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Viscous and Ohmic Heating Effects on Hydromagnetic Convective-Radiative Boundary Layer Flow Due to a Permeable Shrinking Surface with Internal Heat Generation

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ABSTRACT
In this paper, the effects of viscous and Ohmic heating on magneto hydrodynamic forced convection flow due to a permeable shrinking surface in the presence of internal heat generation and radiation is investigated. The shrinking surface subjected to suction is prescribed with non-uniform temperature which varies with the quadratic power of x. Momentum boundary layer equation takes into account of transverse magnetic field effect. Thermal boundary layer equation considered the effects of viscous and Ohmic heating due to transverse magnetic field, radiation and internal heat generation. Numerical results are obtained for various values of governing physical parameters. To access the accuracy of the numerical solution, the present results are checked against previously published work and the analytical solution of the problem obtained using Kummer's function. Table values show that the analytical and numerical solutions are found to be in excellent agreement.

KEYWORDS: *Convection; Internal heat generation; Magneto hydrodynamics; Radiation; shrinking surface*

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1. INTRODUCTION

The combined effects of viscous and Ohmic heating in various flow configurations was considered due to enormous applications in engineering problems. Ohmic heating is an emerging technology with large number of actual and future applications. The possibilities of Ohmic heating include blanching, evaporation, dehydration, fermentation, extraction, sterilization, pasteurization and heating of foods to serving temperature, including in the military field or in long-duration space missions. Javeri and Berlin [1] studied the effect of viscous and Ohmic dissipation on the fully developed MHD flow with heat transfer in a channel. Hossain [2] considered the MHD free convection flow past a semi infinite plate with viscous, Ohmic heating effects and variable plate temperature. Chaim [3] obtained the solution of energy equation for the boundary layer flow of an electrically conducting fluid under the influence of transverse magnetic field over a linear stretching sheet with dissipation and internal heat generation effects. MHD forced convection flow in the presence of viscous and magnetic dissipation along a nonisothermal wedge was carried out by Yih [4]. Amin [5] analyzed the effect of viscous dissipation and Joule heating on forced convection flow from a horizontal circular cylinder under the action of transverse magnetic field. Duwairi [6] investigated the forced convection flow of ionized gases adjacent to isothermal porous surfaces with viscous and Ohmic heating effects.

Barletta and Celli [7] studied the effects of Joule heating and viscous dissipation on mixed convection MHD flow in a vertical channel. Anjali Devi and Ganga [8] have analysed the effect of viscous and Joule's dissipation on MHD flow, heat and mass transfer past a stretching porous surface embedded in a porous medium. Viscous and Joule heating effects on MHD conjugate heat transfer for a vertical flat plate in the presence of heat generation was analysed by Azim et al. [9]. Radiation effects on MHD free convection flow along a vertical flat plate in the presence of Joule's dissipation and heat generation was investigated by Ali et al. [10]. Dual solutions in a boundary layer flow of a power law fluid over a moving permeable flat plate with thermal radiation, viscous dissipation and heat generation/absorption was obtained by Aftab Ahmed et al. [11].

Recently, the boundary layer flow due to shrinking sheet has attracted considerable interest. A shrinking sheet provides a high quality field coating solution for welded pipe ends with Trio coating. Proper application of the shrinking sheet provides excellent tightening of the sheet around the entire circumference of the field welded pipe end good adhesion to the steel pipe and to the industrial coating, as well as a uniform thickness. In the past few years much attention has been focused for the study of different types of flow and heat transfer over a shrinking sheet for various fluids due to its numerous applications. In contrast to stretching sheet, for the shrinking case, the velocity on the boundary is towards a fixed point. It is also shown that the mass suction is required generally to maintain the flow over the shrinking sheet.

