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Linear Programming Problems with Icosikaipentagonal Fuzzy Number

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ABSTRACT
The objective of this paper is to introduce a new fuzzy number with twenty five points called as Icosikaipentagonal fuzzy number. In which Fuzzy numbers develop a membership function where there are no limitations of any specified form. The aim of this paper is to define Icosikaipentagonal fuzzy number and its some arithmetic operations. Fuzzy Linear Programming problem is one of the active research areas in optimization. Many real world problems are modelled as Fuzzy Linear Programming Problems. Icosikaipentagonal fuzzy number proposed a ranking function to solve fuzzy linear programming problems.

KEYWORDS: Alpha cut and Fuzzy operations, Fuzzy Number, Fuzzy Linear Programming Problem, Icosikaipentagonal Fuzzy Number

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1. INTRODUCTION

Fuzzy sets provide machinery for carrying out approximate reasoning processes when its available information is uncertain, in complete, imprecise or vague. The concept of fuzzy set theory was first introduced by Zadeh [25] and also introduces membership function with a range covering the interval (0,1). Didier Dubois and Henri Prade [8,9] have proposed a algebraic operations on real numbers , which are extended to fuzzy number by using the principle of fuzzification and also defined fuzzy number as fuzzy subset of real line. Zadeh [26] proposed an interval arithmetic operations on real numbers can be extended with fuzzy numbers. Akther and Ahamd [3] discussed with computational methods for fuzzy arithmetic operations. Deshrijver [8] has proposed an arithmetic operations in interval valued fuzzy set theory. Abbari et al [1] has proposed elementary fuzzy arithmetic operation on Pseudo-geometric fuzzy numbers. Garg and Ansha have discussed arithmetic operations on parabolic fuzzy numbers. Barnabs and Jnos [5] have proposed product type operations between fuzzy numbers. Chakraborty and Guha [6] have proposed the addition with generalized with fuzzy numbers. Many researchers defined several types of fuzzy numbers like Triangular, Trapezoidal, pentagonal, Hexagonal, Heptagonal, Octagonal, Nanogonal, up to icosikaitetragonal and Icosikaioctogonal fuzzy number and also defined membership functions. These types of membership functions and arithmetic operations applied by the many researchers to solve optimization problems. Panthinathan and K. Ponnovalavan [17] have defined a diamond fuzzy number concepts which helps to solve three types of categorization triangular fuzzy number. Many researchers proposed different methods to found the ranking of fuzzy numbers. The ranking of fuzzy number was first introduced by Jain [14]. In 1980, Yager [24] introduced the concept of centroid fuzzy numbers. Cheng [7] proposed a method for ranking fuzzy number by using distance method. Rahim and Rasoul [18] proposed a method for defuzzified based on the centroid points. M.Adabitabar et al [2] have presented a vague ranking fuzzy numbers which utilize the notion of max and min fuzzy simultaneously in order to determining ambiguity in ranking of two fuzzy numbers. Several ranking function methods are used to solve Fuzzy Linear Programming Problem. Tanaka et al [23] has proposed a concept of Fuzzy Linear Programming Problems. Zimmerman [27] has developed a method for solving Fuzzy Linear Programming Problems using multi objective LP technique. Fang et al introduced Linear Programming Problem with fuzzy constraints. Lotfi et al [15] proposed a new method for find the optimal solution of fully Fuzzy Linear Programming Problems. Allahviranlooet et al [4] have proposed a new method to solve Fully Fuzzy Linear Programming Problems using ranking function. Sudip and

Pinaki [22] has proposed a new technique based on the centroid of triangle and trapezoidal fuzzy numbers to solve a fuzzy linear programming problems. Maleki et al [16] proposed new method to solve Fuzzy Linear Programming Problems using ranking function by the comparison of all decision parameters.

In this paper, Icosikaipentagonal fuzzy number proposed a ranking function which helps to construct the fuzzy linear programming problem into crisp linear programming problem.

2. Preliminaries

In this section, we give some preliminaries are as follows:

Definition 2.1. Let X be a non-empty set. A fuzzy set A of X is defined as $A = \{(x, \mu_A(x): x \in X\}$ where $\mu_A(x)$ is membership function which maps each elements of X to a value between 0 and 1.

Definition 2.2. A fuzzy set A defined on the set of real numbers R is said to be a fuzzy number if its membership function $A: R \to [0,1]$ has the following characteristics:

1. A is convex.
2. A is normal.
3. A is piecewise continuous.

Definition 2.3. The α-cut of a fuzzy set A is the crisp set of all elements of $x \in X$ that belong to the fuzzy set A at least to the degree $\alpha \in [0,1]$.

(i.e.) $\alpha_{[A]} = \{x \in X | \mu_A(x) \geq \alpha\}$.

Definition 2.4. A fuzzy set A is a convex fuzzy set if and only if each α- cuts α_A is a convex fuzzy set.

Definition 2.5. A fuzzy number $A = (a_1, a_2, a_3)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, for\ a_1 \leq x \leq a_2 \\ 1, for\ x = a_2 \\ \frac{a_3 - x}{a_3 - a_2}, for\ a_2 \leq x \leq a_3 \\ 0, otherwise \end{cases}$$

Definition 2.6. A fuzzy number $A = (a_1, a_2, a_3, a_4)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, for\ a_1 \leq x \leq a_2 \\ 1, for\ a_2 \leq x \leq a_3 \\ \frac{a_4 - x}{a_4 - a_3}, for\ a_3 \leq x \leq a_4 \\ 0, otherwise \end{cases}$$

3. Icosikaipentagonal Fuzzy Number

In this section a new form of fuzzy number named Icosikaipentagonal fuzzy number is introduced.

