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Learning of Advanced Mathematics by Chinese Liberal Arts Students: A Study of Developing Applied Mathematical Ability

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ABSTRACT

According to statistics, more than 90% of Chinese universities have offered advanced mathematics for liberal arts students as a general course since 2003 to cultivate their scientific literacy and problem-solving ability. This study aims to provide a feasible approach for liberal arts students to accept the value of mathematics and apply mathematical methods to academic research and real-world problems solution. In the *Applied Probability and Statistics* class with the attendance of 154 liberal arts freshmen, this study is designed to discuss and value on the five mathematics application reports as examples. Then liberal arts students were interviewed on evaluating these 5 mathematics application reports. 28 mathematics application reports were completed by 154 students in groups and cooperation, which were analyzed on how mathematics was incorporated into their reports. The qualitative analysis of the interview results found that the discussion and evaluation of these 5 mathematics application reports about applying mathematics and solving social problems can stimulate liberal arts students' interest in mathematics and realize the value of mathematics application. Therefore, peer mathematics application is a way for liberal arts students to realize mathematics value. Through the classification analysis of the 28 mathematics application reports completed by the liberal arts students, their three forms of mathematics application can be summarized as follows: applying the regularity of mathematical argument instead of the special case of the practice test in social cognition; using data analysis to monitor the probabilities of plausible reasoning; developing their applied mathematical ability to solve daily life problems.

KEYWORDS: *Advanced Mathematics, Applied Mathematical Ability, Liberal Arts Students, Qualitative research method*

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1. INTRODUCTION

Chinese liberal arts majors have been offered advanced mathematics courses for only 40 years. As Yan S.J. (2001) noted, there were two reasons why liberal arts students would learn advanced mathematics. The first reason is the interactions between mathematics and other science fields have led to the production of new ideas and ways of thinking. The second reason is that mathematics plays a crucial role in forming rational thinking. And, since there is a view that many liberal arts students may lack mathematical skills and understandings, that then they should learn and apply mathematics to develop their reasoning ability (Michal, A., Ruhama E., 2010). In addition, when using inductive thinking to study various social problems, they should know that the possibilities afforded by mathematical reasoning. The application of mathematics knowledge is extensive, but the application of advanced mathematics for liberal arts students is waiting to be developed. In the current advanced mathematics teaching for liberal arts, students still learn mathematics passively and are lack of awareness of mathematics application. Therefore, it is necessary to develop liberal arts students' applied mathematics awareness and ability.

As liberal arts students are studying humanities and social sciences, it can be found that social sciences have different features from natural sciences, especially the complexity of the research problems, the causal connection, and the influence of recognition subject. However, the social development law revealed by the social sciences has statistical significance with the unification of probability and inevitability. At this point, the probabilistic statistical method can play an important role in social science research.

What is the attitude of mathematics learning for liberal arts students? It can be seen from the literature that the learning interest of advanced mathematics for liberal arts students needs to be improved. According to Gu Pei's statistics in 2003, more than 90% of Chinese universities have offered advanced mathematics for liberal arts students. Various advanced mathematics textbooks have been published that are suitable for liberal arts students. Sheng L.R.et al. (2005) proposed that an important means to cultivating students' innovative ability in universities is to make liberal arts and mathematics permeate into each other.

Sheng (2006) tried to offer an additional mathematical course for liberal arts students to introduce the theory and method of using mathematics in social science research, e.g. the feasible region in the linear programming problems, the fair social distribution, and the probability model in data analysis, etc. Liberal arts students who have studied this course were shown to think that such a mathematics course is more flexible, easier, and practical.

While filling in the literature gap on developing students' applied mathematics ability in teaching aspect, this paper aims to make a future contribution in several aspects: (1) to provide a feasible approach for liberal arts students to accept the value of mathematics; (2) to help liberal arts students to solve problems by applying probability and statistics methods they have learned.

2. Literature Review

The existing literature has mainly focused on the construction of advanced mathematics courses and teaching materials, the improvement of teaching methods, and the increasing of application examples in mathematics textbooks (Sun Y., 2010), but few focuses on the research of applied mathematical ability of liberal arts students. Nevertheless, it is of vital significance to develop their applied mathematics ability. The advanced mathematics course for liberal arts should not only help students understand the application of mathematics but also cultivate their applied mathematics ability. Here, applied mathematics ability refers to the ability to analyze and solve practical problems, such as problems in students' daily lives and social phenomena, by applying advanced mathematics knowledge, mathematical logic thinking and reasoning methods, and the ability to extract general laws from complicated cases. Advanced mathematics courses are frequently taught in what is colloquially referred to as a 'definition-theorem-proof' (DTP) format. This format has its disadvantages, as Davis and Hersh (1981) asserted that 'a typical lecture in advanced mathematics consists entirely of definition, theorem, proof and so on, which can prevent students from having an intuitive understanding of the truth about theorems (Hersh, 1993) and discourage them from using informal ways of understanding mathematics to produce proofs (e.g., Dreyfus, 1991). Therefore, the abstraction feature of mathematics may be difficult for common liberal arts students to understand, and it is necessary to find suitable ways for liberal arts students to learn advanced mathematics effectively.

