International Journal of Trend in Scientific Research and Development (IJTSRD)
Volume 4 Issue 3, April 2020 Available Online: www.ijtsrd.com e-ISSN: 2456 – 6470

Mixed Convective Hydromagnetic Stagnation Point Flow of Nanofluid over an Inclined Stretching Plate with Prescribed Surface Heat Flux

P. Suriyakumar

Department of Mathematics, Sri Shakthi Institute of Engineering and Technology, Coimbatore, Tamil Nadu, India

ABSTRACT

The aim of this paper is to investigate the two dimensional, nonlinear, steady, mixed convective flow of viscous, incompressible, electrically conducting nanofluid flow over an inclined stretching plate with prescribed heat flux in the presence of uniform transverse magnetic field. The present work concerns with two types of nanofluids such as copper-water and alumina-water nanofluids which analyse the heat transfer due to laminar flow of nanofluids over an inclined stretching plate with angle of inclination α with the horizontal. Using appropriate transformations, the governing partial differential equations are transformed into ordinary differential equations with corresponding conditions and which are solved numerically by MATLAB. Numerical solutions are obtained for the velocity and temperature as well as the skin friction coefficient and Nusselt number for different values of pertinent parameters and the physical aspects of the problem are discussed. The numerical results were validated by comparison with previously published results in the literature. It is found that the effect of nanoparticle volume fraction is to increase the heat transfer and hence enhance the thermal boundary layer thickness.

KEYWORDS: *Nanofluid, MHD, velocity ratio parameter, inclined stretching plate, mixed convection*

How to cite this paper: P. Suriyakumar "Mixed Convective Hydromagnetic Stagnation Point Flow of Nanofluid over an Inclined Stretching Plate with Prescribed Surface Heat Flux" Published in International Journal of Trend in Scientific Research and Development (ijtsrd), ISSN: 2456-6470, Volume-4 | Issue-3, April 2020, pp.905-914, URL: www.ijtsrd.com/papers/ijtsrd30700.pdf

IJTSRD30700

Nomenclature:

A	velocity ratio parameter	Pr	prandtl number
F	dimensionless stream function	$q_w(x)$	prescribed heat flux
G	gravitational acceleration vector	Re_x	local Reynolds number
Gr_x	local Grashof Number	T	Temperature
k_f	thermal conductivity of the base fluid	T_∞	ambient temperature
k_s	thermal conductivity of the nanoparticle	$U_w(x)$	stretching velocity
k_{nf}	thermal conductivity of the nanofluid	$U_\infty(x)$	free stream velocity
M^2	magnetic interaction parameter	(u,v)	velocity components
P	pressure	(x,y)	cartesian coordinates

Greek symbols

ϕ	volume fraction	ν_f	viscosity of the base fluid
ρ_f	density of the base fluid	ν_{nf}	viscosity of the nanofluid
ρ_s	density of the nanoparticle	α	angle of the inclination
ρ_{nf}	density of the nanofluid	μ_f	kinematic viscosity of the base fluid
β_f	volumetric expansion coefficient of the base fluid	μ_{nf}	kinematic viscosity of the nanofluid
β_s	volumetric expansion coefficient of the nanoparticle	λ	mixed convection parameter
β_{nf}	volumetric expansion coefficient of the nanofluid	η	similarity variable
$(\rho C_p)_f$	heat capacity of the base fluid	Ψ	stream function
$(\rho C_p)_s$	heat capacity of the nanoparticle	θ	dimensionless temperature
$(\rho C_p)_{nf}$	heat capacity of the nanofluid	α_{nf}	thermal diffusivity of the nanofluid

1. INTRODUCTION

Nanoparticles are of great scientific interest as they are effectively a bridge between bulk materials and atomic or molecular structures. The term Nanofluid was first coined by Choi (1995) at the Argonne National Laboratory to describe the new class of Nanotechnology-based heat transfer fluids that exhibit thermal properties superior to those of their base fluids or conventional particle fluid suspensions. A very small amount of nanoparticles having dimensions from 1 to 100nm, when dispersed uniformly and suspended stably in base fluids can provide impressive improvements in the thermal properties of base fluid. The thermal conductivity of the ordinary fluids is not adequate to meet today's cooling rate requirements. Nanofluids have been shown to increase the thermal conductivity and convective heat transfer performance of the base fluids. The study of nanofluid increases rapidly due to its several industrial, engineering and technological applications such as ranging from transportation to energy production and supply to electronics, power generation, micro-manufacturing, thermal therapy for cancer treatment, chemical catalytic reactors, grain storage installations, diffusion of medicine in blood veins, ventilation, air-conditioning, welding equipment and high heat flux devices such as high-power microwave tubes and high-power laser diode arrays they can also flow through tiny passages in MEMS.

Besides, the flow near the stagnation point has attracted the attention of many investigators for more than a century because of its wide variety of applications such as cooling of electronic devices by fans, cooling of nuclear reactors during emergency shutdown, solar central receivers exposed to wind currents, and many hydrodynamic processes in engineering applications. The laminar boundary layer flow over a stretching surface is important from theoretical as well as practical point of view because of their wider applications to polymer technology and metallurgy. During many mechanical forming processes such as melt-spinning, aerodynamic extrusion of plastic sheets, cooling of a large metallic plate in a bath, manufacture of plastic and rubber sheets, glass blowing, continuous casting, and spinning of fibers, the extruded material issues through a die.

Further, the research of magnetohydrodynamic (MHD) incompressible viscous flow has many important engineering applications in devices such as power generator, MHD blood flow meters, the design of heat exchangers and MHD accelerators. Moreover, the effects of external magnetic field on magnetohydrodynamic(MHD) flow over a stretching surface are very important due to its many engineering and geophysical applications such as geothermal reservoirs, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors and cooling of nuclear reactors.

Sakiadis (1961) was the first one to analyze the boundary layer flow on continuous surfaces. The dynamics of boundary layer flow over a stretching surface was examined by Crane (1970). He obtained an exact solution for the steady two-dimensional flow of a viscous and incompressible fluid induced by the stretching of an elastic flat sheet in its own plane with a velocity varying linearly with the distance from a fixed point due to the application of a uniform stress. Mucoglu and Chen (1979) is performed to study the mixed convection along an inclined flat plate, with the angle of inclination ranging from 0^o to 90^o from the vertical and the plate is kept at a uniform temperature. Later, many investigations were proposed the hydromagnetic flow over inclined stretching surface considering various physical situations and few of them are Chamkha and Rahim (2001),Ramadan and Chamkha (2003),Alam et al. (2006), Aydin and Kaya (2009) and Noor et al. (2012). However, the stagnation-point flow towards a surface which is moved or stretched, have been considered many researchers such as Chiam (1994), Lok et al.(2007), Wang(2008) and Wong et.al(2013). Mahapatra and Gupta (2001), Abdelkhalek (2006), Anuar Ishak et al. (2009 &2011), Singh et al.(2010)and Lok et al.(2011) studied the effect of magnetic field stagnation point flow over a stretching surface.

