R (r^2 - R^2) r dr \\ &= \frac{\pi \Delta p}{8\mu} R^4 \end{aligned} \quad (10)$$

But volume flow rate Q is constant for all section of tube, then

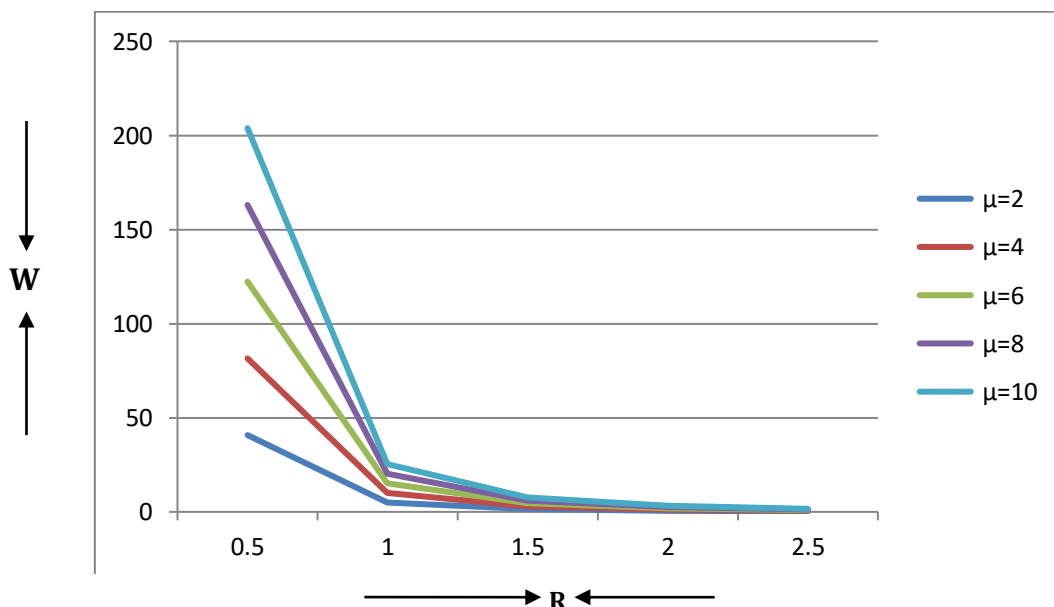
$$\Delta p = \frac{8\mu Q}{\pi R^4} \quad (11)$$

The load capacity is

$$\begin{aligned} W &= \int_0^R \Delta p dr \\ &= \left(\frac{8\mu Q}{\pi R^3} \right) \end{aligned} \quad (12)$$

Values load capacity W for different values μ & R at $Q=1$

$R \backslash \mu$	2	4	6	8	10
0.5	40.80	81.52	122.32	163.04	203.84
1.0	5.10	10.19	15.29	20.38	25.48
1.5	1.51	3.02	4.52	6.03	7.54
2.0	0.64	1.27	1.91	2.55	3.19
2.5	0.33	0.65	0.98	1.30	1.63



Variation of load capacity W for different values μ & R at $Q=1$

Reference

- [1] Downson, D. (1967) Model of lubrication an human Joints Proc. Inst. Mech. Engrs. 183(3).
- [2] Mow, C. W. (1968) the role of lubrication of Biomechanical joints J. lubr. Tech.91,320
- [3] Ogston, A. G. and J. E. Stanier (1953) Viscous elastic lubricant properties Physiol. 119, p244-252
- [4] Tandon, P. N. and S. Jaggi (1977) Lubrication of Hertzian contacts in reference to human joints. Med. Life Sci. Eng. 3(7)
- [5] Tandon, P. N. and S. Jaggi (1977) Lubrication of porous solid in reference to human joints Pro. Ind. Acad. Sci. Sect. A85, 144.
- [6] Yadav A. K. and Pokhriyal S. C. (2000): synovial fluid flow in reference to human joints. Ind. J. Appl. Sci. Periodical vol. 2, p 104-110.
- [7] Yadav, A. K. (2000) Synovial fluid behavior in reference to animal joints. Indian Jr. Pure and Apple. Sci. Vol. 19(E), 311-316
- [8] Yadav A. K and Kumar S. (2016): Synovial fluid flow in reference to animal joints. Intr. Jr. Stream Research Vol. 6 issu-12, p-1-5.
- [9] Yadav A. K. and Kumar S. (2016): Mathematical analysis for the lubrication mechanism of knee joints. Intr. Jr. of orthopaedics Photon. Vol.11, p 120-122.
- [10] Yadav A.K. and Kumar S. (2020): Behaviour of synovial fluid in a channel. IJCRT Vol. 8 issu.- 3 p 2823-2833.

