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RP-58: Formulation of Solutions of Standard Cubic Congruence of Composite Modulus - A Product of Eight-Multiple of an Odd Prime & Nth Power of Three

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ABSTRACT

In this paper, a standard cubic congruence of even composite modulus- a product eight multiple of an odd prime & nth power of three, is considered for solutions. The congruence is studied and formulation of its solution is established. The formulation is done in two cases with different formula of solutions. Using the formula, the finding of solutions becomes easy and time-saving. Formulation is the merit of the paper.

KEYWORDS: *Cubic Congruence, Composite Modulus, Cubic-Residue, Formulation*

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INTRODUCTION:

A cubic congruence is seldom studied in Number Theory. The author started to study the topic and formulated many standard cubic congruence of composite modulus. Here is one of such standard cubic congruence of composite modulus, not formulated earlier. Hence, it is considered for study and formulation of its solutions.

PROBLEM-STATEMENT

"To formulate the solutions of the standard cubic congruence: $x^3 \equiv a \ (mod\ 3^n.8p), p$ being an odd prime

Case-I : $p \equiv 2\ (mod\ 3) and\ a \neq 3l$ is a positive odd integer;

Case-II : $p \equiv 2\ (mod\ 3)\ and\ a \neq 3l$ is an even positive integer.

Case-III : $a = 3l; odd\ multiple\ of\ three$;

Case-IV : $a = 3l; even\ multiple\ of\ three$".

LITERATURE REVIEW

The congruence under consideration has not been formulated earlier by mathematicians.

Actually no detailed study is found in the literature. Zukerman [1] and Thomas Koshy [2] had given a very small start but no formulation is found in the literature. Only the author's formulations of some of the standard cubic congruence of prime and composite modulus are found [3], [4], [5], [6].

ANALYSIS & RESULTS

Consider the congruence $x^3 \equiv a^3\ (mod\ 3^n.8p). Let\ p \equiv 2\ (mod\ 3)$.

Case-I: Also, let $a \neq 3l$ be an odd positive integer.

Then for $x \equiv 3^{n-1}.8pk + a \ (mod\ 3^n.8p)$,
$x^3 \equiv (3^{n-1}.8pk + a)^3 \ (mod\ 3^n.8p)$
$\equiv (3^{n-1}.8pk)^3 + 3.(3^{n-1}8pk)^2.a + 3.3^{n-1}.8pk.a^2 + a^3 \ (mod\ 3^n.8p)$
$\equiv a^3 + 3^n.8pk\{a^2 + 3^{n-1}.8pk.a + 3^{2n-3}.(8pk)^2\}(mod\ 3^n.8p)$
$\equiv a^3 \ (mod\ 3^n.8p)$.

Therefore, it is the solution of the congruence.

But for $k = 3,4,5$ the solution is the same as for $k = 0, 1, 2$.
Thus is can be said that the solutions of the congruence are given by
$x \equiv 3^{n-1}.8pk + a \ (mod\ 3^n.8p), k = 0, 1, 2$.

Case-II:
Also, let $a \neq 3l$ be an even positive integer.
Then for $x \equiv 3^{n-1}.2pk + a \ (mod\ 3^n.8p)$,
$x^3 \equiv (3^{n-1}.2pk + a)^3 \ (mod\ 3^n.8p)$
$\equiv (3^{n-1}.2pk)^3 + 3.(3^{n-1}.2pk)^2.a + 3.3^{n-1}.2pk.a^2 + a^3 \ (mod\ 3^n.8p)$
$\equiv a^3 + 3^n.8pk\{\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\}(mod\ 3^n.8p)$
$\equiv a^3 \ (mod\ 3^n.8p)$, if a.is even positive integer.

Therefore, it is the solution of the congruence.

But for $k = 12, 13, 14, \dots\dots$ the solutions are the same as for $k = 0, 1, 2, \dots\dots$.

Thus it can be said that the solutions of the congruence are given by
$x \equiv 3^{n-1}.2pk + a \ (mod\ 3^n.8p), k = 0, 1, 2, 3, \dots\dots\dots..10, 11$.