The research on nanofluids is gaining a lot of attention in recent years. Heat transfer in nanofluids has been surveyed by review articles by Eastman et al.(2004), Wang and Mujumdar (2007), Das and Choi(2009), and Kakac and Pramuanjaroenkij (2009) and a recent book by Das et al.(2008). These reviews discuss in detail the preparation of nanofluids, theoretical and experimental investigations of thermal conductivity and viscosity of nanofluids and the work done on convective transport in nanofluids. Buongiorno(2006) made a comprehensive survey of convection in nanofluids and wrote down conservation equations for nanofluids based on the effects of Brownian motion and thermophoresis. Santra et al.(2008) presented the effect of copper-water nanofluid as a cooling medium which has been studied to simulate the behavior of heat transfer due to laminar natural convection in a differentially heated square cavity. The effect of inclination angle on natural convection in enclosures filled with Cu-water nanofluid was studied by Abu-Nada and Oztop (2009). Khan and Pop (2010) have proposed the problem of laminar fluid flow over the stretching surface in a nanofluid and they investigated it numerically. Bachok et.al (2010) analysed the boundary layer flow of a nanofluid past a moving plate in a uniform free stream. Nanofluid flow over a flat plate was studied by Anjali Devi and Julie Andrews (2011) and it was found out that suspended nanoparticles enhance the heat transfer capacity of the fluids.

Mustfa et al.(2011) investigated stagnation- point flow of a nanofluid towards a stretching sheet. The steady two-dimensional stagnation-point flow of a nanofluid over a stretching/shrinking sheet in its own plane was carried out by Bachok et al.(2011), different from a stretching sheet, it was found that the solutions for a shrinking sheet are non-unique. Rana et al.(2012)studied the numerical solution for mixed convection boundary layer flow of a nanofluid along an inclined plate embedded in a porous medium.

The heat transfer characteristics of steady two-dimensional stagnation point flow of a copper-water nanofluid over a permeable stretching/shrinking sheet was carried out by Bachok et al.(2013). Das (2013) proposed the numerical simulations of mixed convection stagnation point flow and heat transfer of Cu-water nanofluids impinging normally towards a shrinking sheet. Makinde et al.(2013) analyzed the combined effects of buoyancy force on the MHD stagnation point flow and heat transfer due to nanofluid flow towards a stretching/ shrinking sheet. He reported that both the local Sherwood number and the skin friction coefficient decrease while the local Nusselt number increases with increasing intensity of buoyancy force. Ibrahim et al. (2013) numerically analyzed the effect of magnetic field on the stagnation point flow and heat transfer due to nanofluid towards a stretching sheet.

Malvandi et al. (2014) have proposed the analytical study on boundary layer flow and heat transfer of nanofluid induced by a non-linearly stretching sheet. The magnetic field effect of the boundary layer flow and heat transfer of nanofluids over a nonlinear stretching sheet has been provided by Mabood et al.(2015). The MHD boundary layer flow of alumina water nanofluid over a flat plate under slip conditions was presented by Singh and Kumar (2015). Anjali Devi and Suriyakumar (2016) studied the hydromagnetic convective flow of nanofluid past an inclined stretching plate in the presence of internal heat absorption and suction. Anwar et al. (2017) analysed the numerical study for hydromagnetic stagnation-point flow of a micropolar nanofluid towards a stretching sheet. Mishra et al. (2018) studied the heat and mass transfer on MHD Walters B nanofluid flow induced by a stretching porous surface. Recently, Micropolar nanofluid boundary layer flow over a linear inclined stretching surface with the magnetic effect was investigated by Rafique et al. (2019). The Buongiorno mathematical model of hydromagnetic micropolar nanofluid by using Keller box method was considered by Rafique et al. (2020).

In many applications, the hot surface may be subject to a prescribed heat flux instead of being at a constant temperature. For example, an embedded electronic component on a circuit board can be predefined a heat power, but the temperature of the component is not known. However, the rate of heat generation in the canisters of nuclear waste disposal is known, and hence the heat flux on the surface of these containers is known. Thus, the main focus of the analysis is to investigate the stagnation point flow of nanofluid with mixed convection over an inclined stretching plate within the boundary layer are influenced by the amount of applied magnetic field and prescribed heat flux. To the best of the authors' knowledge no research has been carried out considering the above-stated flow model for a nanofluid. By using similarity approach, the transport equations are transformed into non-linear ordinary differential equations and they are solved by MATLAB. The present results are compared with previously obtained solutions and they are in good agreement. The behaviour of the velocity, temperature, skin-friction and heat transfer rate has been discussed for a range of physical parameters. The accompanying discussion provides physical interpretations of the results.

2. Formulation of the Problem

Consider a two dimensional, steady, mixed convective laminar boundary layer flow of an incompressible, viscous, electrically conducting nanofluid past an inclined stretching plate. The fluid is considered in the influence of transverse magnetic field with prescribed heat flux. The plate is inclined at an angle of inclination α with the horizontal and is of infinite length. The Cartesian coordinates (x, y) are chosen such that x-axis is chosen along the plate and y-axis is chosen perpendicular to the plate. The linear stretching velocity and the prescribed surface heat flux are assumed to be of the forms $U_w(x)=ax$ and $q_w(x)=cx$ where a and c are constants and x is a coordinate measured along the stretching plate. The external flow takes place in the direction parallel to the inclined plate and has the free stream velocity $U_\infty(x) = bx$. The gravitational acceleration g is acting downward. A transverse magnetic field of strength $\overrightarrow{B_0}$ is applied normal to the plate. It is assumed that the magnetic Reynolds number is small and hence the induced magnetic field can be neglected. The induced magnetic field is assumed to be negligible and since being independent of time $rl\,\vec{E} = 0$. Also $div\,\vec{E} = 0$ in the absence of surface charge density. Hence $\vec{E} = 0$ is assumed.The fluid is a water based nanofluid containing two types of nanofluids such as copper-water and alumina-water nanofluids.