Definition 3.1. A fuzzy number $A = (a_1, a_2, \ldots, a_{25})$ is said to be Icosikaipentagonal fuzzy number where $a_1, a_2, \ldots, a_{25}$ are the real numbers which membership function is given by

$(0 \leq k_1 \leq k_2 \leq k_3 \leq k_4 \leq k_5 \leq k_6 \leq 1)$

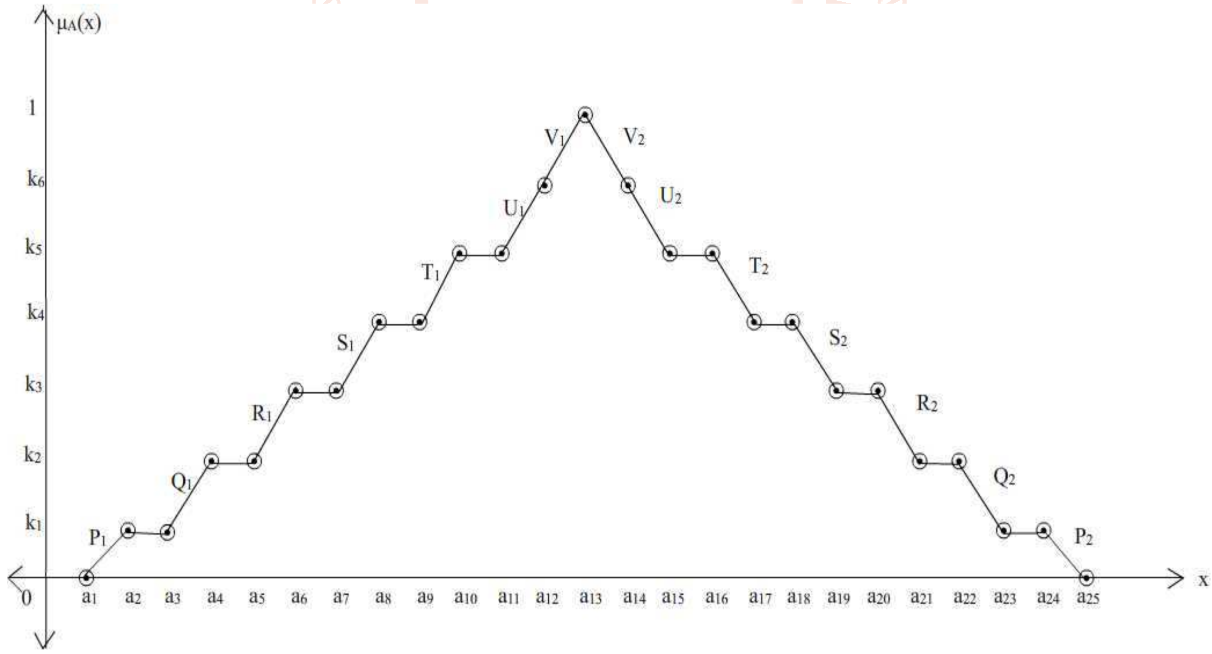

Figure 1: Icosikaipentagonal Fuzzy Number

$$\mu_A(x)=\begin{cases} 0, x<a_1 \\ k_1\left(\dfrac{x-a_1}{a_2-a_1}\right), a_1\le x\le a_2 \\ k_1, a_2\le x\le a_3 \\ (k_2-k_1)\left(\dfrac{x-a_3}{a_4-a_3}\right)+k_1, a_3\le x\le a_4 \\ k_2, a_4\le x\le a_5 \\ (k_3-k_2)\left(\dfrac{x-a_5}{a_6-a_5}\right)+k_2, a_5\le x\le a_6 \\ k_3, a_6\le x\le a_7 \\ (k_4-k_3)\left(\dfrac{x-a_7}{a_8-a_7}\right)+k_3, a_8\le x\le a_9 \\ k_4, a_8\le x\le a_9 \\ (k_5-k_4)\left(\dfrac{x-a_9}{a_{10}-a_9}\right)+k_4, a_9\le x\le a_{10} \\ k_5, a_{10}\le x\le a_{11} \\ (k_6-k_5)\left(\dfrac{x-a_{11}}{a_{12}-a_{11}}\right)+k_5, a_{11}\le x\le a_{12} \\ (1-k_6)\left(\dfrac{x-a_{12}}{a_{13}-a_{12}}\right)+k_6, a_{12}\le x\le a_{13} \\ (1-k_6)\left(\dfrac{a_{14}-x}{a_{14}-a_{13}}\right)+k_6, a_{13}\le x\le a_{14} \\ (k_6-k_5)\left(\dfrac{a_{15}-x}{a_{15}-a_{14}}\right)+k_5, a_{14}\le x\le a_{15} \\ k_5, a_{15}\le x\le a_{16} \\ (k_5-k_4)\left(\dfrac{a_{17}-x}{a_{17}-a_{16}}\right)+k_4, a_{16}\le x\le a_{17} \\ k_4, a_{17}\le x\le a_{18} \\ (k_4-k_3)\left(\dfrac{a_{19}-x}{a_{19}-a_{18}}\right)+k_3, a_{18}\le x\le a_{19} \\ k_3, a_{19}\le x\le a_{20} \\ (k_3-k_2)\left(\dfrac{a_{21}-x}{a_{21}-a_{20}}\right)+k_2, a_{20}\le x\le a_{21} \\ k_2, a_{21}\le x\le a_{22} \\ (k_2-k_1)\left(\dfrac{a_{23}-x}{a_{23}-a_{22}}\right)+k_1, a_{22}\le x\le a_{23} \\ k_1, a_{23}\le x\le a_{24} \\ k_1\left(\dfrac{a_{25}-x}{a_{25}-a_{24}}\right), a_{24}\le x\le a_{25} \end{cases}$$

4. Arithmetic operations on Icosikaipentagonal fuzzy number

4.1. Arithmetic operations on Alpha cut

Definition 4.1. For $\in [0,1]$, the α cut of an Icosikaipentagonal fuzzy number $A=(a_1,a_2,\dots,a_{25})$ is defined as

$$A_\alpha=\begin{cases} [P_1(\alpha),P_2(\alpha)], for\ \alpha\in[0,k_1) \\ [Q_1(\alpha),Q_2(\alpha)], for\ \alpha\in[k_1,k_2) \\ [R_1(\alpha),R_2(\alpha)], for\ \alpha\in[k_2,k_3) \\ [S_1(\alpha),S_2(\alpha)], for\ \alpha\in[k_3,k_4) \\ [T_1(\alpha),T_2(\alpha)], for\ \alpha\in[k_4,k_5) \\ [U_1(\alpha),U_2(\alpha)], for\ \alpha\in[k_5,k_6) \\ [V_1(\alpha),V_2(\alpha)], for\ \alpha\in[k_6,1] \end{cases}$$