Unlike the linear/nonlinear stretching sheet problem, only little work has been done on boundary layer flow and heat transfer induced by a shrinking sheet. This peculiar work started from Micklavcic and Wang [12] to investigate the flow over a shrinking sheet. Hayat et al. [13] reported the analytical solution of magneto hydrodynamic flow of second grade fluid over a shrinking sheet. Sajid et al. [14] analyzed rotating flow of a viscous fluid over a shrinking sheet under the influence of transverse magnetic field. Yao and Chen [15] examined analytical solution branch for the Blasius equation with a shrinking sheet. Thermal boundary layer over a shrinking sheet was investigated by Fang and Zhang [16] for prescribed power law wall temperature and power law wall heat flux case. Later, the effects of suction/blowing on steady boundary layer stagnation point flow and heat transfer towards a shrinking sheet with thermal radiation was carried out by Bhattacharyya and Layek [17]. Effects of heat source/sink on MHD flow and heat transfer over a shrinking sheet with mass suction was analyzed by Bhattacharyya [18]. Javed et al. [19] analyzed the viscous dissipation effect on steady hydromagnetic viscous fluid for nonlinear shrinking sheet. Rohini et al. [20] reported on boundary layer flow and heat transfer over an exponentially shrinking vertical sheet with suction. Nonlinear radiation effects on hydromagnetic boundary layer flow and heat transfer over a shrinking surface was analyzed by Anjali Devi and Wilfred Samuel Raj [21]. MHD flow over a permeable stretching/shrinking sheet was studied by Sandeep and Sulochana [22]. Soid et al. [23] presented unsteady MHD flow over a shrinking sheet with Ohmic heating. Ismail et al. [24] made stability analysis of unsteady MHD stagnation point flow and heat transfer over a shrinking sheet in the presence of viscous dissipation.

To the best of author's knowledge, no studies have been made so far to analyze the effects of viscous and Ohmic heating on MHD boundary layer flow with heat transfer over a linearly shrinking surface in the presence of radiation and internal heat generation. . Highly nonlinear momentum and thermal boundary layer partial differential equations are converted into nonlinear ordinary differential equations using similarity transformations. Later the resultant boundary value problem is converted into initial value problem using the efficient Nachtsheim Swigert shooting iteration scheme which works very well in the case of asymptotic boundary conditions and the numerical solutions are obtained using Runge Kutta Fourth Order method. A parametric study illustrating the influence of various physical parameters on the velocity, temperature, skin friction coefficient and dimensionless rate of heat transfer is conducted.

2. Mathematical Formulation

The problem under investigation comprises a nonlinear, steady, two- dimensional boundary layer forced convection flow of a viscous, incompressible, electrically conducting and radiating fluid over a linear shrinking surface under the influence of uniform transverse magnetic field of strength B_0. The sheet coincides with the plane $y = 0$ and the flow is confined in the region $y > 0$. The x and y axes are taken along and perpendicular to the sheet respectively. A schematic diagram of the problem is given in Figure 1. In the present analysis, x axis is chosen along the shrinking surface in the direction opposite to the motion of the sheet and y axis is taken normal to the surface.

- The properties of the fluid are kept constant.
- Magnetic Reynolds number is assumed to be small so that the induced magnetic field produced by the motion of electrically conducting fluid is negligible.
- Since the flow is steady, *curl* $\mathbf{E} = 0$. Also *div* $\mathbf{E} = 0$ in the absence of surface charge density. Hence $\mathbf{E} = 0$ is assumed.
- The heat transfer takes place in the presence of viscous and Ohmic heating, internal heat generation and thermal radiation.
- The fluid is considered to be viscous, incompressible, electrically conducting, gray, absorbing and emitting radiation but non-scattering medium.
- The radiative heat flux in the energy equation is described using Rosseland approximation and it is assumed to be negligible in x direction when compared to that in the y direction.

