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Behavior of Synovial Fluid in a Circular Rigid Artery

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ABSTRACT

Presented herein are the studies of behavior of synovial fluid in a circular rigid artery. The parameters specified P, a, μ andω. It is clear that the volume flow rate decreases with the increase of viscosity and decreases with the increase of value ωt . it has been observed that volume flow rate increases with the increase of ωt and decreases with the increase the value of viscosity. From the graph of imaginary part it has been observed that volume flow rate increases with the increase of ωt and decreases with the increase the value of viscosity.

KEYWORDS: *synovial fluid, synovial joints, joints cavity, blood flow, cartilage*

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INTRODUCTION

Within the last seven decades sufficient thought has been given to study of lubricant in human joints. The human joint may be visualized. The fluid in the cavity between two mating bones is act as lubricant. A synovial joint may be considered as load carrying system consisting of two mating bones with tangential and/or normal. The bone ends, which are usually globular in appearance, are covered with a soft sponge like material, called articular cartilage. The space between these cartilaginous extremities of the bones, known as joint cavity, is filled with a shear-dependant fluid called synovial fluid.

The behavior of synovial fluid is mainly governed by the characteristics of the articular cartilage. The synovial fluid is a clear yellowish dialyzade of blood plasma with concentration of the hyaluronic acid. When bones approach one another water and other low molecular weight molecules pass through the pores of the cartilage and thehyaluronic acid molecules stay behind.

Scientists have greatly been attracted to the physical problems arising in mechanism of the body functioning and are trying to analysis analytically as well as experimentally[1,2,3]. Biomechanics has attracted engineers mathematicians and other scientists to study the functioning behavior of human skeletal system[4,5,6]. These studies have enabled the researchers to analyze the lubrication mechanism of joint along with the nutrition being transported to the bones and structural behavior of articular and synovial fluid[7,8].

Complex movements of fluids in the biological system demand for their analysis as professional fluid mechanical problems in the biological system in the flow of blood. Many bio-fluid mechanics problems are not only concern with classical fluid mechanics.

Various researchers considered the problem of lubrication approaching porous surfaces in reference to human joint[9,10]. Under the full load conditions the incongruity gets eliminated and large contact area ensued by low modulus of elasticity of the cartilage. More fluid will be trapped in the centre of contact area. The movement of the fluid in and out of the cartilage surfaces contribute considerably to its damped elasticity. The effect of variable viscosity of the lubrication due to filtration action considerd in reference to human joints.

In this paper we have made on attempt to study the behavior of synovial fluid in a circular rigid artery.

Formulation of the problem

Consider the flow of blood in a circular artery of radius a and assuming that only non-zero component of the velocity vector is in the axial direction.

The Navier stokes equation

$$-\frac{\partial p}{\partial r} = 0 \qquad (1)$$

and

$$\rho\frac{\partial q_z}{\partial t} = -\frac{\partial p}{\partial z} + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial q_z}{\partial r}\right) \qquad (2)$$

Equation (1) is not a function of r, then

$$q_z = q_z(r,t) \quad (3)$$

and

$$p = p(z,t) \quad (4)$$

The equation (2) becomes

$$\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial q_z}{\partial r}\right) - \rho\frac{\partial q_z}{\partial t} = \frac{\partial p}{\partial z} \quad (5)$$

We consider a sinusoidal flow, then

$$\frac{\partial p}{\partial z} = -Pe^{i\omega t} \quad (6)$$

and

$$q_z(r,t) = U(r)e^{i\omega t} \quad (7)$$

Where P is constant and $U(r)$ is the velocity profile across the artery.
The slip condition

$$U(r) = 0 \text{ at } r = a \quad (8)$$

Solution of the problem

From Equation (5) and (6), we get;

$$\frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dr} - \frac{i\omega\rho}{\mu}U = -\frac{P}{\mu} \quad (9)$$

The general solution of equation (8) is

$$U(r) = AJ_0\left(i\sqrt{\frac{i\omega\rho}{\mu}}\right)r + BY_0\left(i\sqrt{\frac{i\omega\rho}{\mu}}\right)r + \frac{P}{i\omega\rho} = 0 \quad (10)$$

If q_z and U must be finite on the axis and $Y_0(0)$ is not finite, then B has to be zero.