In this study, it is important to develop students' applied mathematics ability by writing mathematics application reports. In the literature, any kind of connection between mathematics and reality can be regarded as an application of mathematics (Sloyer, Blum and Huntley, 1995) whereas applied mathematics is to explain the real world by using the language of mathematics, answering important questions by using mathematics and applying mathematics to real life. Applied mathematics problems may be from the fields of biology, finance, medicine, business, and so on and require more complex and longer processes relative to real-life problems (Pollak, 1976). In this study, the applied mathematics ability we concern is focused on liberal arts students in the university so that the application of advanced mathematics can apply knowledge in daily life or social science. In these kinds of situations, individuals try to understand the situations they encounter in their lives, analyze and interpret them, and reach certain results or make certain decisions about them (Lubinski, D., Humphreys, L. G., 1990).

In order to develop liberal arts students' applied mathematics ability, it is important to find out a suitable approach to teach and learn. Leung, Low, and Sweller (1997) showed the facts that can help liberal arts students to learn mathematics with high motivation. They found that students benefited from verbal explanations of mathematics problems, at least until they gained a greater facility at solving these problems. Therefore, it is important for the liberal arts students in this study to explain mathematical theorems in their own thinking and language in order to gain a deeper understanding. Bracha and Kramarski (2006) also pointed out that the mathematical literacy was closely related to mathematical problem-solving ability, with the analysis that PISA usually assesses mathematical literacy in relation to mathematics curriculum-based knowledge, mathematical skills, and problem situations in daily life.

With regard to the research method, this study uses a case analysis method to analyze liberal arts students' comments and mathematical application reports, in order to develop students' applied mathematics ability. In fact, strong evidence is available both from mathematics learners' self-reporting (e.g., Halmos, 1994) and from research studies focusing either on interviews with lecturers (e.g., Iannone and Nardi, 2005) or interviews and observation of practice. Additionally, Weber (2004) reported a semester-long case study on how one professor taught real analysis in a traditional manner, regularly interviewing him about his teaching practices. Over the past 20 years, the first author in this paper has taught advanced mathematics to liberal arts students. To develop the applied mathematical ability of liberal arts students and find the links between social science problems and mathematics, this study encouraged liberal arts students to choose their own research topic, collect evidence materials and use the knowledge of advanced mathematics that they have learned in this semester to solve various specific problems, thereby developing their applied mathematical ability.

Based on the literature analysis above, this study aims to provide a feasible approach for liberal arts students to accept the value of mathematics and apply mathematical methods to academic research and real-world problems solution. In the *Applied Probability and Statistics* class with the attendance of 154 liberal arts students, this study is designed to discuss and value on the five mathematics application reports completed by liberal arts students in the previous semester. Then liberal arts students were interviewed on evaluating the topic and method about these 5 mathematics application reports. Finally, mathematics application reports would be completed by 154 liberal arts students in groups and cooperation. The research questions are listed below.

1. What is the feasible approach for liberal arts students to recognize the value of mathematics?
2. How to qualitatively analyze the role of mathematics in mathematical application reports completed by liberal arts students?

To answer these research questions, the case analysis method was applied to analyze the mathematical application

reports of liberal arts students, and they were interviewed to elucidate their understanding of these reports.

3. Research Method

In this study, an *Applied Probability and Statistics* course is selected. Probability theory and statistics are both applied mathematical subjects that study and reveal the statistical laws of uncertain random phenomena. In many fields such as venture capital and management decision-making or in people's daily lives, one will encounter situations requiring the selection of the right decision in an uncertain situation. Probability theory provides the most effective theory and method for solving the uncertainty problem. Statistics is based on using observational data to analyze and judge random phenomena and make a reasonable decision. Probability and statistics are widely used in many fields, such as natural sciences, management, humanities, and social sciences. It is because of the wide application of probability and statistics that we decided that students should propose research questions independently, apply mathematics to solve problems based on the research assumption, and finally write their mathematics application reports.