Under the foregoing assumptions, applying the usual boundary layer and Boussinesq approximations, the governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty(x)\frac{\partial U_\infty}{\partial x} + \nu_{nf}\frac{\partial^2 u}{\partial y^2} + \frac{g(\rho\beta)_{nf}(T-T_\infty)sin\alpha}{\rho_{nf}} - \frac{\sigma B_0^2(u-U_\infty(x))}{\rho_{nf}} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

The boundary conditions for the velocity and temperature of this problem are given by:

$$At\ yu = U_w(x) = ax, v = 0, q_w(x) = -k_{nf}\frac{\partial T}{\partial y}\ at\ y = 0 \tag{4}$$

$$u = U_\infty(x) = bx, T = T_\infty\ as\ y \to \infty$$

For the present study, water has been considered as the base fluid with Pr = 6.2 . The nanofluid considered is water mixed with solid spherical copper and aluminium nanoparticles. The effective density, heat capacity, dynamic viscosity, thermal expansion coefficient, thermal diffusivity and the thermal conductivity of the nanofluids are given by(Brinkman 1952)

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, (\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s, \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$$

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s, \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}$$

3. Method of Solution

In order to seek the solution of the problem, the following dimensionless variables are introduced:

$$\psi(x,y) = x\sqrt{b\nu_f}\,F(\eta), \eta = y\sqrt{\frac{b}{\nu_f}}, T - T_\infty = \frac{q_w(x)}{k_{nf}}\sqrt{\frac{\nu_f}{b}}\,\theta(\eta) \quad (5)$$

where $\psi(x,y)$ is the stream function such that it satisfies Eq.(1) with $u = \frac{\partial\psi}{\partial y}$, $v = -\frac{\partial\psi}{\partial x}$ and θ is the dimensionless temperature. It is obtained that

$$u = bx\,F'(\eta)\,, v = -\sqrt{b\nu_f}\,F(\eta) \quad (6)$$

The momentum and energy equations together with the boundary conditions can be written as

$$F''' + (1-\phi)^{2.5}\left\{\begin{array}{c}(1 + FF'' - F'^2)\left[(1-\phi) + \phi\left(\frac{\rho_s}{\rho_f}\right)\right] - M^2(F'-1) \\ +\frac{k_f}{k_{nf}}\lambda\left[(1-\phi) + \phi\left(\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\right]\theta sin\alpha \\ +\end{array}\right\} = 0 \quad (7)$$

$$\frac{1}{Pr}\frac{k_{nf}}{k_f}\theta'' + \left[(1-\phi) + \phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right](F\theta' - F'\theta) = 0 \quad (8)$$

with the boundary conditions as follows:

$$At\ \eta = 0, F(\eta) = 0, F'(\eta) = A\,, \theta'(\eta) = -1 \quad (9)$$
$$As\ \eta \to \infty, F'(\eta) = 1, \theta(\eta) = 0$$

Here the primes denote differentiations with respect to η. The corresponding dimensionless group that appears in the governing equations are defined by:

$$(Re_x)_f = \frac{U_\infty(x)x}{\nu_f}\,, (Gr_x)_f = \frac{g\beta_f x^4 q_w(x)}{k_f\nu_f^2}\,, \lambda = \frac{(Gr_x)_f}{(Re_x)^{\frac{5}{2}}} = \frac{g\beta_f c\nu_f^{\frac{1}{2}}}{k_f b^{\frac{5}{2}}}$$

$$M^2 = \frac{\sigma B_0^2}{b\rho_f}, A = \frac{a}{b}\,, \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, Pr = \frac{\nu_f}{\alpha_f}$$

where $(Re_x)_f$ is the local Reynolds number, $(Gr_x)_f$ is the local Grashof number, λ is the mixed convection parameter, M^2 is the magnetic interaction parameter, A is the velocity ratio parameter , α_{nf} is the thermal diffusivity of the nanofluid and Pr is the Prandtl number.

4. Numerical solution

The set of nonlinear coupled differential equations (7) and (8) along with the boundary conditions Eq.(9) constitute a boundary value problem. This boundary value problem cannot be solved analytically and hence are solved numerically using MATLAB for several values of the physical parameters such as the angle of inclination, volume fraction, magnetic interaction parameter, Prandtl number, velocity ratio parameter and mixed convection parameter. The asymptotic boundary conditions given by Eq.(9) were replaced by using a value of 15 for the similarity variable η_{max} as follows.

$$\eta_{max} = 15, F'(15) = 0, \theta(15) = 0$$

The choice of $\eta_{max} = 15$ ensured that all numerical solutions approached the asymptotic values correctly. The absolute error tolerance for this method is 10^{-6}. The numerical values for skin friction coefficient and the Nusselt number are also obtained and are tabulated for different values of M^2, ϕ, A and α.

Concerning this study, the physical quantities of practical interest are the local Skin friction coefficient C_f and the local Nusselt number Nu_x and are defined as

$$C_f = \frac{\tau_w}{\rho_f(bx)^2} \text{ where } \tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (10)$$

$$Nu_x = \frac{xq_w}{k_{f(T_w - T_\infty)}} \text{ where } q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (11)$$

Using Eq.(5) into Eqs.(10) and (11), we get

$$C_f(Re_x)_f^{1/2} = \frac{1}{(1-\phi)^{2.5}}F''(0), \qquad Nu_x(Re_x)_f^{-1/2} = \frac{k_{nf}}{k_f}\frac{1}{\theta(0)} \quad (12)$$

5. Results and Discussion

The two dimensional steady, laminar, mixed convective stagnation point flow of nanofluid over an inclined stretching plate in the presence of a uniform transverse magnetic field with prescribed heat flux has been analysed. Numerical solutions of the problem are obtained for various values of physical parameters involved in the study such as α, A , M^2, Pr and λ. The values of different physical parameters considered here are as follows: $\alpha = 0^0, 30^0, 45^0, 60^0$, A = 0.0,0.5,1.0,2.0, M^2 = 0,1,2,4 and ϕ = 0.00,0.01,0.05,0.1 for fixed Pr = 6.2 and $\lambda = 1.5$. Numerical results thus obtained are represented by means of graphs and tables. Further, skin friction coefficient and the local Nusselt number are found out and are presented by means of tables.

In order to highlight the validity of the numerical computations adopted in the present investigation, the numerical values are obtained in the absence of the magnetic interaction parameter, volume fraction and the angle of inclination with Pr = 1 and compared with those of Wang(2008) and Wong et al.(2013). The comparative study is made through results presented in Table 2, which elucidates an excellent agreement with previous investigations for the case m=1 to that of Wang(2008) and Wong et al.(2013).

Figures 1 to 12 represent typical numerical results based on the numerical solution of equations (7)-(9). These results are obtained to illustrate the influence of the magnetic interaction parameter, volume fraction, angle of inclination and velocity ratio parameter on nondimensional velocity and temperature in both the cases of copper-water nanofluid and alumina-water nanofluid.

Figures (1) and (2) represent the longitudinal velocity for different magnetic interaction parameter M^2 for both copper-water and alumina-water nanofluids, when A =0.5 and A =2. The effect of magnetic field parameter (M^2) has been explained from Eq.(2). The last term in the right hand side of Eq.(2) is $\frac{\sigma B_0^2(u - U_\infty(x))}{\rho_{nf}}$. This term has been resolved into two components. The first component, $\frac{\sigma B_0^2 U_\infty(x)}{\rho_{nf}}$, represents the imposed pressure force, whereas the second component, $\frac{\sigma B_0^2 u}{\rho_{nf}}$, represents the Lorentz force which slows down the fluid motion in the boundary layer region. When the imposed pressure force overcomes the Lorentz force $(U_\infty > u)$, the effect of the magnetic parameter enhance the velocity as shown in Fig.1 for both copper-water and alumina-water nanofluids. Similarly, when the Lorentz force which slows the fluid motion dominates the imposed pressure force $(U_\infty < u)$, the effect of the magnetic parameter (M^2) suppresses the velocity flow and hence it decreases the momentum boundary layer thickness as displayed in Fig.2 for both types of nanofluids.