Definition 4.2. The α cut of Icosikaipentagonal fuzzy number operations interval A_α is defined as

$$A_\alpha=\begin{cases} [a_1+7\alpha(a_2-a_1), a_{25}-7\alpha(a_{25}-a_{24})], for\ \alpha\in[0,0.14) \\ [a_3+(7\alpha-1)(a_4-a_3), a_{23}-(7\alpha-1)(a_{23}-a_{22}], for\ \alpha\in[0.14,0.28) \\ [a_5+(7\alpha-2)(a_6-a_5), a_{21}-(7\alpha-2)(a_{21}-a_{20})], for\ \alpha\in[0.28,0.42) \\ [a_7+(7\alpha-3)(a_8-a_7), a_{19}-(7\alpha-3)(a_{19}-a_{18})], for\ \alpha\in[0.42,0.56) \\ [a_9+(7\alpha-4)(a_{10}-a_9), a_{17}-(7\alpha-4)(a_{17}-a_{16})], for\ \alpha\in[0.56,0.7) \\ [a_{11}+(7\alpha-5)(a_{12}-a_{11}), a_{15}-(7\alpha-5)(a_{15}-a_{14}), for\ \alpha\in[0.7,0.84) \\ [a_{12}+(7\alpha-6)(a_{13}-a_{12}), a_{14}-(7\alpha-6)(a_{14}-a_{13})], for\ \alpha\in[0.84,1] \end{cases}$$

Theorem 4.3. If $A=(a_1,a_2,\dots,a_{25})$ and $B=(b_1,b_2,\dots,b_{25})$ are Icosikaipentagonal fuzzy numbers, then $C=A+B$ is also a Icosikaipentagonal fuzzy numbers $A+B=(a_1+b_1,a_2+b_2,\dots,a_{25}+b_{25})$

Proof: Let us add the α – cut of A and B Icosikaipentagonal fuzzy number is defined as $C = A_\alpha + B_\alpha$

$$C = \begin{cases} [a_1 + 7\alpha(a_2 - a_1), a_{25} - 7\alpha(a_{25} - a_{24})] + \\ [b_1 + 7\alpha(b_2 - b_1), b_{25} - 7\alpha(b_{25} - b_{24})], for\ \alpha \in [0,0.14) \\ [a_3 + (7\alpha - 1)(a_4 - a_3), a_{23} - (7\alpha - 1)(a_{23} - a_{22}] + \\ [b_3 + (7\alpha - 1)(b_4 - b_3), b_{23} - (7\alpha - 1)(b_{23} - b_{22})], for\ \alpha \in [0.14,0.28) \\ [a_5 + (7\alpha - 2)(a_6 - a_5), a_{21} - (7\alpha - 2)(a_{21} - a_{20})] + \\ b_5 + (7\alpha - 2)(b_6 - b_5), b_{21} - (7\alpha - 2)(b_{21} - b_{20}), for\ \alpha \in [0.28,0.42) \\ [a_7 + (7\alpha - 3)(a_8 - a_7), a_{19} - (7\alpha - 3)(a_{19} - a_{18})] + \\ [b_7 + (7\alpha - 3)(b_8 - b_7), b_{19} - (7\alpha - 3)(b_{19} - b_{18})], for\ \alpha \in [0.42,0.56) \\ [a_9 + (7\alpha - 4)(a_{10} - a_9), a_{17} - (7\alpha - 4)(a_{17} - a_{16})] + \\ [b_9 + (7\alpha - 4)(b_{10} - b_9), b_{17} - (7\alpha - 4)(b_{17} - b_{16})], for\ \alpha \in [0.56,0.7) \\ [a_{11} + (7\alpha - 5)(a_{12} - a_{11}), a_{15} - (7\alpha - 5)(a_{15} - a_{14}) + \\ [b_{11} + (7\alpha - 5)(b_{12} - b_{11}), b_{15} - (7\alpha - 5)(b_{15} - b_{14}), for\ \alpha \in [0.7,0.84) \\ [a_{12} + (7\alpha - 6)(a_{13} - a_{12}), a_{14} - (7\alpha - 6)(a_{14} - a_{13})] \\ [b_{12} + (7\alpha - 6)(b_{13} - b_{12}), b_{14} - (7\alpha - 6)(b_{14} - b_{13})], for\ \alpha \in [0.84,1] \end{cases}$$

For $0 \leq \alpha \leq 0.14$, $x \in [a_1 + b_1, a_2 + b_2]$,

Then $x = a_1 + b_1 + 7\alpha\big(a_2 + b_2 - (a_1 + b_1)\big)$

$$7\alpha = \frac{x-(a_1+b_1)}{a_2+b_2-(a_1+b_1)}$$

$$\alpha = \frac{1}{7}\left(\frac{x-(a_1+b_1)}{a_2+b_2-(a_1+b_1)}\right)$$

Similarly for the remaining intervals. Therefore addition of membership function is given by