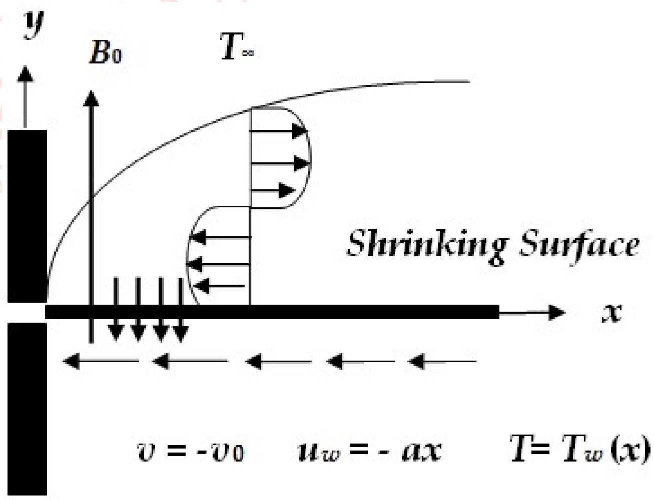

Figure1. Schematic diagram of the problem

From the above assumptions and the boundary layer approximations, the equations based on law of conservation of mass, momentum and energy can be written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}+Q(T-T_\infty)+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_0^2 u^2 \quad (3)$$

where u and v are the flow velocities along the x and y axes, respectively, ν is the kinematic viscosity, σ is electrical conductivity of the fluid, B_0 is the magnetic field, ρ is the fluid density, C_p is the specific heat at constant pressure, T is the fluid temperature, k is the thermal conductivity of the fluid, T_∞ is temperature of the fluid far away from the sheet, μ is the coefficient of viscosity, q_r is the radiative heat flux and Q is the volumetric rate of heat generation.

By using the Rosseland approximation, [Brewster [25]], the radiative heat flux is given by

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad (4)$$

where σ^* is the Stefan Boltzmann constant and k^* denotes the mean absorption coefficient. It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids.

If temperature differences within the flow are sufficiently small, then equation (4) can be linearized by expanding T^4 in Taylor series about T_∞ [Raptis et al. [26]], which after neglecting higher order terms take the form

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (5)$$

Using equations (4) and (5), equation (3) reduces to

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}+Q(T-T_\infty)+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_0^2 u^2 \quad (6)$$

The boundary condition subjected to the velocity and temperature are given by

$$u = u_w(x) = -ax,\ v = -v_0,\ T=T_w(x) = T_\infty + Ax^2 \ \text{ at } y = 0 \quad (7)$$

$$u \to 0,\ T \to T_\infty \quad \text{as} \quad y \to \infty \quad (8)$$

where u_w is the shrinking sheet velocity, $a > 0$ is a dimensional constant called as shrinking rate, v_0 is the constant suction velocity, T_w is the variable wall temperature, T_∞ is the temperature outside the dynamic region. The constant A depends on thermal property of the fluid.

2.1. Flow and heat transfer analysis

To facilitate the analysis, the following similarity transformations are introduced

$$\eta=y\left(\frac{a}{\nu}\right)^{\frac{1}{2}}, \quad \psi(x,y)=\sqrt{a\nu}\, x F(\eta) \quad (9)$$

The horizontal and vertical velocity components are given by,

$$u=\frac{\partial \psi}{\partial y}=a x F'(\eta), \quad v=-\frac{\partial \psi}{\partial x}=-\sqrt{a\nu}\, F(\eta) \quad (10)$$

Equation (10) automatically satisfies the continuity equation (1). Substituting equation (10), the momentum equation (2) reduces to the following nonlinear ordinary differential equation

$$F'''+FF''-(F')^2-(M^2)F'=0 \quad (11)$$

with the corresponding boundary conditions

$$F(\eta) = S,\ F'(\eta)=-1 \quad \text{at} \quad \eta = 0 \quad \text{and}$$
$$F'(\eta)=0 \quad \text{as} \quad \eta \to \infty \quad (12)$$

In terms of the dimensionless temperature,

$$\theta(\eta)=\frac{T-T_\infty}{Ax^2} \quad (13)$$

the energy equation (6) reduces to the ordinary differential equation

$$\left(\frac{3Rd+4}{3Rd\Pr}\right)\theta''+F\theta'-(2F'-Hs)\theta=-Ec\left[(F'')^2+M^2(F')^2\right] \quad (14)$$

The corresponding thermal boundary conditions become

$$\theta(\eta) = 1 \text{ at } \eta = 0 \quad \text{and} \quad \theta(\eta) \to 0 \text{ as } \eta \to \infty \quad (15)$$