It is noted from Fig. 3 that the temperature distribution decreases with increasing magnetic interaction parameter M^2 for both copper-water nanofluid and alumina-water nanofluid when A=0.5. For this case, the thermal boundary layer thickness decreases for increasing magnetic interaction parameter. On the other hand, when A=2, the temperature distribution increases with increasing magnetic interaction parameter for both copper-water and alumina-water nanofluids which is plotted through Fig. 4. At any point of the boundary layer, the temperature of the nanofluid increases with the increase of the magnetic interaction parameter. This is due to the fact that the application of the magnetic field on the flow domain creates a Lorentz force which acts like strings to retard the fluid motion and as a consequence the temperature of the fluid within the boundary layer increases. The thickness of the thermal boundary layer also increases with the increase in the strength of the applied magnetic field. Thus the temperature of the plate can be controlled by controlling the strength of the applied magnetic field.

Figs.5 and 6 illustrate the dimensionless velocity for different typical angle of inclination $\alpha = 0^0, 30^0, 45^0, 60^0$ for both copper-water and alumina-water nanofluids when A=0.5 and A=2. It is noted that the dimensionless velocity gets accelerated due to increasing angle of inclination α for both types of nanofluids. The case of $\alpha = 90^0$ corresponds to the vertical plate configuration and for this scenario, the velocity is found to be maximum. In the case of $= 0^0$, $sin 0^0 \to 0$, i.e., buoyancy effects vanish(horizontal plate scenario, where the gravity field is normal to the plate surface and exerts no effect on the flow). As α increases, $sin\alpha$ increases. This is the fact that the angle of inclination increases the effect of the buoyancy force increases due to gravity components of $sin\alpha$. For both copper-water and alumina-water nanofluids the values of α increases the longitudinal velocity also increases accompanied with the rise in the momentum boundary layer thickness in the case of A=2 as vividly shown through Fig.6.

Fig.7 depicts the effect of nanoparticle volume fraction ϕ on the non-dimensional velocity in both the cases when A =0.5 and A = 2 for copper-water nanofluid (ϕ = 0.00, 0.01, 0.05, 0.1). It is seen from the figure that the velocity gets accelerated with the increase of volume fraction parameter ϕ for Cu- water nanofluid when the stretching plate velocity is less than the free stream velocity(A =0.5). On the other hand, when the stretching plate velocity is greater than the free strem velocity, the fluid velocity gets decelerated with increasing number of copper nanoparticles. It is noticed that the increasing the volume fraction parameter ϕ leads to decrease in the momentum boundary layer thickness and also the presence of the magnetic field leads to thinning of the boundary layer in both the cases.

The effect of nanoparticle volume fraction parameter ϕ on non dimensional velocity distribution for alumina - water nanofluid in both the cases is shown in Fig.8. When the stretching plate velocity is less than the free stream velocity, it is found that the velocity reduces due to increasing volume fraction ϕ and it is reversed effect in the case of stretching plate velocity is greater than the free stream velocity. It is inferred from the Fig.8 that the increasing volume fraction of alumina-water nanofluid leads to enhanced the momentum boundary layer thickness in the case of A =2 and it is reduced in the case of A = 0.5.

Figures 9 and 10 demonstrate the variation of the temperature distribution for different values of the nanoparticle volume fraction for both copper and alumina water nanofluids in both the cases when A=0.5 and A=2. The conductivity of the nanofluid increases as the nanoparticle volume fraction increases, leading to a broadening of the thermal boundary layer thickness. This agrees with the physical behavior in that due to the addition of copper and alumina particles the thermal conductivity of the fluid increases, and consequently the thickness of the thermal boundary layer also increases.

Fig.11 illustrates the influence of velocity ratio parameter A(which denotes the ratio of the velocity of the stretching plate to the free stream velocity) over the velocity distribution for both copper and alumina water nanofluids. There is an appreciable increase in the velocity and the momentum boundary layer thickness when free stream velocity is less than the velocity of the stretching plate for both types of nanofluids. However, when free stream velocity exceeds the velocity of the stretching plate the velocity increases and the boundary layer thickness decreases with an increase in the velocity ratio parameter. Fig.12 shows the variation of temperature distribution in response to changes in the values of velocity ratio parameter A. It is observed that the effect of velocity ratio parameter is to reduce the temperature, elucidating the fact that the thermal boundary layer thickness decreases for both types of nanofluids.

The variation of skin friction coefficient and the local Nusselt number for various values of physical parameters for both copper-water and alumina- water nanofluid are shown in Table 3 and 4. As the magnetic interaction parameter and volume fraction increase, the skin friction coefficient increases in magnitude for both copper-water and alumina-water nanofluids when A=2 as well as for A=0.5. However, this is not in the case A=0.5 for the variation of velocity ratio parameter. For increasing values of angle of inclination the skin friction coefficient increases for both copper-water and alumina-water nanofluids for both the cases when A=0.5 and A=2. All these are noticed through Table 3.

Table 4 demonstrates the result of the local Nusselt number of both types of nanofluids for different values of M^2, A, α and ϕ for fixed values of Pr=6.2 and $\lambda = 1.5$. It is clear that the local nusselt number increases for increasing values of ϕ, α and A for both the cases A=2 and A=0.5. The nusselt number at the surface increases due to increasing values of magnetic interaction parameter in the case of A=0.5 and it gets decreased when A=2.

6. Conclusion

Numerical solutions are obtained for the nonlinear, steady, two dimensional, mixed convective flow of an incompressible, viscous, electrically conducting nanofluid over an inclined stretching plate with prescribed heat flux in the presence of a uniform transverse magnetic field. Two types of nanofluids such as copper-water and alumina - water nanofluids respectively, are considered for the prescribed heat flux case and the effects of various non-dimensional parameters are analyzed over the flow field of velocity and temperature. Numerical values of skin friction coefficient and the local Nusselt number are also obtained.

The following conclusions are arrived from all the results and discussion of numerical computations

- The effect of magnetic interaction parameter M^2 is to accelerate the dimensionless velocity, skin friction coefficient and the local Nusselt number, but the temperature decreases for both copper-water and alumina-water nanofluids when A =0.5. The dimensionless velocity, skin friction coefficient, local Nusselt number and the momentum boundary layer thickness decrease and the dimensionless temperature increases in the case of A =2.0 for both types of nanofluids when the magnetic interaction parameter increases.
- It is interesting to note that the temperature, local Nusselt number and the thermal boundary layer thickness enhance for increasing values of volume fraction for both types of nanofluids in both the cases. The skin friction coefficient increases with increasing ϕ in the case A =0.5 and it gets decreased when A =2.0.The effect of volume fraction is to accelerate the velocity for Cu-water nanofluid while its effect is to decelerate the velocity for alumina-water nanofluid when A =0.5. All these effects are reversed while the stretching velocity is greater than that of the free stream velocity.
- For both the copper and alumina-water nanofluids, increasing values of the angle of inclination α enhances the dimensionless velocity, skin friction coefficient and the non-dimensional rate of heat transfer in both the cases when A =2.0 and A =0.5.
- For increasing values of velocity ratio parameter A, both the dimensionless velocity and the rate of heat transfer enhance while the temperature, skin friction coefficient and the thermal boundary layer thickness reduce for both types of nanofluids when A =2.0 and A =0.5.