$$\mu_{(A+B)}(x) = \begin{cases} 0, x < a_1 + b_1 \\ k_1\left(\frac{x-(a_1+b_1)}{a_2+b_2-(a_1+b_1)}\right), a_1 + b_1 \leq x \leq a_2 + b_2 \\ k_1, a_2 + b_2 \leq x \leq a_3 + b_3 \\ (k_2 - k_1)\left(\frac{x-(a_3+b_3)}{a_4+b_4-(a_3+b_3)}\right) + k_1, a_3 + b_3 \leq x \leq a_4 + b_4 \\ k_2, a_4 + b_4 \leq x \leq a_5 + b_5 \\ (k_3 - k_2)\left(\frac{x-(a_5+b_5)}{a_6+b_6-(a_5+b_5)}\right) + k_2, a_5 + b_5 \leq x \leq a_6 + b_6 \\ k_3, a_6 + b_6 \leq x \leq a_7 + b_7 \\ (k_4 - k_3)\left(\frac{x-(a_7+b_7)}{a_8+b_8-(a_7+b_7)}\right) + k_3, a_8 + b_8 \leq x \leq a_9 + b_9 \\ k_4, a_8 + b_8 \leq x \leq a_9 + b_9 \\ (k_5 - k_4)\left(\frac{x-(a_9+b_9)}{a_{10}+b_{10}-(a_9+b_9)}\right) + k_4, a_9 + b_9 \leq x \leq a_{10} + b_{10} \\ k_5, a_{10} + b_{10} \leq x \leq a_{11} + b_{11} \\ (k_6 - k_5)\left(\frac{x-(a_{11}+b_{11})}{a_{12}+b_{12}-(a_{11}+b_{11})}\right) + k_5, a_{11} + b_{11} \leq x \leq a_{12} + b_{12} \\ (1 - k_6)\left(\frac{x-(a_{12}+b_{12})}{a_{13}+b_{13}-(a_{12}+b_{12})}\right) + k_6, a_{12} + b_{12} \leq x \leq a_{13} + b_{13} \\ (1 - k_6)\left(\frac{a_{14}+b_{14}-x}{a_{14}+b_{14}-(a_{13}+b_{13})}\right) + k_1, a_{13} + b_{13} \leq x \leq a_{14} + b_{14} \\ (k_6 - k_5)\left(\frac{a_{15}+b_{15}-x}{a_{15}+b_{15}-(a_{14}+b_{14})}\right) + k_5, a_{14} + b_{14} \leq x \leq a_{15} + b_{15} \\ k_5, a_{15} + b_{15} \leq x \leq a_{16} + b_{16} \\ (k_5 - k_4)\left(\frac{a_{17}+b_{17}-x}{a_{17}+b_{17}-(a_{16}+b_{16})}\right) + k_4, a_{16} + b_{16} \leq x \leq a_{17} + b_{17} \\ k_4, a_{17} + b_{17} \leq x \leq a_{18} + b_{18} \\ (k_4 - k_3)\left(\frac{a_{19}+b_{19}-x}{a_{19}+b_{19}-(a_{18}+b_{18})}\right) + k_3, a_{18} + b_{18} \leq x \leq a_{19} + b_{19} \\ k_3, a_{19} + b_{19} \leq x \leq a_{20} + b_{20} \\ (k_3 - k_2)\left(\frac{a_{21}+b_{21}-x}{a_{21}+b_{21}-(a_{20}+b_{20})}\right) + k_2, a_{20} + b_{20} \leq x \leq a_{21} + b_{21} \\ k_2, a_{21} + b_{21} \leq x \leq a_{22} + b_{22} \\ (k_2 - k_1)\left(\frac{a_{23}+b_{23}-x}{a_{23}+b_{23}-(a_{22}+b_{22})}\right) + k_1, a_{22} + b_{22} \leq x \leq a_{23} + b_{23} \\ k_1, a_{23} + b_{23} \leq x \leq a_{24} + b_{24} \\ k_1\left(\frac{a_{25}+b_{25}-x}{a_{25}+b_{25}-(a_{24}+b_{24})}\right), a_{24} + b_{24} \leq x \leq a_{25} + b_{25} \end{cases}$$

Thus if $A = (a_1, a_2, \dots, a_{25})$ and $B = (b_1, b_2, \dots, b_{25})$ are Icosikaipentagonal fuzzy number, then$C = A + B = (a_1 + b_1, a_2 + b_2, \dots, a_{25} + b_{25})$.

Theorem 4.4. If $A = (a_1, a_2, \dots, a_{25})$ and $B = (b_1, b_2, \dots, b_{25})$ are Icosikaipentagonal fuzzy numbers, then $C = A + B$ is also a Icosikaipentagonal fuzzy numbers $A - B = (a_1 - b_1, a_2 - b_2, \dots, a_{25} - b_{25})$

Proof: Let us add the α –cut of A and B Icosikaipentagonal fuzzy number is defined as $C = A_\alpha - B_\alpha$

$$C = \begin{cases} [a_1 + 7\alpha(a_2 - a_1), a_{25} - 7\alpha(a_{25} - a_{24})] - \\ [b_1 + 7\alpha(b_2 - b_1), b_{25} - 7\alpha(b_{25} - b_{24})], for\ \alpha \in [0,0.14) \\ [a_3 + (7\alpha - 1)(a_4 - a_3), a_{23} - (7\alpha - 1)(a_{23} - a_{22}] - \\ [b_3 + (7\alpha - 1)(b_4 - b_3), b_{23} - (7\alpha - 1)(b_{23} - b_{22})], for\ \alpha \in [0.14,0.28) \\ [a_5 + (7\alpha - 2)(a_6 - a_5), a_{21} - (7\alpha - 2)(a_{21} - a_{20})] - \\ b_5 + (7\alpha - 2)(b_6 - b_5), b_{21} - (7\alpha - 2)(b_{21} - b_{20}), for\ \alpha \in [0.28,0.42) \\ [a_7 + (7\alpha - 3)(a_8 - a_7), a_{19} - (7\alpha - 3)(a_{19} - a_{18})] - \\ [b_7 + (7\alpha - 3)(b_8 - b_7), b_{19} - (7\alpha - 3)(b_{19} - b_{18})], for\ \alpha \in [0.42,0.56) \\ [a_9 + (7\alpha - 4)(a_{10} - a_9), a_{17} - (7\alpha - 4)(a_{17} - a_{16})] - \\ [b_9 + (7\alpha - 4)(b_{10} - b_9), b_{17} - (7\alpha - 4)(b_{17} - b_{16})], for\ \alpha \in [0.56,0.7) \\ [a_{11} + (7\alpha - 5)(a_{12} - a_{11}), a_{15} - (7\alpha - 5)(a_{15} - a_{14})] - \\ [b_{11} + (7\alpha - 5)(b_{12} - b_{11}), b_{15} - (7\alpha - 5)(b_{15} - b_{14}), for\ \alpha \in [0.7,0.84) \\ [a_{12} + (7\alpha - 6)(a_{13} - a_{12}), a_{14} - (7\alpha - 6)(a_{14} - a_{13})] - \\ [b_{12} + (7\alpha - 6)(b_{13} - b_{12}), b_{14} - (7\alpha - 6)(b_{14} - b_{13})], for\ \alpha \in [0.84,1] \end{cases}$$