In the above equation, $F'=\frac{dF}{d\eta}$ and $\theta'=\frac{d\theta}{d\eta}$, S, M^2, Pr, Rd, Hs, Ec denote the Suction parameter, Magnetic parameter, Prandtl number, Radiation parameter, Heat generation parameter and Eckert number respectively. They are defined as

$$S=\frac{v_0}{\sqrt{a\nu}}, \quad M^2=\frac{\sigma B_0^2}{\rho a},$$
$$Pr=\frac{\mu C_p}{k}, \quad Rd=\frac{kk^*}{4\sigma^* T_\infty^3},$$
$$Hs=\frac{Q}{a\rho C_p}, \quad Ec=\frac{a^2}{C_p A}.$$

2.1.1. Skin friction coefficient and Non-dimensional rate of heat transfer

The important physical quantity of interest is skin friction coefficient C_f, which is defined as $C_f=\frac{\tau_w}{\rho u_w^2/2}$ and the wall shear stress appeared in the above is given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

After using the dimensionless quantities given by equation (10), the skin friction coefficient is obtained as follows:

$$\sqrt{\mathrm{Re}_x}\frac{C_f}{2}=F''(0) \quad (16)$$

Non-dimensional rate of heat transfer in terms of local Nusselt number is given by

$Nu_x=\frac{xq_w}{k(T_w-T_\infty)}$ where $q_w=\left(-k\frac{\partial T}{\partial y}+q_r\right)_{y=0}$ is the net heat flux at the wall.

The local Nusselt number is obtained as

$$\frac{Nu_x}{\sqrt{\mathrm{Re}_x}}=-\left(1+\frac{4}{3Rd}\right)\theta'(0) \quad (17)$$

where $Re_x=\frac{ax^2}{\nu}$ is the local Reynolds number.

3. Numerical solution

Numerical solution of MHD boundary layer forced convection flow over a shrinking surface in the presence of uniform magnetic field, radiation, internal heat generation, viscous and Ohmic heating effects is obtained. The third order momentum equation and second order energy equation is reduced into a system of five first order ordinary differential equations. To solve this system of equations, five initial conditions are needed of which $F''(0)$ and $\theta'(0)$ are not known. These unknown values are obtained using Nachtsheim Swigert shooting iteration scheme. The essence of the shooting method to solve a boundary value problem is to convert it into a system of initial value problems. Once all the initial values are known, the system of first order differential equations is then solved using Fourth Order Runge Kutta method. In order to access the accuracy of the numerical results obtained, the validity of the numerical code developed has been checked for a limiting case and are shown in graphs. Numerical results are obtained for the range of values of Suction parameter, Magnetic parameter, Prandtl number, Radiation parameter, Heat generation parameter and Eckert number.

4. Analytical Solution

The analytical solution of the problem is obtained in order to inspect the accuracy of numerical results.

4.1. Momentum and Energy equation

The analytical solution of equation (11) is of the form

$$F(\eta)=S-\frac{1}{\alpha}(1-e^{-\alpha\eta}) \text{ and } F'(\eta)=-e^{-\alpha\eta} \quad (18)$$

where $\alpha=\frac{S+\sqrt{S^2+4(M^2-1)}}{2}$.

To solve equation (14), introduce a new variable $\xi=\frac{P}{\alpha^2}e^{-\alpha\eta}$. Now, the energy equation becomes

$$\xi\frac{d^2\theta}{d\xi^2}+(1-a_0-\xi)\frac{d\theta}{d\xi}+\left(2+\frac{P\beta}{\alpha^2\xi}\right)\theta=-Ec\frac{\alpha^2\xi}{P}(\alpha^2+M^2) \quad (19)$$

where $a_0=\frac{P}{\alpha^2}(\alpha^2-M^2)$ and $P=\frac{3Rd\Pr}{3Rd+4}$.