Table - 1 Thermo physical properties of base fluid water, copper and alumina

	ρ (Kg/m³)	C_p (J/Kg.K)	K (W/m.K)	β x10⁻⁵ k^{-1}
Water	997.1	4179	0.613	21
Copper	8933	385	400	1.67
Alumina	3970	765	40	0.85

Table 2 : Comparison Table for $F''(0)$ for different A when $\phi = 0$, $M^2 = 0$, $\alpha = 0$, and Pr = 1

A	Wang (2008)	Wong et al. (2013)	Present results
0	1.232588	1.23259	1.23258
0.1	1.14656	1.14656	1.14656
0.2	1.05113	1.05113	1.05113
0.5	0.71330	0.71329	0.71329
1	0.00000	0.00000	0.00000
2	-1.88731	-1.88730	-1.88731
5	-10.26475	-10.26475	-10.26475

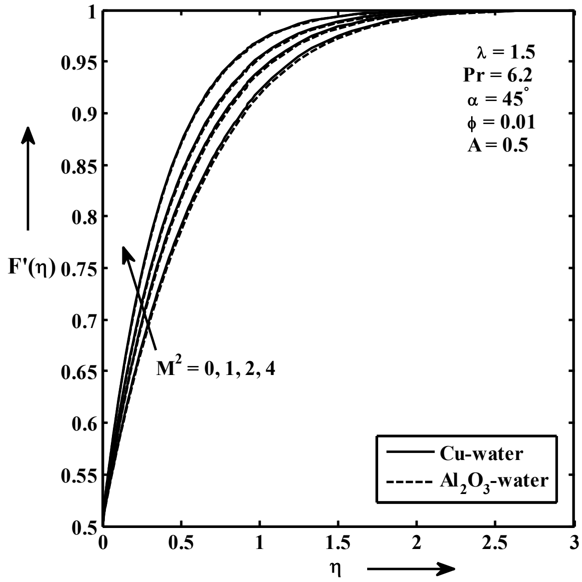

Fig. 1 Dimensionless velocity profiles for different M² at A=0.5

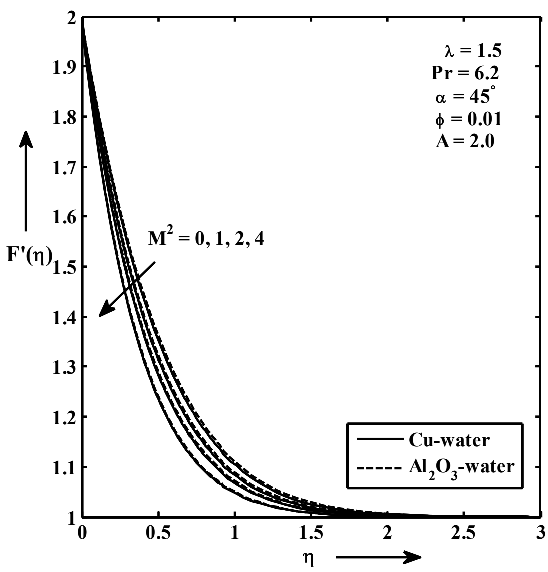

Fig. 2 Dimensionless velocity profiles for different M² at A=2.0

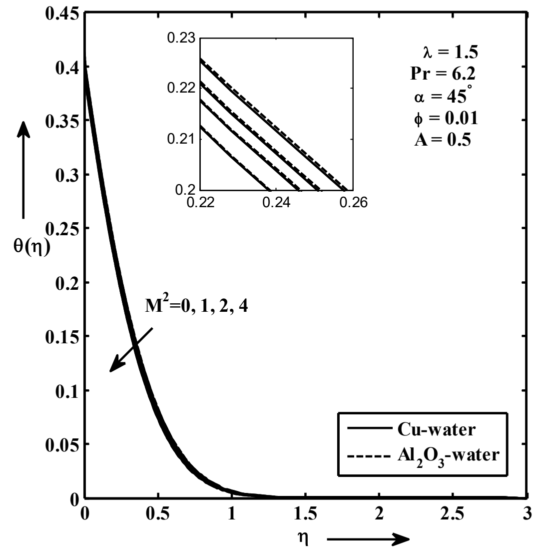

Fig.3 Temperature distribution for different M^2 at A=0.5

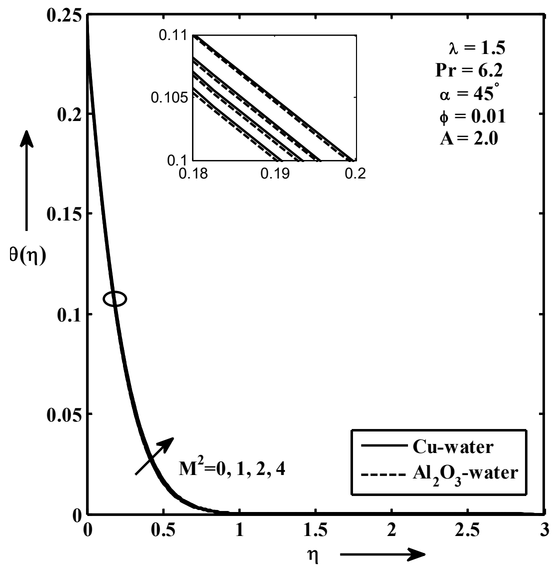

Fig.4 Temperature distribution for different M^2 at A=2.0