For $0 \le \alpha \le 0.14$, $x \in [a_1 + b_1, a_2 + b_2]$,

Then $x = a_1 - b_1 + 7\alpha\big(a_2 - b_2 - (a_1 - b_1)\big)$

$$7\alpha = \frac{x-(a_1-b_1)}{a_2-b_2-(a_1-b_1)}$$

$$\alpha = \frac{1}{7}\left(\frac{x-(a_1-b_1)}{a_2-b_2-(a_1-b_1)}\right)$$

Similarly for the remaining intervals. Therefore addition of membership function is given by

$$\mu_{(A+B)}(x) = \begin{cases} 0, x < a_1 - b_1 \\ k_1\left(\dfrac{x-(a_1-b_1)}{a_2-b_2-(a_1-b_1)}\right), a_1 - b_1 \le x \le a_2 - b_2 \\ k_1, a_2 - b_2 \le x \le a_3 - b_3 \\ (k_2 - k_1)\left(\dfrac{x-(a_3-b_3)}{a_4-b_4-(a_3-b_3)}\right) + k_1, a_3 - b_3 \le x \le a_4 - b_4 \\ k_2, a_4 - b_4 \le x \le a_5 - b_5 \\ (k_3 - k_2)\left(\dfrac{x-(a_5-b_5)}{a_6-b_6-(a_5-b_5)}\right) + k_2, a_5 - b_5 \le x \le a_6 - b_6 \\ k_3, a_6 - b_6 \le x \le a_7 - b_7 \\ (k_4 - k_3)\left(\dfrac{x-(a_7-b_7)}{a_8-b_8-(a_7-b_7)}\right) + k_3, a_8 - b_8 \le x \le a_9 - b_9 \\ k_4, a_8 - b_8 \le x \le a_9 - b_9 \\ (k_5 - k_4)\left(\dfrac{x-(a_9-b_9)}{a_{10}-b_{10}-(a_9-b_9)}\right) + k_4, a_9 - b_9 \le x \le a_{10} - b_{10} \\ k_5, a_{10} - b_{10} \le x \le a_{11} - b_{11} \\ (k_6 - k_5)\left(\dfrac{x-(a_{11}-b_{11})}{a_{12}+b_{12}-(a_{11}+b_{11})}\right) + k_5, a_{11} - b_{11} \le x \le a_{12} - b_{12} \\ (1 - k_6)\left(\dfrac{x-(a_{12}-b_{12})}{a_{13}-b_{13}-(a_{12}-b_{12})}\right) + k_6, a_{12} - b_{12} \le x \le a_{13} - b_{13} \\ (1 - k_6)\left(\dfrac{a_{14}-b_{14}-x}{a_{14}-b_{14}-(a_{13}-b_{13})}\right) + k_1, a_{13} - b_{13} \le x \le a_{14} - b_{14} \\ (k_6 - k_5)\left(\dfrac{a_{15}-b_{15}-x}{a_{15}-b_{15}-(a_{14}-b_{14})}\right) + k_5, a_{14} - b_{14} \le x \le a_{15} - b_{15} \\ k_5, a_{15} - b_{15} \le x \le a_{16} - b_{16} \\ (k_5 - k_4)\left(\dfrac{a_{17}-b_{17}-x}{a_{17}-b_{17}-(a_{16}-b_{16})}\right) + k_4, a_{16} - b_{16} \le x \le a_{17} - b_{17} \\ k_4, a_{17} - b_{17} \le x \le a_{18} - b_{18} \\ (k_4 - k_3)\left(\dfrac{a_{19}-b_{19}-x}{a_{19}-b_{19}-(a_{18}-b_{18})}\right) + k_3, a_{18} - b_{18} \le x \le a_{19} - b_{19} \\ k_3, a_{19} - b_{19} \le x \le a_{20} - b_{20} \\ (k_3 - k_2)\left(\dfrac{a_{21}-b_{21}-x}{a_{21}-b_{21}-(a_{20}-b_{20})}\right) + k_2, a_{20} - b_{20} \le x \le a_{21} - b_{21} \\ k_2, a_{21} - b_{21} \le x \le a_{22} - b_{22} \\ (k_2 - k_1)\left(\dfrac{a_{23}-b_{23}-x}{a_{23}-b_{23}-(a_{22}-b_{22})}\right) + k_1, a_{22} - b_{22} \le x \le a_{23} - b_{23} \\ k_1, a_{23} - b_{23} \le x \le a_{24} - b_{24} \\ k_1\left(\dfrac{a_{25}-b_{25}-x}{a_{25}-b_{25}-(a_{24}-b_{24})}\right), a_{24} - b_{24} \le x \le a_{25} - b_{25} \end{cases}$$

Thus if $A = (a_1, a_2, \dots, a_{25})$ and $B = (b_1, b_2, \dots, b_{25})$ are Icosikaipentagonal fuzzy number, then$C = A - B = (a_1 - b_1, a_2 - b_2, \dots, a_{25} - b_{25})$.

Theorem 4.5. If $A = (a_1, a_2, \dots, a_{25})$ and $B = (b_1, b_2, \dots, b_{25})$ are Icosikaipentagonal fuzzy numbers, then $C = A * B$ is also a Icosikaipentagonal fuzzy numbers $A * B = (a_1 * b_1, a_2 * b_2, \dots, a_{25} * b_{25})$

Proof: Let us add the α – cut of A and B Icosikaipentagonal fuzzy number is defined as $C = A_\alpha * B_\alpha$ and $z = A * B$

$$C = \begin{cases} [a_1 + 7\alpha(a_2 - a_1), a_{25} - 7\alpha(a_{25} - a_{24})] * \\ [b_1 + 7\alpha(b_2 - b_1), b_{25} - 7\alpha(b_{25} - b_{24})], for\ \alpha \in [0,0.14) \\ [a_3 + (7\alpha - 1)(a_4 - a_3), a_{23} - (7\alpha - 1)(a_{23} - a_{22}] * \\ [b_3 + (7\alpha - 1)(b_4 - b_3), b_{23} - (7\alpha - 1)(b_{23} - b_{22})], for\ \alpha \in [0.14,0.28) \\ [a_5 + (7\alpha - 2)(a_6 - a_5), a_{21} - (7\alpha - 2)(a_{21} - a_{20})] * \\ b_5 + (7\alpha - 2)(b_6 - b_5), b_{21} - (7\alpha - 2)(b_{21} - b_{20}), for\ \alpha \in [0.28,0.42) \\ [a_7 + (7\alpha - 3)(a_8 - a_7), a_{19} - (7\alpha - 3)(a_{19} - a_{18})] * \\ [b_7 + (7\alpha - 3)(b_8 - b_7), b_{19} - (7\alpha - 3)(b_{19} - b_{18})], for\ \alpha \in [0.42,0.56) \\ [a_9 + (7\alpha - 4)(a_{10} - a_9), a_{17} - (7\alpha - 4)(a_{17} - a_{16})] * \\ [b_9 + (7\alpha - 4)(b_{10} - b_9), b_{17} - (7\alpha - 4)(b_{17} - b_{16})], for\ \alpha \in [0.56,0.7) \\ [a_{11} + (7\alpha - 5)(a_{12} - a_{11}), a_{15} - (7\alpha - 5)(a_{15} - a_{14})] * \\ [b_{11} + (7\alpha - 5)(b_{12} - b_{11}), b_{15} - (7\alpha - 5)(b_{15} - b_{14}), for\ \alpha \in [0.7,0.84) \\ [a_{12} + (7\alpha - 6)(a_{13} - a_{12}), a_{14} - (7\alpha - 6)(a_{14} - a_{13})] * \\ [b_{12} + (7\alpha - 6)(b_{13} - b_{12}), b_{14} - (7\alpha - 6)(b_{14} - b_{13})], for\ \alpha \in [0.84,1] \end{cases}$$