The sample for this study was selected from a general education course at a university in China. At this university (and almost all universities in China), liberal arts students must study advanced mathematics in general education. The first-year general education compulsory course *Applied Probability and Statistics* was selected in this study. For the post-class mathematics homework, the liberal arts students were required to complete mathematics problem-solving assignments after each class. In addition, they were required to complete mathematical application reports by group discussion and cooperation throughout the semester.

This research is based on the previous work lasting for over 10 years, the purpose of which is trying to encourage liberal arts students to develop their mathematics application reports. During this period, 205 math application reports developed by 1123 liberal arts students have been collected (Authors, 2018). Before encouraging liberal arts students to develop mathematics application reports, examples of previous mathematics application reports among different subjects in the early stage were presented to make sure that liberal arts students can fully discuss the content and characteristics of these example reports. On the basis of previous experience, the high quality of mathematics application reports written by liberal arts students is guaranteed. Therefore, the procedure of this study is as *Fig 1*.

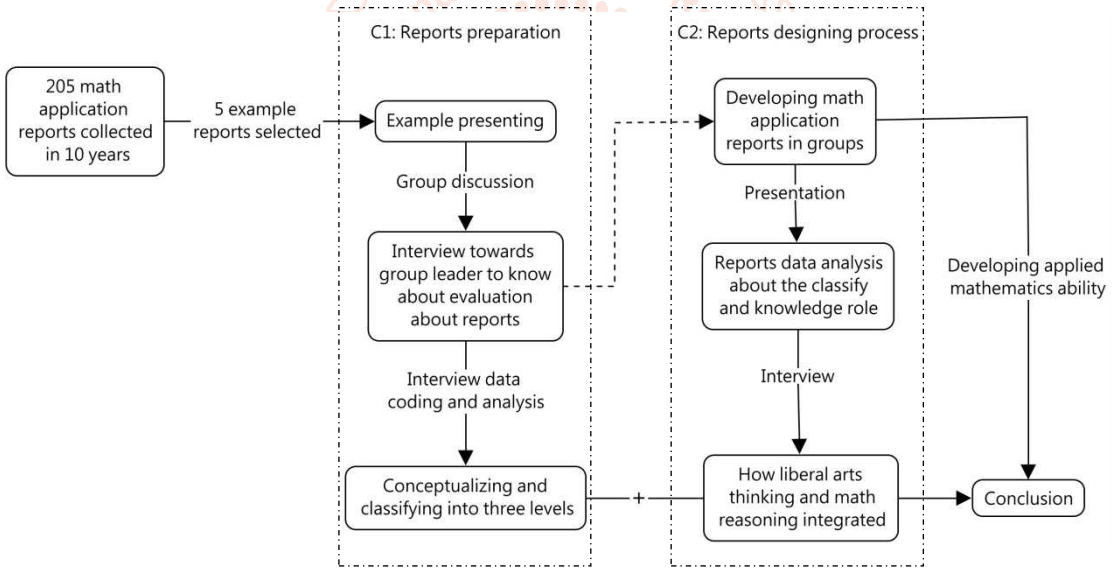

Figure 1 Research path of this study

This study was conducted in the *Applied Probability and Statistics* course in one semester spanning February to July 2019. The subjects were 154 freshmen who majored in pedagogy, education management, preschool education, and special education. Among them, 80% were female, and 20% were male; 76% of them had studied algebra, trigonometric function, preliminary solid geometry, plane analytic geometry, calculus, statistical cases, reasoning and proof, and expansion of the number system in high school while others had studied extra mathematics topics such as vector and solid geometry, permutation and combination, and probability. In *Applied Probability and Statistics* course, students need to study probabilistic and statistical knowledge, such as classical probability, geometry approach, conditional probability, total probability Formula, Bayes Formula, Bernoulli probability, binomial distribution, normal distribution, mathematical expectation, variance, central limit theorem; descriptive statistics, correlation analysis, sampling, point estimation, interval estimation, etc. (Authors, 2016). At the beginning of the new semester in this study, five of these mathematical application reports were selected and presented to the 154 subjects in class in order to give them a demonstration and inspiration. Regarding the fields covered in the five reports, two belonged to science fields, two belonged to social sciences, and one pertained to students' daily lives. The 154 liberal arts students were divided into 28 groups to discuss the features of the 5 presentation reports. Each group had a group leader selected by the members, and they would be taken an interview to make comments on the 5 example reports. Then, each group discussed its own research topic and implemented the process of writing the mathematical application report. The groups were required to submit their mathematical application reports as group assignments within one semester. During the semester, the organization of data collection could be divided into two cycles, as described below. The first author was the teacher of this course and had a history of involvement in such endeavors dating back 20 years.