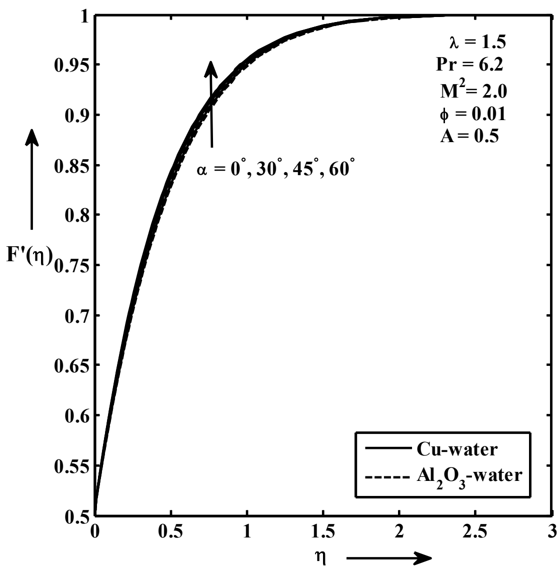

Fig.5 Dimensionless velocity profiles for different α at A = 0.5

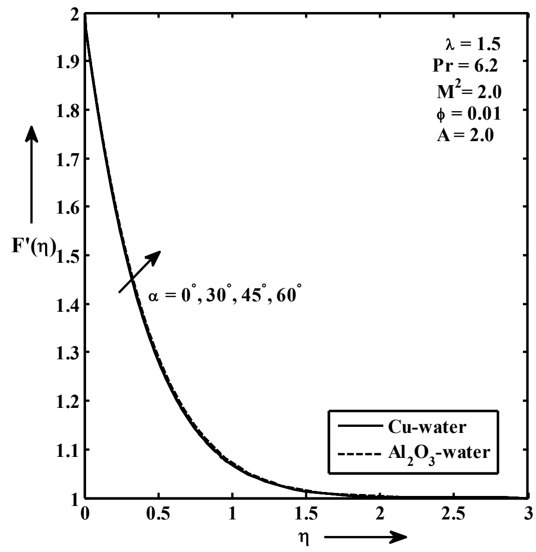

Fig.6 Dimensionless velocity profiles for different α at A = 2.0

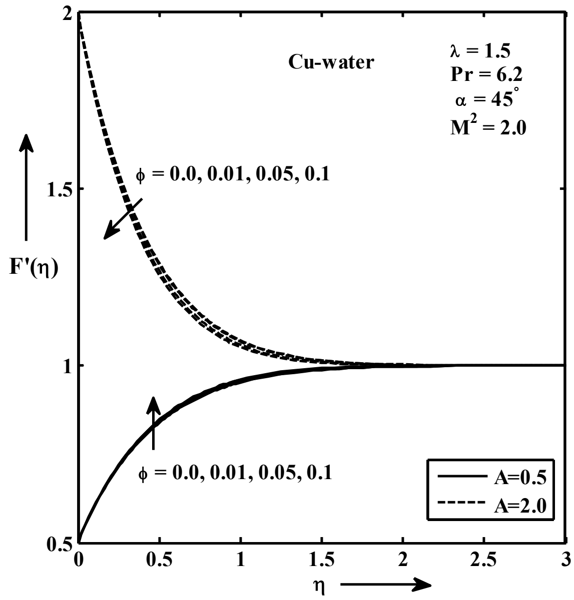

Fig.7 Dimensionless velocity profiles for different ϕ for Cu-water nanofluid

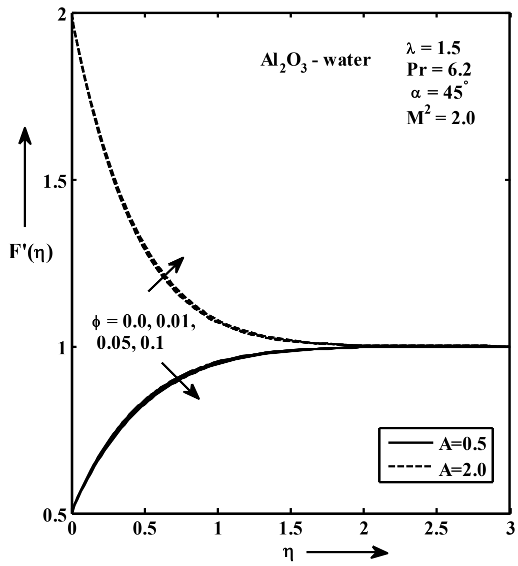

Fig.8 Dimensionless velocity profiles for different ϕ for Al_2O_3-water nanofluid

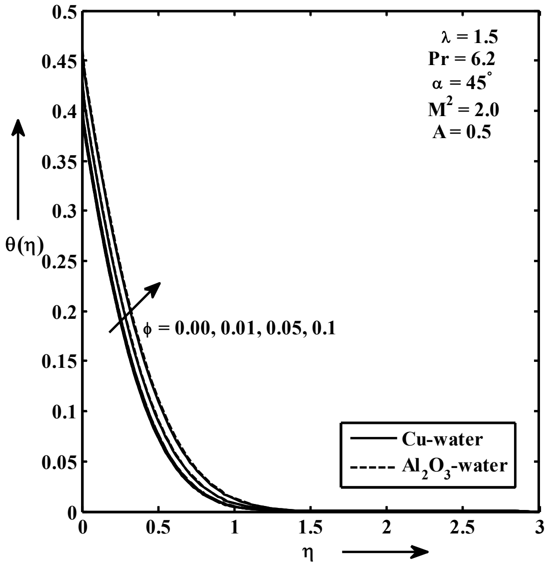

Fig.9 Temperature distribution for different ϕ at A=0.5

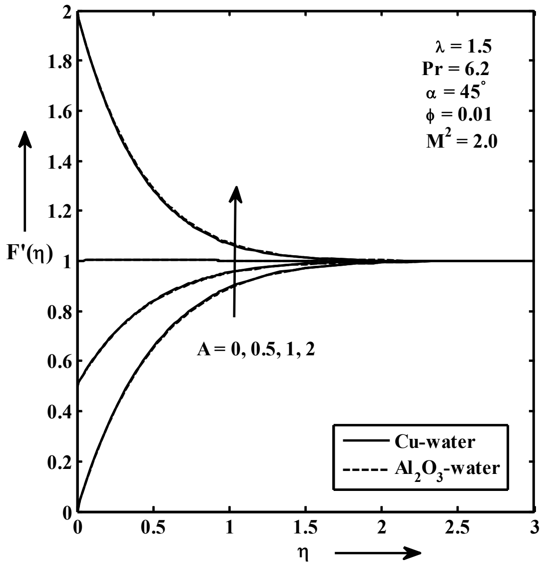

Fig.11 Dimensionless velocity profiles for different values of A

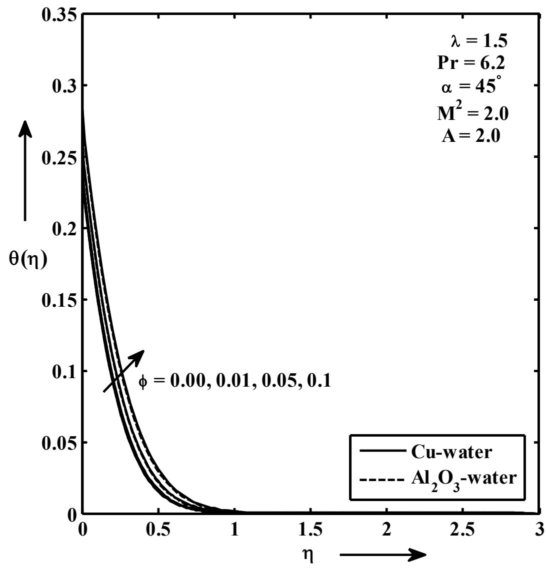

Fig.10 Temperature distribution for different ϕ at A=2.0

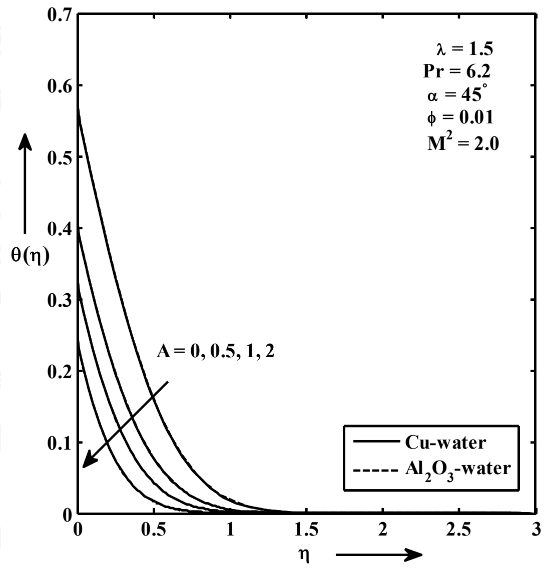

Fig.12 Temperature distribution for different values of A

Table 3: Variation in $\frac{1}{(1-\phi)^{2.5}}F''(0)$ for different values of $M^2, \alpha, \phi, and\ A$ when λ = 1.5 and Pr = 6.2

