Behavior of Synovial Fluid in a Circular Rigid Artery

Dr. A. K. Yadav¹, Dr. Sushil Kumar², C. S Yadav¹

¹Department Mathematics Govt. P.G. College, Datia, Madhya Pradesh, India
²Department Mathematics C.C.S.P.G. College, Heonra Etawah, Uttar Pradesh, India

ABSTRACT

Presented herein are the studies of behavior of synovial fluid in a circular rigid artery. The parameters specified P, a, μ and ωt. It is clear that the volume flow rate decreases with the increase of viscosity and decreases with the increase of value ωt. It has been observed that volume flow rate increases with the increase of ωt and decreases with the increase of viscosity. From the graph of imaginary part it has been observed that volume flow rate increases with the increase of ωt and decreases with the increase of the value of viscosity.

KEYWORDS: synovial fluid, synovial joints, joints cavity, blood flow, cartilage

INTRODUCTION

Within the last seven decades sufficient thought has been given to study of lubricant in human joints. The human joint may be visualized. The fluid in the cavity between two mating bones is act as lubricant. A synovial joint may be considered as load carrying system consisting of two mating bones with tangential and/or normal. The bone ends, which are usually globular in appearance, are covered with a soft sponge like material, called articular cartilage. The space between these cartilaginous extremities of the bones, known as joint cavity, is filled with a shear-dependant fluid called synovial fluid.

The behavior of synovial fluid is mainly governed by the characteristics of the articular cartilage. The synovial fluid is a clear yellowish dialyzade of blood plasma with concentration of the hyaluronic acid. When bones approach one another water and other low molecular weight molecules pass through the pores of the cartilage and thehyaluronic acid molecules stay behind.

Scientists have greatly been attracted to the physical problems arising in mechanism of the body functioning and are trying to analysis analytically as well as experimentally¹,²,³. Biomechanics has attracted engineers mathematicians and other scientists to study the functioning behavior of human skeletal system⁴,⁵,⁶. These studies have enabled the researchers to analyze the lubrication mechanism of joint along with the nutrition being transported to the bones and structural behavior of articular and synovial fluid⁷,⁸,⁹.

How to cite this paper: Dr. A. K. Yadav | Dr. Sushil Kumar | C. S Yadav "Behavior of Synovial Fluid in a Circular Rigid Artery" Published in International Journal of Trend in Scientific Research and Development (ijtsrd), ISSN: 2456-6470, Volume-4 | Issue-4, June 2020, pp.652-655, URL: www.ijtsrd.com/papers/ijtsrd31132.pdf

Copyright © 2020 by author(s) and International Journal of Trend in Scientific Research and Development Journal. This is an Open Access article distributed under the terms of the Creative Commons Attribution License (CC BY 4.0) (http://creativecommons.org/licenses/by/4.0)
Equation (1) is not a function of r, then

$$q_z = q_z(r, t)$$

and

$$p = p(z, t)$$

The equation (2) becomes

$$\frac{\mu}{r} \frac{\partial}{\partial r} \left( r \frac{\partial q_z}{\partial r} \right) - p \frac{\partial q_z}{\partial z} = \frac{\partial p}{\partial z}$$

We consider a sinusoidal flow, then

$$\frac{\partial p}{\partial z} = -P e^{i\omega t}$$

and

$$q_z(r, t) = U(r) e^{i\omega t}$$

Where $P$ is constant and $U(r)$ is the velocity profile across the artery.

The slip condition

$$U(r) = 0 \text{ at } r = a$$

**Solution of the problem**

From equation (5) and (6), we get;

$$\frac{d^2 U}{dr^2} + \frac{1}{r} \frac{dU}{dr} - \frac{i\omega p}{\mu} U = -\frac{P}{\mu}$$

The general solution of equation (8) is

$$U(r) = A J_0 \left( i \sqrt{\frac{i\omega p}{\mu}} r \right) + B Y_0 \left( i \sqrt{\frac{i\omega p}{\mu}} r \right) + \frac{P}{i\omega p} = 0$$

If $q_z$ and $U$ must be finite on the axis and $Y_0(0)$ is not finite, then $B$ has to be zero.