Case-III: let $a = 3l; l = 1, 3, 5 \dots\dots\dots$.
Consider $x \equiv 8p.3^{n-2}k + 3l \ (mod\ 8p.3^n)$,
Then $x^3 \equiv (8p.3^{n-2}k + 3l)^3 \ (mod\ 8p.3^n)$
$\equiv (8p.3^{n-2}k)^3 + 3.(8p.3^{n-2}k)^2.3l + 3.8p.3^{n-2}k.(3l)^2 + (3l)^3 \ (mod\ 8p.3^n)$
$\equiv 8p.3^n k\{64p^2k^2 3^{2n-4} + 8pkl + 3l\} + (3l)^3 \ (mod\ 8p.3^n)$
$\equiv (3l)^3 \ (mod\ 8p.3^n)$, for a for any positive integer.

Thus, $x \equiv 8p.3^{n-2}k + 3l \ (mod\ 8p.3^n)$ satisfies the said congruence.

Hence, it is a solution of the congruence.

But for $k = 9$, the solution becomes $x \equiv 2.3^{n-2}.9 + 3 \ (mod\ 8p.3^n)$
$\equiv 2.3^n + 3 \ (mod\ 8p.3^n)$
$\equiv 3 \ (mod\ 8p.3^n)$

Which is the same as for the solution for $k = 0$.

For $k = 10, 11, \dots\dots\dots$ the solutions are also the same as for $k = 1, 2 \dots\dots$

Thus, the congruence has exactly nine incongruent solutions
$x \equiv 8p.3^{n-2}k + 3 \ (mod\ 8p.3^n)$, for any integer a, $k = 0, 1, 2, 3, 4, 5, 6, 7, 8$.

Case-IV: But if $a = 3l, l = 2, 4, \dots\dots$, then for $x \equiv 4p.3^{n-2}k + 3l \ (mod\ 8p.3^n)$,
$x^3 \equiv (4p.3^{n-2}k + 3l)^3 \ (mod\ 8p.3^n)$
$\equiv (4p.3^{n-2}k)^3 + 3.(4p.3^{n-2}k)^2.3l + 3.4p.3^{n-2}k.(3l)^2 + (3l)^3 \ (mod\ 8p.3^n)$
$\equiv (3l)^3 + 4p.3^n k(4^2.3^{2n-6}.p^2k^2 + 4pkl + 3l^2) \ (mod\ 8p.3^n)$
$\equiv (3l)^3 \ (mod\ 8p.3^n)$ as l is even.

Therefore, $x \equiv 4p.3^{n-2}k + 3l \ (mod\ 8p.3^n)$ satisfies the cubic congruence and hence it is a solution of it.
But for $k = 18 = 2.3^2$, the solution reduces to $x \equiv 4p.3^{n-2}.2.3^2 + 3l \ (mod\ 8p.3^n)$
$\equiv 8p.3^n + 3l \ (mod\ 8p.3^n)$
$\equiv 3l \ (mod\ 8p.3^n)$

Thus, it is the same solution as for $k = 0$.

Similarly, it is also seen that for $k = 19, 20 \dots\dots$., the solutions repeat as for $k = 1, 2, \dots..$

Therefore, the congruence has eighteen solutions: $x \equiv 4.3^{n-2}k + 3l \ (mod\ 8p.3^n)$;

$k =, 1, 2, \dots \dots \dots .., 17.$

Sometimes, in the cubic congruence, the integer a may not be a perfect cube. The readers have to make it so, by adding multiples of the modulus [1].