Φ	M^2	α	$\frac{1}{(1-\phi)^{2.5}}F''(0)$ (A=0.5)			$\frac{1}{(1-\phi)^{2.5}}F''(0)$ (A= 2)		
			A	Cu-water	Al_2O_3 – water	A	Cu-water	Al_2O_3 – water
0.00				1.086600	1.086600		-2.318243	-2.318243
0.01	2	45^0		1.118521	1.106019		-2.408311	-2.370964
0.05			0.5	1.251102	1.188298	2	-2.777384	-2.592751
0.1				1.430293	1.302767		-3.266892	-2.897902
	0			0.847058	0.830005		-1.949478	-1.903207
0.01	1	45^0		0.992721	0.978461	2	-2.190174	-2.149041
	2		0.5	1.118521	1.106019		-2.408311	-2.370964
	4			1.333360	1.323020		-2.795983	-2.763908
		0^0		1.035270	1.022693		-2.443462	-2.406051
	2	30^0		1.094289	1.081768	2	-2.418597	-2.381231
0.01		45^0	0.5	1.118521	1.106019		-2.408311	-2.370964
		60^0		1.137029	1.124544		-2.400424	-2.363092
			0.0	2.069222	2.049546	1.5	-1.119121	-1.102336
	2	45^0	0.3	1.510670	1.494584	2.0	-2.408311	-2.370964
0.01			0.5	1.118521	1.106019	2.5	-3.803783	-3.742529
			0.7	0.708297	0.700218	3.0	-5.299813	-5.211620

Table 4 : Variation in $\frac{k_{nf}}{k_f}\frac{1}{\theta(0)}$ for different values of M^2, α, ϕ and A when λ = 1.5 and Pr = 6.2

Φ	M²	α	$\frac{k_{nf}}{k_f}\frac{1}{\theta(0)}$ (A=0.5)			$\frac{k_{nf}}{k_f}\frac{1}{\theta(0)}$ (A=2.0)		
			A	Cu-water	Al_2O_3 – water	A	Cu-water	Al_2O_3 – water
0.00	2	45^0	0.5	2.505813	2.505813	2	4.161950	4.161950
0.01				2.545270	2.540131		4.213896	4.213040
0.05				2.704220	2.678187		4.424408	4.418972
0.1				2.906406	2.853284		4.695359	4.681372
0.01	0	45^0	0.5	2.495731	2.489218	2	4.256589	4.256771
	1			2.523393	2.517742		4.233916	4.233495
	2			2.545270	2.540131		4.213896	4.213040
	4			2.578998	2.574437		4.179866	4.178107
0.01	2	0^0	0.5	2.532936	2.527725	2	4.211364	4.210506
		30^0		2.541698	2.536536		4.213155	4.212299
		45^0		2.545270	2.540131		4.213896	4.213040
		60^0		2.547996	2.542868		4.214465	4.213610
0.01	2	45^0	0.0	1.815246	1.807988	1.5	3.720840	3.718834
			0.3	2.267573	2.261622	2.0	4.213896	4.213040
			0.5	2.545270	2.540131	2.5	4.663395	4.663489
			0.7	2.806356	2.801951	3.0	5.078321	5.079270

References:

[1] Abdelkhalek, M. M. (2006) The skin friction in the MHD mixed convection stagnation point with mass transfer, *International Communication in Heat and mass Transfer*, 33, 249-258.

[2] Abu-Nada, E. and Oztop, H.F. (2009) Effects of inclination angle on natural convection in enclosures filled with Cu-water nanofluid, *International Journal of Heat and fluid flow,* 30, 669-678.

[3] Alam, M. S., M. M. Rahman, and M. A. Sattar (2006). MHD free convective heat and mass transfer flow past an inclined surface with heat generation, *Thammasat International Journal of Science and Technology*, 11(4), 1- 8.

[4] Anjali Devi, S.P. and Julie Andrews(2011) Laminar boundary layer flow of nanofluid over a flat plate, *International Journal of Applied Mathematics and Mechanics,* 7, 52-71.

[5] Anjali Devi S. P., Suriyakumar P., (2016) Hydromagnetic mixed convective nanofluid slip flow past an inclined stretching plate in the presence of internal heat absorption and suction, *Journal of Applied Fluid Mechanics*, 9, 1409-1419.

[6] Anuar Ishak, Khamisah Jafar, Roslinda Nazar and Ioan Pop (2009) MHD stagnation point flow towards a stretching sheet, *Physica A*, 388, 3377 -3383.

[7] Anuar Ishak, Roslinda Nazar, Norihan M. Arifin, Fadzilah M. Ali and Ioan Pop (2011) MHD stagnation-point flow towards a stretching sheet with prescribed surface heat flux, *Sains Malaysiana*, 40(10), 1193-1199.

[8] Anwar M.I, Shafie S, Hayat T, Shehzad S. A, Salleh M. Z (2017) Numerical study for MHD stagnation-point flow of a micropolar nanofluid towards a stretching sheet, *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 39(1), 89-100.

[9] Aydin, O. and Kaya, A. (2009) MHD mixed convective heat transfer flow about an inclined plate, *Heat and Mass Transfer*, 46, 129-136.

[10] Bachok, N., Ishak, A. and Pop, I. (2010) Boundary layer flow of nanofluids over a moving surface in a flowing fluid. *International Journal of Thermal Science*, 49, 1663-1668.

[11] Bachok,N., Ishak, A. and Pop,I. (2011) Stagnation-point flow over a stretching/shrinking sheet in a nanofluid, Nanoscale Research Letters, 6, 623 -632.

[12] Bachok. N, Anuar Ishak, Roslinda Nazar and Norazak Senu (2013) Stagnation-point flow over a permeable stretching/shrinking sheet in a copper-water nanofluid, *Boundary value problems*, 39, 1-10.

[13] Brinkman, H.C.(1952) The Viscosity of Concentrated Suspensions and Solutions, *J. Chem. Phys.,*, 20, 571.

[14] Buongiorno, J. (2006) Convective transport in nanofluids., *ASME Journal of Heat transfer*, 128, 240-250.