$$\mu_{(A*B)}(x) = \begin{cases} 0, for\ x < a_1 * b_1 \\ \dfrac{-B_1 \pm \sqrt{B_1^2 - 4A_1(a_1b_1 - z)}}{2A_1}, for\ a_1 * b_1 \le x \le a_2 * b_2 \\ k_1, for\ a_2 * b_2 \le x \le a_3 * b_3 \\ \dfrac{-B_2 \pm \sqrt{B_2^2 - 4A_2(4a_3b_3 - 2(a_3b_4 + a_4b_3) + a_4b_4 - z)}}{2A_2}, for a_3 * b_3 \le x \le a_4 * b_4 \\ k_2, for\ a_4 * b_4 \le x \le a_5 * b_5 \\ \dfrac{-B_3 \pm \sqrt{B_3^2 - 4A_3(9a_5b_5 - 6(a_5b_6 + a_6b_5) + 4a_6b_6 - z)}}{2A_3}, for\ a_5 * b_5 \le x \le a_6 * b_6 \\ k_3, for\ a_6 * b_6 \le x \le a_7 * b_7 \\ \dfrac{-B_4 \pm \sqrt{B_4^2 - 4A_4(16a_7b_7 - 12(a_7b_8 + a_8b_7) + 9a_8b_8 - z)}}{2A_4}, for\ a_7 * b_7 \le x \le a_8 * b_8 \\ k_4, for\ a_8 * b_8 \le x \le a_9 * b_9 \\ \dfrac{-B_5 \pm \sqrt{B_5^2 - 4A_5(25a_9b_9 - 20(a_9b_{10} + a_{10}b_9) + 16a_{10}b_{10} - z)}}{2A_5}, for\ a_9 * b_9 \le x \le a_{10} * b_{10} \\ k_5, for\ a_{10} * b_{10} \le x \le a_{11} * b_{11} \\ \dfrac{-B_6 \pm \sqrt{B_6^2 - 4A_6(31a_{11}b_{11} - 30(a_{11}b_{12} + a_{12}b_{11}) + 25a_{12}b_{12} - z)}}{2A_6}, for\ a_{11} * b_{11} \le x \le a_{12} * b_{12} \\ \dfrac{-B_7 \pm \sqrt{B_7^2 - 4A_7(49a_{12}b_{12} - 42(a_{13}b_{12} + a_{12}b_{13}) + 36a_{13}b_{13} - z)}}{2A_7}, for\ a_{12} * b_{12} \le x \le a_{13} * b_{13} \\ \dfrac{-B_8 \pm \sqrt{B_8^2 - 4A_8(49a_{13}b_{13} - 42(a_{13}b_{14} + a_{14}b_{13}) + 36a_{14}b_{14} - z)}}{2A_8}, for\ a_{13} * b_{13} \le x \le a_{14} * b_{14} \\ \dfrac{-B_9 \pm \sqrt{B_9^2 - 4A_9(31a_{14}b_{14} - 30(a_{15}b_{14} + a_{14}b_{15}) + 25a_{15}b_{15} - z)}}{2A_9}, for\ a_{14} * b_{14} \le x \le a_{15} * b_{15} \\ k_5, for\ a_{15} * b_{15} \le x \le a_{16} * b_{16} \\ \dfrac{-B_{10} \pm \sqrt{B_{10}^2 - 4a_{10}(25a_{16}b_{16} - 20(a_{17}b_{16} + a_{16}b_{17}) + 16a_{17}b_{17} - z)}}{2A_{10}}, for\ a_{16} * b_{16} \le x \le a_{17} * b_{17} \\ k_4, for\ a_{17} * b_{17} \le x \le a_{18} * b_{18} \\ \dfrac{-B_{11} \pm \sqrt{B_{11}^2 - 4A_{11}(16a_{18}b_{18} - 12(a_{18}b_{19} + a_{19}b_{18}) + 9a_{19}b_{19} - z)}}{2A_{11}}, for a_{18} * b_{18} \le x \le a_{19} * b_{19} \\ k_3, for a_{19} * b_{19} \le x \le a_{20} * b_{20} \\ \dfrac{-B_{12} \pm \sqrt{B_{12}^2 - 4A_{12}(9a_{20}b_{20} - 6(a_{20}b_{21} + a_{21}b_{20}) + 4(a_{21}b_{21}) - z)}}{2A_{12}}, for\ a_{20} * b_{20} \le x \le a_{21} * b_{21} \\ k_2, for\ a_{21} * b_{21} \le x \le a_{22} * b_{22} \\ \dfrac{-B_{13} \pm \sqrt{B_{13}^2 - 4A_{13}(4a_{22}b_{22} - 2(a_{22}b_{23} + a_{23}b_{22}) + a_{23}b_{23} - z)}}{2A_{13}}, for\ a_{22} * b_{22} \le x \le a_{23} * b_{23} \\ k_1, for\ a_{23} * b_{23} \le x \le a_{24} * b_{24} \\ \dfrac{-B_{14} \pm \sqrt{B_{14}^2 - 4A_{14}(a_{25}b_{25} - z)}}{2A_{14}}, for\ a_{24} * b_{24} \le x \le a_{25} * b_{25} \end{cases}$$

Where $A_i = 49(a_{i+1} - a_i)(b_{i+1} - b_i)$ and $B_j = 7(a_{i+1} - a_i)(jb_j - b_{j+1}) + (b_j - 1)\left(ja_j - (j-1)\right)$ for $i = 1,2,\dots,24$ *and* $j = 1,2,\dots,24$. Thus if $A = (a_1, a_2, \dots, a_{25})$ and $B = (a_1, b_2, \dots, B_{25})$ are Icosikaipentagonal fuzzy numbers, then $C = A * B = (a_1 * b_1, a_2 * b_2, \dots, a_{25} * b_{25})$.