3.1. Five Reports as Examples of Mathematical Application

At the beginning of the semester, five presentations were selected from the previous reports based on the creation of topic selection, the proper application of mathematical knowledge, the logic of argument and the essential role of mathematics in unexpected conclusions over the past 10 years. These five reports were recorded as R1, R2, R3, R4, and R5. Their titles are as follows.

R1: Which one is more difficult, receiving an offer from Tsinghua University or winning the first-class lottery?
It is the dream for most Chinese high school students to pass the entrance examination of Tsinghua University, and it is also human nature to desire to win the lottery to become wealthy. Six girls as a group in this study investigated the difficulty of receiving an offer from Tsinghua University versus winning the first-class lottery. This group collected the data about the winning probability of this lottery and calculated that the first-class probability is 1/17,720,000. As a comparison, this group collected the mean and variance data of passing the entrance examination of Tsinghua University and found out that the probability of a high-school student getting the offer from Tsinghua University is about 1.5%. Apparently, the probability of getting an offer from Tsinghua University is higher than the probability of winning the first-class lottery.

R2: What is the use of the Bayesian formula?
The Bayesian formula will be taught in *Applied Probability and Statistics*. Three example questions in the textbook use the Bayesian formula to solve applied problems. However, a group in this study has solved three example questions without using the Bayesian formula but using "addition principle" and "multiplication principle" in an easier way. The group leader believes that the Bayesian formula could be too complicated to use, and some problems could also be solved without it.

R3: It is hard for the “3-high” female to get married.
There is a specialized social problem in some big cities in China where the "3-high" female, which means women with high IQ, high education, and high salary, are less likely to get married. A group in this study collected the birth ratio data of men and women in China. This group applied the normal distribution knowledge learned from the *Applied Probability and Statistics* course and calculated that according to the principle of *3-σ* principle of normal distribution, people with normal IQ accounts for account for 34.1%, people with high IQ accounts for 13.6%, people with higher IQ accounts for 2.14%, and people with super higher IQ accounts for 0.16%. The “3-high” female are already in the 13.6% of the total people and they would like to marry men with higher IQ than themselves in China, which can only be found in the 2.14% of the total male people. The gender ratio of China's population at birth comes from the data of the China Statistical Yearbook. The average IQ of people is quoted from professional literature data, while *high IQ* is the value defined by liberal arts students themselves that are higher than the average IQ.

R4: Star-chasing students’ winning-ticket process
This report has a background about a famous Chinese singer, Yi, who was 18 years old in 2018, the same age as freshmen in this study. A group of 6 fans completed this report to calculate the probability of getting a concert ticket of Yi in different ways. They collected related data and calculated the probability of getting the ticket. The number of tickets in a different way is: 1,200 from the performance company, 100 from microblogs of Yi, 300 from *Taobao.com*, and 200 from *Tmall.com*. Then data showed that the number of buyers is: 30,158 from the performance company, 910,000 from microblogs of Yi, 161,000 from *Taobao.com*, and 340,000 from *Tmall.com*. Assuming the probability of a fan buying this ticket is 0.3 at the performance company, 0.1 at microblogs of Yi, 0.3 at *Taobao.com*, and 0.3 at *Tmall.com*, the probability of this fan buying this ticket successfully is 0.0127. It is noted that the data are collected from the performance company website, microblog of Yi, *Taobao.com* and *Tmall.com* by this group.

R5: Reasonable blood test issues
The mathematical knowledge in R5 is the mathematical expectation. It draws lessons from a large number of conscriptions in the United States during World War II. To check whether the candidates suffer from certain diseases, group blood tests are used. If the blood of a group (k candidates) is qualified, only one test will be performed. If it is not qualified, a total of *k+1* tests will be conducted, one for each of the candidates. This is a grouping inspection method that both ensures the elimination of unqualified candidates and reduces the workload of the method. The liberal arts students in this study applied this method to other problems, such as detecting whether water from different sources is polluted. According to the method mentioned above, it is possible to draw reliable conclusions without adding experimental equipment.

The five cases were divided into scientific situations (R2, R5), social science situations (R1, R3), and students’ daily life situations (R4).

3.2. Cycle 1: The Purpose of Presenting Reports as Examples to Participants

In the first class, 154 liberal arts students received five examples of mathematical applications and were divided into 28 groups to discuss the five reports. The first cycle of data collection consisted of lecture observations and the email collection of subjects’ comments about the 5 reports presented in class. Table 1 summarizes the data collected in this cycle.