Subsequently, the corresponding boundary conditions take the form

$$\theta\left(\xi=\frac{P}{\alpha^2}\right)=1 \text{ and } \theta(\xi=0)=0 \quad (20)$$

Utilizing Kummer's function, the solution of equation (19) is attained in terms of ξ in the presence of viscous dissipation, Joules dissipation, radiation and internal generation as

$$\theta(\xi)=C_1\xi^{\frac{a_0+b_0}{2}}\Phi\left(\frac{a_0+b_0-4}{2},1+b_0;\xi\right)+C_2\xi^2$$

In terms of similarity variable η, the analytical solution of energy equation is specified as

$$\theta(\eta)=C_1e^{-\alpha\left(\frac{a_0+b_0}{2}\right)\eta}\Phi\left(\frac{a_0+b_0-4}{2},1+b_0;\frac{P}{\alpha^2}e^{-\alpha\eta}\right)+C_2\left(\frac{P}{\alpha^2}\right)^2e^{-2\alpha\eta} \quad (21)$$

where $\Phi(a,b;x)$ is the confluent hypergeometric function of first kind. The constants involved in the solution can be represented as shown below

$$a_0=\frac{P}{\alpha^2}(\alpha^2-M^2),$$

$$b_0=\sqrt{a_0^2-\frac{4\beta P}{\alpha^2}},$$

$$C_1=\frac{1-C_2\frac{P^2}{\alpha^4}}{\Phi\left(\frac{a_0+b_0-4}{2},1+b_0;\frac{P}{\alpha^2}\right)} \quad \text{and}$$

$$C_2=\frac{-Ec\,\alpha^2(\alpha^2+M^2)}{P\left(4-2a_0+\frac{P\beta}{\alpha^2}\right)}.$$

In the absence of magnetic field, radiation, internal heat generation, viscous and Ohmic heating effects, the analytical solutions obtained above are identical to the closed form solution of Fang and Zhang [16].

5. Results and discussion

In order to gain the physical insight of the problem, velocity and temperature distribution have been discussed by assigning various numerical values to the physical parameters encountered in the problem. The numerical results are tabulated and displayed with the graphical illustrations. Fourth Order Runge Kutta based Nachtsheim Swigert shooting iteration scheme is used to find the numerical solution. Comparison of the numerical results obtained has been made with the existing literature and are found to be in good agreement.

Figure 2 illustrates the comparison graph for dimensionless temperature for different values of Prandtl number. It is observed from the figure that the present results for dimensionless temperature when the shrinking surface has constant temperature are identical to that of Bhattacharyya [18] in the absence of Radiation parameter and Eckert number.

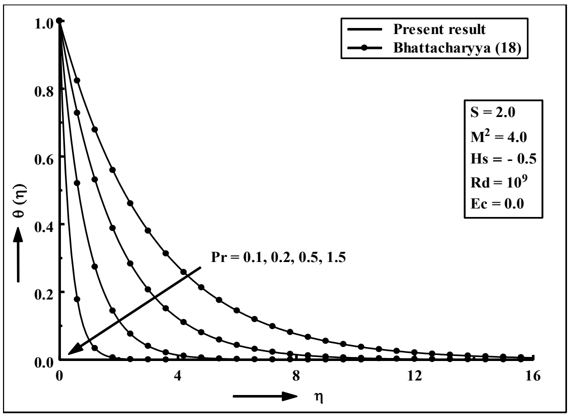

Figure 2. Comparison graph showing dimensionless temperature for various *Pr*

5.1. Effect of physical parameters over Velocity distribution and skin friction coefficient

Figure 3 discloses the effect of Suction parameter over the velocity distribution. It is noted that the longitudinal velocity is accelerated by the influence of Suction parameter and consequently, the momentum boundary layer thickness decreases.