ILLUSTRATIONS

Example-1: Consider the congruence: $x^3 \equiv 1^3 \ (mod\ 3^2.8.5)$

It is of the type
$x^3 \equiv a^3 \ (mod\ 3^n.8p) with\ a = 1, an\ odd\ integer, p = 5 \equiv 2\ (mod\ 3).$

Such types of congruence has exactly three solutions given by
$x \equiv 3^{n-1}.8pk + a\ (mod\ 3^n.8p)$
$\equiv 3^1.8.5k + 1\ (mod\ 3^2.8.5)$
$\equiv 120k + 1\ (mod\ 360); k = 0, 1, 2.$
$\equiv 1, 121, 241\ (mod\ 360).$

Example-2: Consider the congruence: $x^3 \equiv 4^3\ (mod\ 3^2.8.11)$

It is of the type
$x^3 \equiv a^3\ (mod\ 3^n.8.p) with\ a = 4, an\ even\ integer, p = 11 \equiv 2\ (mod\ 3).$

Such types of congruence has exactly twelve solutions given by
$x \equiv 3^{n-1}.2pk + a\ (mod\ 3^n.8.p)$
$\equiv 3^1.2.11k + 4\ (mod\ 3^2.8.11)$
$\equiv 66k + 4\ (mod\ 396); k = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.$
$\equiv 4, 70, 136, 202, 268, 334, 466, 400, 532, 598, 664, 730\ (mod\ 792).$

Example-3: Consider the
congruence: $x^3 \equiv 216\ (mod\ 1080)$
It can be written as $x^3 \equiv 6^3\ (mod\ 3^4.8.5)$

It is of the type
$x^3 \equiv a^3\ (mod\ 3^n.8p) with\ a = 6, an\ even\ integer, p = 5 \equiv 2\ (mod\ 3).$

Such types of congruence has exactly eighteen solutions given by
$x \equiv 3^{n-2}.4pk + a\ (mod\ 3^n.8p)$
$\equiv 3^1.4.5k + 6\ (mod\ 3^3.8.5)$
$\equiv 60k + 6\ (mod\ 1080); k = 0, 1, 2, 3, 4, 5, \dots \dots \dots .., 16, 17.$
$\equiv 4, 66, 126, 186, 246, 306, 366, 426, 486, 546, 606, 666, 726,$
$786, 846, 906, 966, 1026\ (mod\ 1080).$

Example-4: Consider the congruence: $x^3 \equiv 729\ (mod\ 2376)$
It can be written as $x^3 \equiv 9^3\ (mod\ 3^3.8.11)$
It is of the type
$x^3 \equiv a^3\ (mod\ 3^n.8.p) with\ a = 9, an\ odd\ integer, p = 11 \equiv 2\ (mod\ 3).$

Such types of congruence has exactly nine solutions given by
$x \equiv 3^{n-2}.8pk + a\ (mod\ 3^n.8.p)$
$\equiv 3^1.8.11k + 9\ (mod\ 3^3.8.11)$
$\equiv 264k + 9\ (mod\ 2376); k = 0, 1, 2, 3, 4, 5, 6, 7, 8.$
$\equiv 9, 273, 537, 801, 1065, 1329, 1593, 1857, 2121\ (mod\ 2376).$

CONCLUSION

Therefore, it is concluded that the cubic congruence under consideration:
$x^3 \equiv a^3\ (mod\ 3^n.8p)$; p $\equiv 2\ (mod\ 3)$ has exactly three solutions, if a is an odd positive integer, given by $x \equiv 3^{n-1}.8pk + a\ (mod\ 3^n.8p); k = 0, 1, 2;$

while the congruence has exactly twelve solutions if a is an even positive integer, given by $x \equiv 3^{n-1}.2pk + a\ (mod\ 3^n.8p); k = 0, 1, 2, \dots \dots .., 11.$

If $a = 3l, an\ odd\ multiple\ of\ three$, the congruence has exactly nine solutions given by
$x \equiv 3^{n-2}.8pk + a\ (mod\ 3^n.8.p); k = 0, 1, 2, \dots \dots \dots .8.$

If $a = 3l, an\ even\ multiple\ of\ three$, the congruence has exactly eighteen solutions given by $x \equiv 3^{n-2}.4pk + a\ (mod\ 3^n.8.p); k = 0, 1, 2, \dots \dots \dots .17.$

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