Table 1 Breakdown of data collected for cycle 1

Step	Data collected	Purpose
1	5 of the previous mathematical application reports were selected and presented to the 154 subjects in the class	To evaluate 5 examples of mathematical application reports and find out the connection with the probability and statistics knowledge
2	The groups were required to discuss the 5 presented reports	To analyze the role of mathematics in the 5 mathematical application reports
3	28 groups submitted their comments by email, and some of them were interviewed	Liberal arts students are expected to write their own mathematical application reports

It is necessary to note that cycle 1 of the data collection included the discussion of the five typical reports and subsequent interviews with students and that the reports were NOT written that semester but rather in the previous semester. At the beginning of the semester, the liberal arts students had not learned the mathematical knowledge contained in the five reports, but they can see the arguments and demonstrations of the mathematical formulas and calculations in the reports and also evaluate the impression that humanities and social sciences can be integrated into mathematics.

3.3. Cycle 2: Design Process of Mathematical Application Reports

The first cycle of data collection consisted of the mathematical application reports created by the 154 liberal arts students themselves and the interview records of some subjects. During the semester, the liberal arts students cooperated within groups to discuss their ideas, design their reports, and finally submit their reports to their math teacher. Most of them presented their reports in front of the whole class. It is necessary to note that mathematics knowledge in these reports mostly covered the first half of the semester because most of the subjects submitted and presented their mathematical application reports after the middle of the semester. Table 2 summarizes the data collected in this cycle.

Table 2 Breakdown of data collected for cycle 2

Step	Data collected	Purpose
1	Topics of the 28 mathematics application reports	To classify the topics of 28 mathematics application reports
2	The applied mathematics cases from 28 mathematics application reports	To analyze the role of mathematics in the 28 mathematical application reports
~~3~~	Reflections on the process of report creation	To understand and master mathematical method in solving problems

In addition, for this cycle, the mathematical application reports were text recorded, and interviews were audio-recorded by email with verbatim transcription.

This section is divided into two parts: 154 liberal arts students were divided into 28 groups to evaluate five examples of mathematical application reports, and 28 groups wrote new math application reports during the semester.

4. Findings

4.1. Evaluation of the Five Reports by 28 Student Representatives

In this study, 5 previous mathematical application reports were presented to 154 students in the first lesson of advanced mathematics for liberal arts courses as an example model. The students were divided into 28 groups to discuss these reports in groups, and they then submitted their comments. Finally, each group elected a representative whose comments were received by e-mail, and they were recorded as S1, S2, etc. NVivo 10® software was used to edit the 28 written documents and classify them into nodes. Each node corresponds to a topic from a category of co-participants.

The research path of grounded theory was used in this study to import open interview problems and report texts into NVivo 10® software for sorting and coding. This study has repeatedly and continuously compared, analyzed, and generalized the mathematical application reports developed by liberal arts students as well as the interviews based on students' self-evaluation about their mathematical application reports. The theory has finally constructed after gradually conceptualizing and classifying them from low level to high level. The coding of this study can be divided into three levels: the first level open coding, the second level relational coding, and the third level core coding.

First level open coding: interview and report texts were decomposed, compared, conceptualized, and categorized to keep the coding open and close to the data. The two researchers first encoded all the items independently and then tested the consistency of the codes. The results showed that the consistency ratio reached 92%, and the coding methods and rules were clear and definite in general. For the inconsistencies, the coder could be discussed again until an agreement was reached. In the process of first level open coding, researchers have tried to keep the key elements of the text, but at the same time, some colloquial expression was removed, so that a total of 28 first-level codes were formed.

Second level relational coding: based on the first level open coding, genera, and dimensions could be formed to develop and test the relationship between genera and forms related categories. Thus, 8 relational codes were formed.
Third level core coding: after a systematic analysis of all the discovered conceptual genera, the core genera with high generality and integrity were selected, thus forming 4 core codes., or 4 themes, as shown in Table 3.

Table 3 Student's comments about 5 presented reports

Theme	Comments
Theme 1: Liberal arts students' understanding of applied mathematics	(1) The results applied with mathematics are more objective compared with human's subjective wishes (2) Mathematical knowledge can be meaningful only if applied in the real world.
Theme 2: The connections between mathematics and topics from mathematical application reports	(1) Social cognitive problems. (2) Social phenomena.
Theme 3: The role of mathematics in the mathematical application reports	(1) Mathematics can provide objective judgment for social cognition. (2) Mathematical methods can be regarded as tools for explaining social phenomena.
Theme 4: The challenge of mathematics knowledge from liberal arts students	(1) It is unfeasible to blindly follow the complex mathematical formula. (2) Learn to pass on some opinions through deduction, explanation, and recitation of mathematical formulas in real situations.

4.1.1. Theme 1: Liberal Arts Students' Understanding of Applied Mathematics

S10: In report R4, celebrity-chasing girls quoted online sales statistics about celebrity birthday tickets and then used conditional probability and full probability to calculate that the probability of purchasing tickets successfully is very small. The conclusion is regrettable but acceptable given the data support behind it. Some liberal arts students realized that the results applied in mathematics are more objective compared with our subjective wishes.