[15] Chamkha, A. J. and Rahim A.K.A(2001). Similarity solutions for hydromagnetic simultaneous heat and mass transfer by natural convection from an inclined plate with internal heat generation or absorption, *Heat and Mass Transfer*, 37, 117-123.

[16] Chiam, T.C. (1994) Stagnation point flow towards a stretching plate, *J Phys Soc Japan,* 63, 2443-2444.

[17] Choi, S. (1995)Enhancing thermal conductivity of fluids with nanoparticle, in development and applications on non-Newtonian flow. *Proceedings of the 1995 ASME International Mechanical Engineering Congress and Exposition, American Society of Mechanical Engineers, New York, FED 231/MD,* 66, 99-105.

[18] Crane, L.J.(1970) Flow past a stretching plate, *J.Appl. Math Phys(ZAMP)*, 21, 645-647.

[19] Das S.K. and Choi S.U.S. (2009) A review of heat transfer in nanofluids , *Advanced Heat transfer*, 41, 81-97.

[20] Das S.K. and Choi S.U.S., Yu W., and Pradeep T. (2008). Nanofluids: Science and Technology. *Inc., Hoboken, New Jersey, USA: John Wiley and Sons.*

[21] Das K. (2013) Mixed Convection Stagnation Point Flow and Heat Transfer of Cu-Water Nanofluids Towards a Shrinking Sheet, *Heat Transfer-Asian Research*, 423, 230-242.

[22] Eastman J.A., Phillpot S.R., Choi S.U.S. and Keblinski P.(2004)Thermal transport in nanofluids, *Annu. Rev. Mater. Res.*, 34, 219 - 246.

[23] Ibrahim, W., Shankar, B. and Nandeppanavar, M.M. (2013) MHD stagnation point flow and heat transfer due to nanofluid towards a stretching sheet, *International Journal of Heat and Mass Transfer*, 56, 1-9.

[24] Kakac S. and Pramaumjaroenkij A. (2009) Review of convective heat transfer enhancement with nanofluids, *International Journal of Heat and Mass Transfer*, 52, 3187 -3196.

[25] Khan, W.A. and Pop, I. (2010) Boundary layer flow of a nanofluid past a stretching sheet, *International journal of heat and mass transfer*, 53, 2477-2483.

[26] Lok Yian Yian, Norsarahaida Amin and Pop I. (2007) Mixed convection flow near a non-orthogonal stagnation point towards a stretching vertical plate, *International Journal of Heat and Mass Transfer*, 50, 4855-463.

[27] Lok yian yian, Anuar Ishak and Pop I. (2011) MHD Stagnation point flow with suction towards a shrinking Sheet, *Sains Malaysiana*, 40(10), 1179-1186 .

[28] Mabood F., Khan W. A., Ismail A. I. M., (2015) MHD Boundary layer flow and heat transfer of nano-fluids over an non- linear stretching sheet: A numerical study, *Journal of Magnetism and Magnetic Materials ,* 374, 569-576.

[29] Mahapatra, T. R. and Gupta, A.S.(2001) Magnetohydrodynamic stagnation-point flow towards a stretching sheet, *Acta Mechanica*, 152, 191-196.

[30] Makinde, O. D., Khan, W. A. and Khan, Z. H.(2013) Buoyancy effects on MHD stagnation point flow and heat transfer due to nanofluid past a convectively heated stretching/ shrinking sheet, *International Journal of Heat and Mass Transfer,* 62, 526-533.

[31] Malvandi A., Hedayati F., Nobari M. R. H., (2014) An Analytical Study on Boundary Layer Flow and Heat Transfer of Nanofluid Induced by a Non-Linearly Stretching Sheet, *Journal of Applied Fluid Mechanics*, 7, 375-384.

[32] Mishra, S. R., Baag, S., Bhatti, M. M (2018) Study of heat and mass transfer on MHD Walters B nanofluid flow induced by a stretching porous surface, *Alexandria Engineering Journal*, 57, 2435-2443.

[33] Mucoglu, A. and Chen, T.S. (1979) Mixed Convection on Inclined Surfaces, *Journal of Heat Transfer*, 101, 422-426.

[34] Mustfa, M., Hayat, T., Pop, I., Asghar, S. and Obaidat, S. (2011)Stagnation- point flow of a nanofluid towards a stretching sheet, *International Journal of Heat and Mass transfer*, 54, 5588-5594.

[35] Noor, N. F. M., S. Abbasbandy, and I. Hashim (2012). Heat and mass transfer of thermophoretic MHD flow over an inclined radiate isothermal permeable surface in the presence of heat source/sink, *International Journal of Heat and Mass Transfer*, 55, 2122- 2128.

[36] Rafique, K, Anwar, M. I., Misiran, M. (2019) Numerical Study on Micropolar Nanofluid Flow over an Inclined Surface by Means of Keller-Box, *Asian J. Probab. Stat*, 4, 1-21.

[37] Rafique K et al.(2020) Hydromagnetic Flow of Micropolar Nanofluid , *Symmetry*, 12, 251.

[38] Rana, P, Bhargava, R and Beg, O.A .(2012) Numerical solution for mixed convection boundary layer flow of a nanofluid along an inclined plate embedded in a porous medium, *Computers and Mathematics with Applications*, 64 , 816-2832.

[39] Ramadan, H. M. and A. J. Chamkha (2003). Hydromagnetic free convection of a particulate suspension from a permeable inclined plate with heat absorption for non-uniform particle -phase density, *Heat and Mass Transfer*, 39, 367-374.

[40] Sakiadis BC (1961). Boundary layer behaviour on a continuous solid surface: II- The boundary layer on a continuous flat surface, *AIChe Journal,* 7, 221-225.

[41] Santra, A. K., Sen, S. and Chakraborthy, N. (2008) Study of heat transfer augmentation in a differentially heated square cavity using Copper-water nanofluid, *International Journal of Thermal Sciences*, 47, 1113-1122.

[42] Singh P., Kumar M., (2015) Mass transfer in MHD flow of alumina water nanofluid over a flat plate under slip conditions, *Alexandria Engineering Journal*, 54, 383-387.

[43] Singh,P., Tomer, N. S.,Kumar, S. and Sinha, D.(2010) MHD oblique stagnation-point flow towards a stretching sheet with heat transfer, *International Journal of Applied Mathematics and Mechanics*, 6(13), 94-111.

[44] Wang, C. Y. (2008) Stagnation flow towards a shrinking sheet, *International Journal of Non-Linear Mechanics,* 43, 377-382.

[45] Wang X. Q. and Majumdar A. S. (2007) Heat transfer characteristics of nanofluids: a review, *International Journal of thermal Sciences*, 46, 1 - 19.

[46] Wong, S. W. Omar Awang, M. A. and Anuar Ishak.(2013) Stagnation-Point Flow toward a Vertical, Nonlinearly Stretching Sheet with Prescribed Surface Heat Flux, *Journal of Applied Mathematics*, ArticleID528717, 6pages