5. Ranking of Icosikaipentagonal Fuzzy Number

In this section, we proposed a new ranking function with measure of Icosikaipentagonal fuzzy number. The measure of Icosikaipentagonal fuzzy number is defined as average of the two fuzzy side areas(left side area and right side area), from membership function to α axis of the fuzzy interval.

Definition 5.1. Let $A = (a_1, a_2, \dots, a_{25})$ be a normal Icosikaipentagonal fuzzy number. Then the measure of Icosikaipentagonal fuzzy number is calculated as follows:

$$R(A) = \frac{1}{2}[\int_0^{k_1} \left(P_1(\alpha) + P_2(\alpha)\right)d\alpha + \int_{k_1}^{k_2} \left(Q_1(\alpha) + Q_2(\alpha)\right)d\alpha + \int_{k_2}^{k_3} \left(R_1(\alpha) + R_2(\alpha)\right)d\alpha + \int_{k_3}^{k_4} \left(S_1(\alpha) + S_2(\alpha)\right)d\alpha]$$
$$+ \int_{k_4}^{k_5} \left(T_1(\alpha) + T_2(\alpha)\right)d\alpha + \int_{k_5}^{k_6} \left(U_1(\alpha) + U_2(\alpha)\right)d\alpha + \int_{k_6}^{1} \left(V_1(\alpha) + V_2(\alpha)\right)d\alpha]$$

$$R(A) = \frac{1}{4}[k_1(a_1 + a_2 + a_{24} + a_{25}) + (k_2 - k_1)(a_3 + a_4 + a_{22} + a_{23}) + (k_3 - k_2)(a_5 + a_6 + a_{20} + a_{21}) + (k_4 - k_3)(a_7 + a_8 + a_{18} + a_{19}) + (k_5 - k_4)(a_9 + a_{10} + a_{16} + a_{17}) + (k_6 - k_5)(a_{11} + a_{12} + a_{14} + a_{15}) + (1 - k_6)(a_{12} + a_{13} + a_{14} + a_{15})]$$

Where $k_1 = \frac{1}{7}, k_2 = \frac{2}{7}, k_3 = \frac{3}{7}, k_4 = \frac{4}{7}, k_5 = \frac{5}{7}, k_6 = \frac{6}{7}$ *and* $(k_1, k_2, k_3, k_4, k_5, k_6) \in [0,1]$

6. Fuzzy Linear Programming Problems with Icosikaipentagonal Fuzzy Number

Consider the standard form of fuzzy linear programming problem as follows:
Maximize (Minimize)$Z = \sum_{j=1}^{n} \tilde{c}_j x_j$
Subject to $\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq (\geq, =)\tilde{b}_i$
$x_j \geq 0, i = 1,2,\dots,m, j = 1,2,\dots,n$
Where $\tilde{c}_j, \tilde{a}_{ij}, \tilde{b}_i$ are Icosikaipentagonal fuzzy numbers.

6.1. Proposed Method

In this section to find an optimal solution for fuzzy linear programming problem with Icosikaipentagonal fuzzy number has been proposed. The method as follows:

Step 1: The Fuzzy Linear programming problem is represented as follows:
Maximize (Minimize)$Z = \sum_{j=1}^{n} \tilde{c}_j x_j$
Subject to $\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq (\geq, =)\tilde{b}_i$
$x_j \geq 0, i = 1,2,\dots,m, j = 1,2,\dots,n$
Where $\tilde{c}_j, \tilde{a}_{ij}, \tilde{b}_i \in F(R), x_j \in R$.

Step 2: By applying ranking function with Icosikaipentagonal fuzzy number $\tilde{c}_j, \tilde{a}_{ij}, \tilde{b}_i$ they can be defuzzified.

Step 3: Using the Icosikaipentagonal fuzzy number ranking function, the fuzzy linear programming problem can be written as follows:
Maximize (Minimize)$Z = \sum_{j=1}^{n} R(\tilde{c}_j) x_j$
Subject to $\sum_{j=1}^{n} R(\tilde{a}_{ij}) x_j \leq (\geq, =)R(\tilde{b}_i)$
$x_j \geq 0, i = 1,2,\dots,m, j = 1,2,\dots,n$
Where $\tilde{c}_j, \tilde{a}_{ij}, \tilde{b}_i$ are Icosikaipentagonal fuzzy number and $R(\tilde{c}_j), R(\tilde{a}_{ij}), R(\tilde{b}_i), x_j \in R$,
$i = 1,2,\dots,m\ , j = 1,2,\dots,n$.

Step 4: Then solve the Crisp Linear programming problem using the simplex method and obtain a fuzzy optimal solution.

6.2. Illustrative Examples

Consider the fuzzy linear programming problem with Icosikaipentagonal fuzzy number.

6.2.1. Example

Maximize $Z = \tilde{c}_1 x_1 + \tilde{c}_2 x_2$
Subject to $\tilde{a}_{11} x_1 + \tilde{a}_{12} x_2 \leq \tilde{b}_1$
$\tilde{a}_{21} x_1 + \tilde{a}_{22} x_2 \leq \tilde{b}_2$
$x_1, x_2 \geq 0$

Where $\tilde{a}_{11} = (-8,-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16)$

$\tilde{a}_{12} = (1,2,3,4,5,8,9,12,13,15,17,18,20,21,24,25,26,37,28,29,30,31,33,34,35)$
$\tilde{a}_{21} = (4,5,6,7,8,9,10,12,14,16,18,20,21,22,23,24,25,26,29,30,31,32,33,34,35)$
$\tilde{a}_{22} = (-1,0,1,2,3,5,7,8,9,11,12,13,15,16,18,19,21,23,25,26,28,29,30,31,32)$
$\tilde{b}_1 = (-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19)$
$\tilde{b}_2 = (2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,33,34,35,36,37,38,39,40,41)$
$\tilde{c}_1 = (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25)$
$\tilde{c}_2 = (-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,18,20)$