S11: Without a deep understanding of the topic itself, the solution to the problem will be too rigid; without the ability to grasp the key points, the problem solving will become complicated. In fact, every person's brain stores a considerable number of problem-solving methods, but these methods are often disorganized. When you need to solve a specific problem, due to the confusion of these methods, we prioritize the most familiar methods. Although mathematics method is not a familiar method for liberal arts students themselves, they can see from the 5 reports that mathematics is still available, and is only useful in the process of solving problems.

Therefore, liberal arts students agreed that mathematical knowledge can be meaningful only if applied in the real world.

4.1.2. Theme 2: The Connections Between Mathematics and Topics from Mathematical Application Reports

S5: The report took admission to Tsinghua University and 5 million yuan to be the desires of our youth. These goals attract a very high degree of attention such that they easily to resonate. College students have tried hard to get admitted to prestigious schools and know the difficulties. Such topics give us a sense of substitution. This case is different from the general discussion in society. It applies the probability calculation as the entry point, uses the data to express a viewpoint, and inspires deep thinking with its conclusion. This case shows the connection between social cognitive problems and mathematics.

S9: The reason why it is impressive that *3-high women* have difficulty marrying is that this case reflects the application of statistical probability problems to a general phenomenon of society, indicating that mathematics is not just for calculus or numerical formulas but also relevant to real-life for solving problems. This case shows the connection between social phenomena and mathematics.

4.1.3. Theme 3: The Role of Mathematics in the Mathematical Application Reports

S15: The expressions of social hotspots are diverse, and when they are analyzed by statistics, their essence is mostly the same. Some time ago, a commercial company held an activity about a large-scale lottery "Koi", which became a hot topic. Through the case analysis of small-probability events, people can have a more rational understanding of such activities, and they will not blindly believe that small-probability events come true easily. Therefore, mathematics can provide objective judgment for social cognition.

S14: Report R4 is closely related to the students, so it is easy to attract students' attention. The analysis process of report R4 is as follows: asking questions → collecting data → calculating probability → relying on the results to answer the questions raised before→ rethinking the application of conclusions. Most of the data are from an official source, which is more authoritative, and the probability calculation method is appropriate. The results are not only quite accurate and convincing but also valuable. Therefore, mathematical methods can be regarded as tools for explaining social phenomena.

4.1.4. Theme 4: The challenge of mathematics knowledge from liberal arts students

S10: Thinking stereotypes have a great influence on people's problem-solving. Sometimes they are a good thing to help us solve common problems as soon as possible, but sometimes they limit the divergence of people's thinking, which makes it difficult for us to solve problems creatively. When R2 solves the specific situation, the author can solve the problem by choosing suitable math methods. This is what we students should learn: the mathematical application should not blindly follow knowledge from textbooks but should create our own innovative thinking.

S24: The case *What is the use of the Bayesian formula* can let us break through the shackles of thinking and dare to question authority, which is a kind of courage and praiseworthy; this case is good at thinking rather than blindly excluding with its own theoretical methods, solution system, and valuable innovative thinking. Many students may have glimpsed doubts about the use of Bayesian formula in their minds, but most of them do not have enough confidence to compete with authority, or they do not think deeply and try to solve problems in their own way. Innovation is a kind of knowledge transfer and application. Innovation on the basis of a deep understanding of existing mathematical knowledge is a new way to explore and solve problems. This author's example realized me that liberal arts students should not learn mathematical theories in an abstract way, but should transfer some opinions through deduction, explanation and recitation of mathematical knowledge in real situations.

4.2. The Evaluation of 28 Mathematical Application Reports in This Semester

In the *Applied Probability and Statistics* course in the 2018 semester, 154 liberal arts students were required to divide into 28 cooperative groups and complete a mathematical application report within one semester. Among 28 mathematical application reports, there were 3 scientific situations (7%), 2 social situations (11%) and 23 life situations (82%), as shown in Table 4.