From equation (9), we have

$$AJ_0\left(i^{\frac{3}{2}}\sqrt{\frac{\omega\rho}{\mu}}\right)a + \frac{P}{\omega\rho i} = 0 \quad (11)$$

Then

$$A = \frac{ip}{\omega\rho}\frac{1}{J_0\left(i^{\frac{3}{2}}\alpha\right)} \quad (12)$$

$$\text{Where } \alpha = a\sqrt{\frac{\omega\rho}{\mu}} \quad (13)$$

and

$$q_r(r,t) = -\frac{iP}{\omega\rho}\left[1 - \frac{J_0\left(i^{\frac{3}{2}}\frac{\alpha r}{a}\right)}{J_0\left(i^{\frac{3}{2}}\alpha\right)}\right]e^{i\omega t} \quad (14)$$

The volume flow rate

$$Q = \int_0^a q_z(r,t)2\pi r dr$$

$$Q = -\frac{\pi iP}{\omega\rho}e^{i\omega t}\alpha^2\left[1 - \frac{2}{\alpha^3 J_0\left(i^{\frac{3}{2}}\alpha\right)}\int_0^a rJ_0\left(\frac{i^{\frac{3}{2}}\alpha r}{a}\right)dr\right] \quad (15)$$

But

$$\int_0^a rJ_0\left(i^{\frac{3}{2}}\frac{\alpha r}{a}\right)dr = \frac{\alpha^2}{\beta}J_1(\beta) \quad (16)$$

$$\text{Where } \beta = i^{\frac{3}{2}}\alpha \quad (17)$$

From equation (15) & (16), we have

$$Q = -\frac{\pi iP}{\omega\rho}e^{i\omega t}\alpha^2\left[1 - \frac{2J_1(\beta)}{\beta J_0(\beta)}\right] \quad (18)$$

and

$$Q = \left[\frac{\pi P a^4}{8\mu} + o(\alpha^4)\right]e^{i\omega t} \quad (19)$$