From equation (9), we have

$$A J_0 \left( \frac{3i}{\sqrt{\frac{i\omega p}{\mu}}} a \right) + \frac{P}{i\omega p} = 0$$

Then

$$A = \frac{ip}{\omega p} J_0 \left( \frac{3i}{\sqrt{\frac{i\omega p}{\mu}}} a \right)$$

Where $\alpha = a \sqrt{\frac{\omega p}{\mu}}$

and

For real part $(\cos)p=1, a=0.5$

<table>
<thead>
<tr>
<th>$\omega t$</th>
<th>$\pi/6$</th>
<th>$\pi/5$</th>
<th>$\pi/4$</th>
<th>$\pi/3$</th>
<th>$\pi/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.25</td>
<td>0.08487</td>
<td>0.07928</td>
<td>0.06929</td>
<td>0.04900</td>
<td>0.00000</td>
</tr>
<tr>
<td>.50</td>
<td>0.04243</td>
<td>0.03964</td>
<td>0.03464</td>
<td>0.02450</td>
<td>0.00000</td>
</tr>
<tr>
<td>.75</td>
<td>0.02829</td>
<td>0.02642</td>
<td>0.02309</td>
<td>0.01633</td>
<td>0.00000</td>
</tr>
<tr>
<td>1.00</td>
<td>0.02121</td>
<td>0.01982</td>
<td>0.01732</td>
<td>0.01225</td>
<td>0.00000</td>
</tr>
<tr>
<td>1.25</td>
<td>0.01697</td>
<td>0.01585</td>
<td>0.01385</td>
<td>0.00980</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

The present paper proposes a more realistic model for explaining the behavior of synovial fluid in a circular rigid artery. The volume flow rate variation depends on various of parameter. It is clear that the volume flow rate of synovial fluid decreases with the increases of viscosity. From the graph of real part, it is clear that the volume flow rate decreases with the increase of viscosity and decreases with the increase the value of $\omega t$.

From the graph of imaginary part the increase of value of $\omega t$, it has been observed that volume flow rate increases with the increase of $\omega t$ and decreases with the increase the value of viscosity.
Graph of volume flow rate of (real part) synovial fluid for different values of $\omega t$ and $\mu$

For imaginary part ($\sin$) $p=1$, $a=0.5$

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\omega t = \pi/6$</th>
<th>$\pi/5$</th>
<th>$\pi/4$</th>
<th>$\pi/3$</th>
<th>$\pi/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.04900</td>
<td>0.05760</td>
<td>0.06929</td>
<td>0.08787</td>
<td>0.09800</td>
</tr>
<tr>
<td>0.50</td>
<td>0.02450</td>
<td>0.02880</td>
<td>0.03464</td>
<td>0.04243</td>
<td>0.04900</td>
</tr>
<tr>
<td>0.75</td>
<td>0.01633</td>
<td>0.01920</td>
<td>0.02309</td>
<td>0.02829</td>
<td>0.03266</td>
</tr>
<tr>
<td>1.00</td>
<td>0.01225</td>
<td>0.01440</td>
<td>0.01732</td>
<td>0.02121</td>
<td>0.02450</td>
</tr>
<tr>
<td>1.25</td>
<td>0.00980</td>
<td>0.01152</td>
<td>0.01385</td>
<td>0.01697</td>
<td>0.01960</td>
</tr>
</tbody>
</table>

Graph of volume flow rate of (real part) synovial fluid for different values of $\omega t$ and $\mu$
Reference


[3] Ogston, A. G. and J. E. Stanier (1953) "Viscous elastic lubricant properties" Physiol. 119,244-252