Solution:
Using the measure of ranking function as FLPP as
Maximize $Z = R(\tilde{c}_1)x_1 + R(\tilde{c}_2)x_2$
Subject to $R(\tilde{a}_{11})x_1 + R(\tilde{a}_{12})x_2 \le R(\tilde{b}_1)$
$R(\tilde{a}_{21})x_1 + R(\tilde{a}_{22})x_2 \le R(\tilde{b}_2)$
$x_1, x_2 \ge 0$

The measure of ranking function with Icosikaipentagonal fuzzy number is

$$R(A) = \frac{1}{4}[k_1(a_1 + a_2 + a_{24} + a_{25}) + (k_2 - k_1)(a_3 + a_4 + a_{22} + a_{23}) + (k_3 - k_2)(a_5 + a_6 + a_{20} + a_{21}) + (k_4 - k_3)(a_7 + a_8 + a_{18} + a_{19}) + (k_5 - k_4)(a_9 + a_{10} + a_{16} + a_{17}) + (k_6 - k_5)(a_{11} + a_{12} + a_{14} + a_{15}) + (1 - k_6)(a_{12} + a_{13} + a_{14} + a_{15})]$$

Where $k_1 = \frac{1}{7}, k_2 = \frac{2}{7}, k_3 = \frac{3}{7}, k_4 = \frac{4}{7}, k_5 = \frac{5}{7}, k_6 = \frac{6}{7}$ *and* $(k_1, k_2, k_3, k_4, k_5, k_6) \in [0,1]$

If $R(-8,-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16) = \frac{1}{4}[0.14\,(-8 - 7 + 15 + 16) + 0.14(-6 -$ *4+13+14+0.14−4−3+11+12+0.14−2−1+9+10+0.140+1+7+8+0.142+3+5+6+0.143+4+5+6]*

$= \frac{1}{4}[0.14(16 + 16 + 16 + 16 + 16 + 16 + 18)]$
$= \frac{1}{4}(15.96) = 3.99$

The crisp linear programming problem with Icosikaipentagonal fuzzy number is given by
Maximize $Z = 12.81x_1 + 6.055x_2$
Subject to $3.99x_1 + 18.66x_2 \le 6.93$
$19.57x_1 + 15.05x_2 \le 24.05$
$x_1, x_2 \ge 0$

Using Mathematical Linear Programming Problem, we obtain the optimal solution is $x_1 = 1.229, x_2 = 0$ and Max $Z = 15.743$

6.2.2. Example
Maximize $Z = \tilde{c}_1x_1 + \tilde{c}_2x_2$
Subject to $\tilde{a}_{11}x_1 + \tilde{a}_{12}x_2 \le \tilde{b}_1$
$\tilde{a}_{21}x_1 + \tilde{a}_{22}x_2 \le \tilde{b}_2$
$x_1, x_2 \ge 0$

Where
$\tilde{a}_{11} = (-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17)$ $\tilde{a}_{12} =$ $(1,2,3,4,5,6,7,8,9,10,11,12,13,,14,15,16,17,18,19,20,21,22,23,24,25)$
$\tilde{a}_{21} = (-20,-18,-16,-14,-12,-10,-8,-6,-4,-2,0,2,4,6,8,10,12,14,16,18,20,22,24,26,28)$
$\tilde{a}_{22} = (-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9.10,11,12,13,14)$
$\tilde{b}_1 = (-7,-5,-3,-1,0,2,4,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,22,23)$
$\tilde{b}_2 = (-2,0,2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36,38,40,42,44,46)$
$\tilde{c}_1 = (-1,0,1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37,39,41,43,45)$
$\tilde{c}_2 = (-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,1)$

Solution:
Using the measure of ranking function as FLPP as
Maximize $Z = R(\tilde{c}_1)x_1 + R(\tilde{c}_2)x_2$
Subject to $R(\tilde{a}_{11})x_1 + R(\tilde{a}_{12})x_2 \le R(\tilde{b}_1)$
$R(\tilde{a}_{21})x_1 + R(\tilde{a}_{22})x_2 \le R(\tilde{b}_2)$
$x_1, x_2 \ge 0$

The measure of ranking function with Icosikaipentagonal fuzzy number is

$$R(A) = \frac{1}{4}[k_1(a_1 + a_2 + a_{24} + a_{25}) + (k_2 - k_1)(a_3 + a_4 + a_{22} + a_{23}) + (k_3 - k_2)(a_5 + a_6 + a_{20} + a_{21}) + (k_4 - k_3)(a_7 + a_8 + a_{18} + a_{19}) + (k_5 - k_4)(a_9 + a_{10} + a_{16} + a_{17}) + (k_6 - k_5)(a_{11} + a_{12} + a_{14} + a_{15}) + (1 - k_6)(a_{12} + a_{13} + a_{14} + a_{15})]$$

Where $k_1=\frac{1}{7}, k_2=\frac{2}{7}, k_3=\frac{3}{7}, k_4=\frac{4}{7}, k_5=\frac{5}{7}, k_6=\frac{6}{7}$ *and* $(k_1,k_2,k_3,k_4,k_5,k_6)\in[0,1]$

If $R(-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17)=\frac{1}{4}[0.14\,(-7-6+16+17)+0.14(-5-4+$ *14+15+0.14−3−2+12+13+0.14−1+0+10+11+0.141+2+8+9+0.143+4+6+7+0.144+5+6+7]*

$$=\frac{1}{4}[0.14(20+20+20+20+20+20+22)]$$
$$=\frac{1}{4}(19.88)=4.97$$

The crisp linear programming problem with Icosikaipentagonal fuzzy number is given by

Maximize $Z=20.83x_1+4.97x_2$

Subject to $4.97x_1+12.81x_2\le 10.01$

$4.06x_1+2.03x_2\le 21.7$

$x_1,x_2\ge 0$

Using Mathematical Linear Programming Problem, we obtain the optimal solution is $x_1=2.01, x_2=0$ and Max $Z=41.95$

7. Conclusion

In this paper, we introduced a new form fuzzy with twenty five points called as Icosikaipentagonal fuzzy number and defining some arithmetic operations using with Icosikaipentagonal membership function. Icosikaipentagonal fuzzy number applied with fuzzy linear programming problem to finding an optimal solution using the measure of ranking function.

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