As $\alpha \to 0$ and $\omega \to 0$ then $\to Q_0$, we have

$$Q_0 = \frac{\pi pP}{8\mu}e^{i\omega t} \quad (20)$$

$$Q_0 = \frac{\pi P a^4}{8\mu}[cos\omega t + isin\omega t] \quad (21)$$

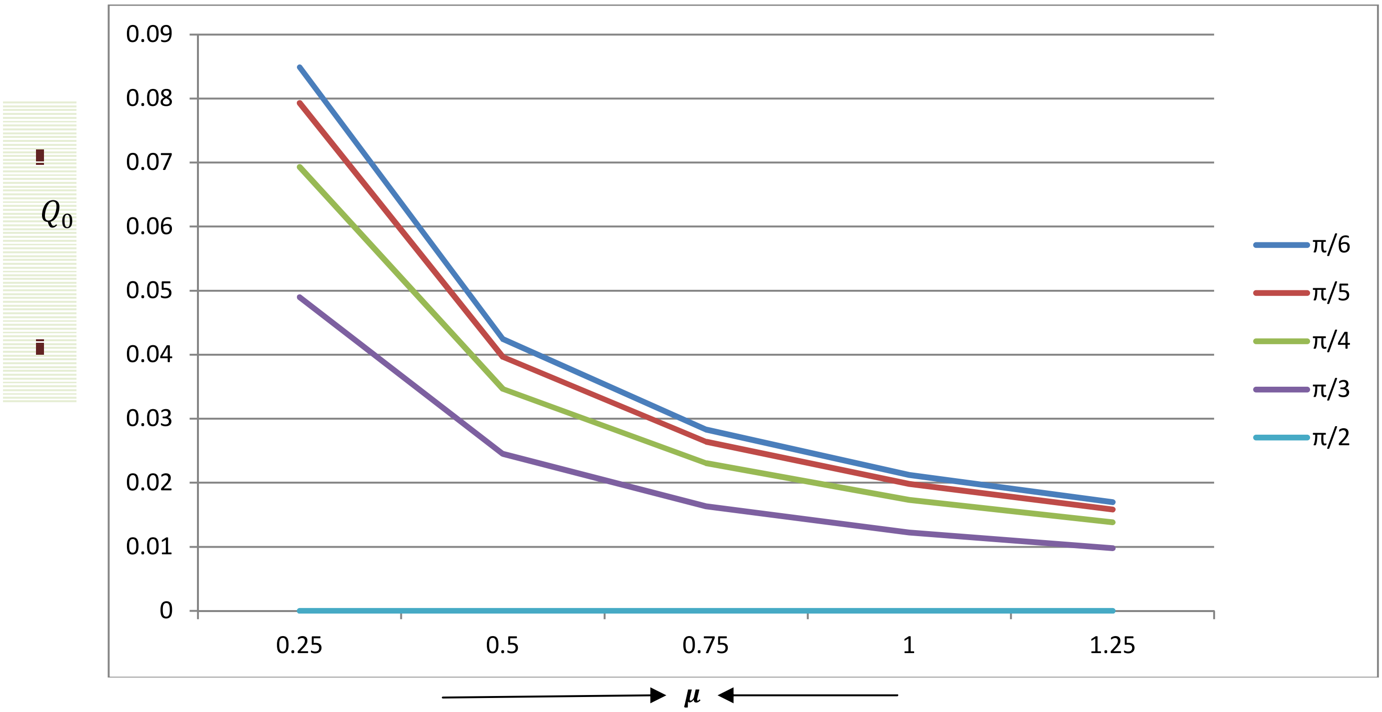
Result and discussion

The present paper proposes a more realistic model for explaining the behavior of synovial fluid in a circular rigid artery. The volume flow rate variation depends on various of parameter. It is clear that the volume flow rate of synovial fluid decreases with the increases of viscosity. From the graph of real part, it is clear that the volume flow rate decreases with the increase of viscosity and decreases with the increase the value of ωt .

From the graph of imaginary part the increase of value of ωt, it has been observed that volume flow rate increases with the increase of ωt and decreases with the increase the value of viscosity.

For real part (cos) p=1 , a=0.5

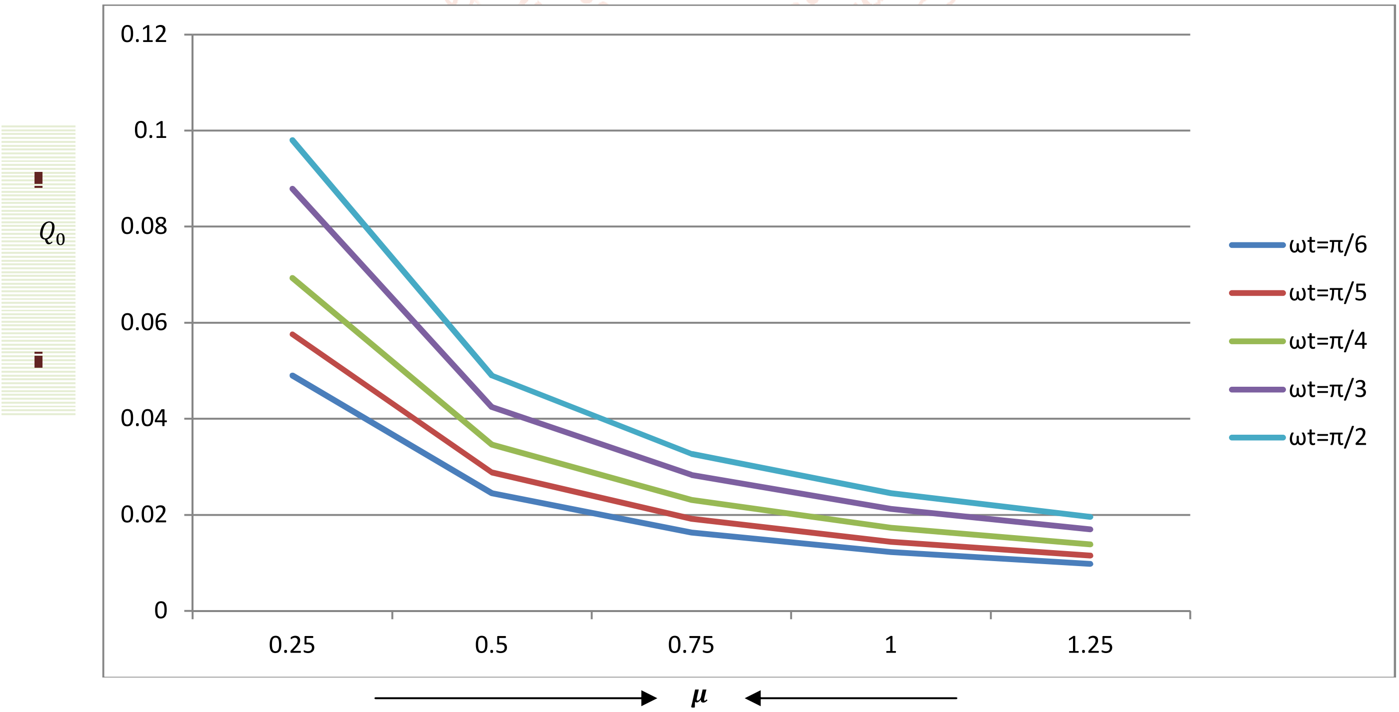
μ \ ωt	$\pi/6$	$\pi/5$	$\pi/4$	$\pi/3$	$\pi/2$
.25	0.08487	0.07928	0.06929	0.04900	0.00000
.50	0.04243	0.03964	0.03464	0.02450	0.00000
.75	0.02829	0.02642	0.02309	0.01633	0.00000
1.00	0.02121	0.01982	0.01732	0.01225	0.00000
1.25	0.01697	0.01585	0.01385	0.00980	0.00000