Table 4 Question Situations in 28 Completed Mathematics Application Reports

Question Situations (number, percent)	Example Topics	The Role of Mathematics
Scientific Situations (3, 11%)	Probability interpretation of Murphy's law	Applying the regularity of mathematical argument instead of the special case of the practice test in social cognition
Social situations (2, 7%)	Probability thinking from a physical education examination	Using data analysis to monitor the probabilities of plausible reasoning
Daily life situations (23, 82%)	The choice of lunch using probability	Developing their applied mathematical ability to solve daily life problems

In the first question situations, scientific situations, liberal arts students applied the regularity of mathematical argument instead of the special case of the practice test in social cognition. For instance, Edward A. Murphy's Law (1949) shows that if bad things have the possibility to happen, it will happen and cause the most possible loss, no matter how small the possibility is. Most liberal arts students agree with Murphy's law based on real cases in their lives. This group assumed that the completion of some complex event included a total of m steps and n results. They calculated out that the probability of a good outcome is much lower than that of the bad outcome by applying the addition principle and the multiplication principle. They concluded with logical inference if we want to achieve something successfully, we must eliminate all the bad possibilities and avoid all the pits perfectly. We can see that this group can apply probability calculation as proof of their social cognition and logical inference.

The second question situations are social situations. Taking the reports of *the probability thinking from a physical education examination* as an example, liberal arts students believed that the scoring standard of college basketball course, which is evaluated with the number of goals shot in 10 times, is unreasonable. They questioned that the final score could not represent students' usual percentage of pitches? They calculated that if someone's hitting probability was 0.6 in normal times, then the probability of 6 pitches was 0.25082 by using the Bernoulli formula, which is smaller than we had expected. Therefore, reasonable reasoning also needs data analysis and monitoring. Liberal arts students made a conclusion that reasonable reasoning is necessary for problems solution, but there may be deviations due to various unpredictable reasons. Therefore, data analysis can monitor the probability of reasonable reasoning.

The third question situations, daily life situations, are to use mathematics as a tool to solve problems in students' daily life. For example, the case *the choice of lunch* has used probability knowledge to optimize the choice of lunch in university life. Through data collection, they found out that students' rest time at noon is limited. In order to ensure punctual attendance in the afternoon class, this group would like to know about lunchtime in the different canteen with a 95% confidence interval. The data of the lunch period of 20 students in each canteen was collected, and the range of the average lunch period was estimated by using the interval estimation method. Specifically, after data collection, they calculated the mean value and standard deviation, and then concluded that the 0.95 confidence level of lunch period in the 5^{th} canteen is (29.12min, 44.48min) by using the confidence interval of the overall mean μ; the 0.95 confidence level of lunch period in the 4^{th} canteen is (39.42min, 49.59min); and so on. If only considering the time factor, it is more appropriate to choose the 5^{th} canteen for lunch in order to guarantee punctual attendance in the afternoon class.

The qualitative analysis of the interview results found that the discussion and evaluation of these 5 mathematics application reports about applying mathematics and solving social problems. When discussing the mathematical application reports on solving some social and daily-life problems by applying mathematical knowledge, liberal arts students' interest in mathematics and realize the value of mathematics application can be simulated. Therefore, peer mathematics application is a way for liberal arts students to realize mathematics value. Through the classification analysis of the 28 mathematics application reports completed by the liberal arts students, their three forms of mathematics application can be summarized as follows: applying the regularity of mathematical argument instead of the special case of the practice test in social cognition; using data analysis to monitor the probabilities of plausible reasoning; developing their applied mathematical ability to solve daily life problems.

5. Discussions

The 28 mathematical application reports were the result of interaction between liberal arts students and social

situations. Laplace (1829), a French mathematician, always emphasized the practicability of probability theory and mentioned more than once that the delicate calculation in probability theory can be applied to the most important problems in daily life. Among practical sciences, there are uncertainties that we cannot understand and describe, but probability theory is just a mathematical tool to make up for our lack of knowledge. Therefore, *Applied Probability and Statistics* as a general university course has both the scientific nature of mathematical knowledge and the sociality of content situations, and it can satisfy people's curiosity about predicting the future. 82% of the 28 mathematical application reports in the 2019-semester *Applied Probability and Statistics* course focused on daily situations, and the students see the wisdom of the probability and statistics with small scenarios so that people can understand the complex mathematics knowledge while paying attention to the reality of life.

Of relevance to the present study, it is noteworthy that the 28 mathematics application reports developed by the liberal arts students in this study took into account the characteristics of both mathematics and literature because their research problems were specific and practical, their research ideas had mathematical rational thinking, and their research conclusions were based on evidence and easily relatable to other students.

It can be seen from the mathematical application reports and interviews of liberal arts students that they don't value math as a problem-solving tool, but value mathematics application. Liberal arts students have opened up a new field of applied mathematics, and they have applied mathematics to social cognition, social phenomena, and other issues. These reports may open up a new direction of mathematical application, and applied mathematics can also enrich the social research methods of liberal arts students.