Graph of volume flow rate of(real part)synovial fluid for different values of ωt and μ

For imaginary part (sin) p=1 , a=0.5

μ \ ωt	$\pi/6$	$\pi/5$	$\pi/4$	$\pi/3$	$\pi/2$
.25	0.04900	0.05760	0.06929	0.08787	0.09800
.50	0.02450	0.02880	0.03464	0.04243	0.04900
.75	0.01633	0.01920	0.02309	0.02829	0.03266
1.00	0.01225	0.01440	0.01732	0.02121	0.02450
1.25	0.00980	0.01152	0.01385	0.01697	0.01960



Graph of volume flow rate of(real part)synovial fluid for different values of ωt and μ

Reference

[1] Downson, D. (1967) "Model of lubrication an human Joints" (Proc. Inst. Mech. Engrs. 183(3).

[2] Mow, C. W. (1968) "The role of lubrication in Biomechanical joints" J. Iubr..; pbr. Technl.9 1,320

[3] Ogston, A. G. and J. E. Stanier (1953) "Viscous elastic lubricant properties" Physiol. 119,244-252

[4] Tandon, P. N. and S. Jaggi (1977) "Lubrication of Hertzian contacts in reference to human joints."Med. Life Sci. Eng. 3(7)

[5] Tandon, P. N. and S. Jaggi (1977) "Lubrication of porous solid in reference to human joints" Pro. Ind. Acad. Sci. Sect. A85, 144.

[6] Yadav A. K. and Pokhriyal S. C. (2000): synovial fluid flow in reference to human joints. Intr. J. Appl. Sc. Periadical vol. 11, p 104-110.

[7] Yadav, A. K. (2000) "Synovial fluid behavior in reference to animal joints." Indian Jr. Pure and Apple. Sci. Vol. 16, 311-316

[8] Yadav A. K and Kumar S. (2016): Synovial fluid flow in reference to animal joints. Intr. Jr. Stream Research Jr. Vol. 6 issu-12, p-1-5.

[9] Yadav A. K. and Kumar S.(2016): Mathematical analysis for the lubrication mechanism of knee joints. Intr. Jr. of orthoperdics Photon. Vol.11, p 120-122.

[10] Yadav A. K. and Kumar S. (2020): Behaviour of synovial fluid in a channel. IJCRT Vol. 8 issu.- 3 p 2823-2833.