6. Research Limitations and Future Research Directions

6.1. Research Limitations

There are three limitations to the intent and the scope of the study. Firstly, regarding a small number of participants, the participants are only limited to liberal arts students who majored in pedagogy, education management, preschool education, special education and other limited majors. Whether the research results are suitable for other liberal arts students or not needs further verification. Secondly, liberal arts students have completed their mathematics application reports by group cooperation and each member's contribution to their report is required to be noted. However, how the mathematical application reports are designed, how the students actively interact and develop with each other and between individuals and groups, remain to be further investigated.

6.2. Future Research Directions

We suggest three broad ideas in future research. The first is to collect more cases in a variety of liberal arts majority and universities in different countries to help distinguish between local cultures and global cultures about the mathematical application. It is also important to find out the difference in the mathematical application process between different majority and universities. The second is to explore the application in other mathematical fields, such as the application of statistics for liberal arts students. Finally, this is a question that is still open yet important to investigate liberal arts students' learning styles and mental frames interact when applying mathematics in groups.

REFERENCES

[1] Authors. (Eds.). (2016). Beijing: Beijing Normal University Press.

[2] Authors. (2018). *Pure and Applied Mathematics*.

[3] Davis, P. J., & Hersh, R. (1981). *The mathematical experience*. New York: Viking Penguin Inc.

[4] Dreyfus, T. (1991). Advanced mathematical thinking processes. In D. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 25-41). Dordrecht, The Netherlands: Kluwer.

[5] 顾沛, & 戴瑛. (2004). 人文学科类数学课程设置和教学内容、体系改革的研究和实践[Research and practice on the reform of the mathematics curriculum, teaching content, and system in humanities]. *大学数学, 20*(2), 10-16.

[6] Halmos, P. R. (1994). What is teaching? *The American Mathematical Monthly*, *101*(9), 848–854.

[7] Hersh, R. (1993). Proving is convincing and explaining. *Educational Studies in Mathematics*, *24*(4), 389-399.

[8] Kramarski, B., Mizrachi N. (2006). Online discussion and self-regulated learning: effects of instructional methods on mathematical literacy. *Journal of Educational Research, 99(4)*, 218-230.

[9] Iannone, P., & Nardi, E. (2005). On the pedagogical insight of mathematicians: Interaction and transition from the concrete to the abstract. *The Journal of Mathematical Behavior*, *24*(2), 191–215.

[10] Intaros, P., Inprasitha, M., & Srisawadi, N. (2014). Students' problem-solving strategies in problem solving-mathematics classroom. *Procedia-Social and Behavioral Sciences*, *116*(10), 4119-4123.

[11] Laplace, P. S. (1829).*Essai philosophique sur les probabilitiés.* H. Remy.

[12] Leung, M., Low, R., & Sweller, J. (1997). Learning from equations or words. *Instructional Science, 25*, 37–70.

[13] Lubinski, D., & Humphreys, L. G. (1990). Assessing spurious "moderator effects": illustrated substantively with the hypothesized ("synergistic") relation between spatial and mathematical ability. *Psychological Bulletin, 107*(3), 385.

[14] Michal, A., Ruhama E. (2010). Mathematics Educators' Views on the Role of Mathematics Learning in Developing Deductive Reasoning. *International Journal of Science and Mathematics Education, 8*(6), 1131-1154.

[15] Moschkovich, J. N. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics, 19*(1), 11–19.

[16] Pollak, H. (1976). The Interaction between Mathematics and other School Subjects. *New Trends in Mathematics Teaching, 4,* 232-240.

[17] Sloyer, C., Huntley, I., & Blum, W. (1995). *Advances and perspectives in the teaching of mathematical modeling and applications*. Water Street Mathematics.

[18] 盛立人, 胡卫群, 肖箭, 杨明辉, & 蒋华松. (2006). *社会科学中的数学[Mathematics in social sciences]*. 北京: 科学出版社.

[19] 孙勇. (2010). 关于数学应用能力若干问题的探讨[Study on some issues of mathematics application ability]. *课程·教材·教法, 30*(8), 19 & 54-57.

[20] Weber, K. (2003). A procedural route toward understanding the concept of proof. In: *Proceedings of the Twenty-third Conference of the International Group for the Psychology of Mathematics Education, Honolulu, HI*.

[21] Weber, K. (2004). Traditional instruction in advanced mathematics courses: A case study of one professor's lectures and proofs in an introductory real analysis course. *The Journal of Mathematical Behavior*, *23*(2), 115-133.

[22] Wiggins, G., Wiggins, G. P., & McTighe, J. (2005). *Understanding by design*. Ascd.

[23] 严士健. (2002). *大学文科数学[Liberal arts mathematics in universities]*. 北京: 高等教育出版社.

[24] 翟文明. (2006). *不可不知的50个生活法则[50 rules of life we must know]*. 北京: 蓝天出版社.