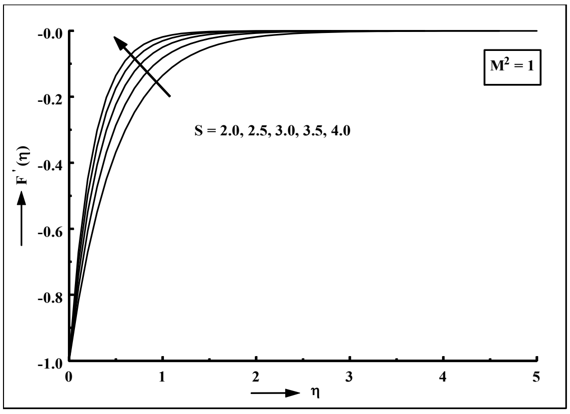

Figure 3. Effect of Suction parameter over the dimensionless velocity

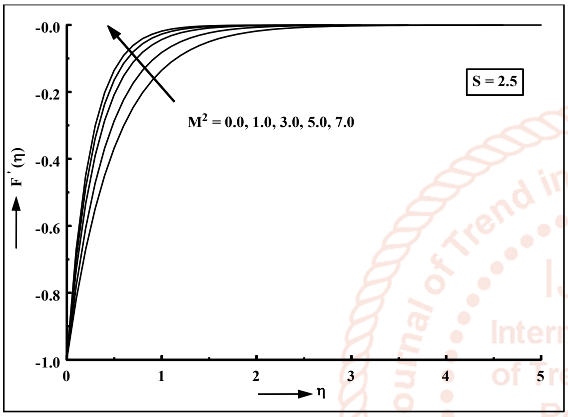

Figure 4. Effect of Magnetic parameter over the dimensionless velocity

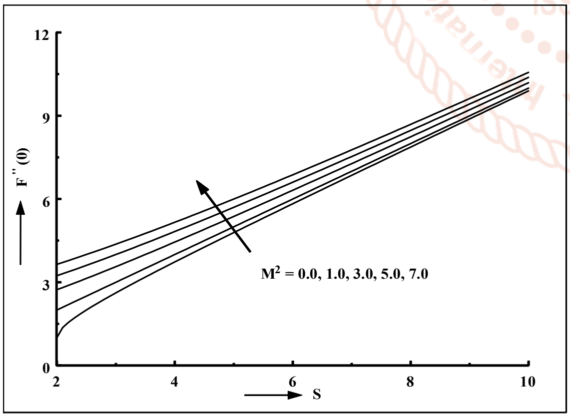

Figure 5. Effect of Magnetic parameter over skin friction coefficient

Variation in dimensionless longitudinal velocity for various values of Magnetic parameter is portrayed in Figure 4. The effect of Magnetic parameter is to accelerate the dimensionless longitudinal velocity. This happens due to Lorentz force arising from the interaction of magnetic and electric fields during the motion of electrically conducting fluid. The momentum boundary layer thickness is reduced consequently.

Figure 5 portrays the effect of Magnetic parameter over the skin friction coefficient against Suction parameter. The skin friction at the shrinking sheet is felt more for higher values of Magnetic parameter. It is noted from the figure that the skin friction coefficient ascends as the Suction parameter along the horizontal axis increases.

5.2. Effect of physical parameters on Temperature distribution

The increase in magnitude of Suction parameter is to decrease the temperature of the fluid is depicted in Figure 6. Further, the thermal boundary layer thickness declines as the Suction parameter increases. The temperature at the wall is found to be higher than temperature away from the wall.

Figure 7 implies that the temperature is suppressed owing to the increase in value of Magnetic parameter. It is further noted that the thickness of thermal boundary layer diminishes with an enhancement of Magnetic parameter.

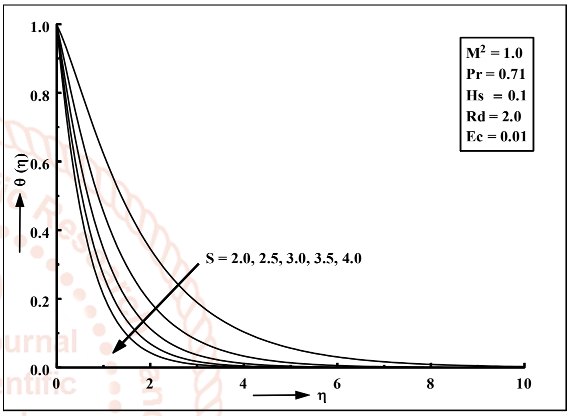

Figure 6. Dimensionless temperature profiles for various values of *S*

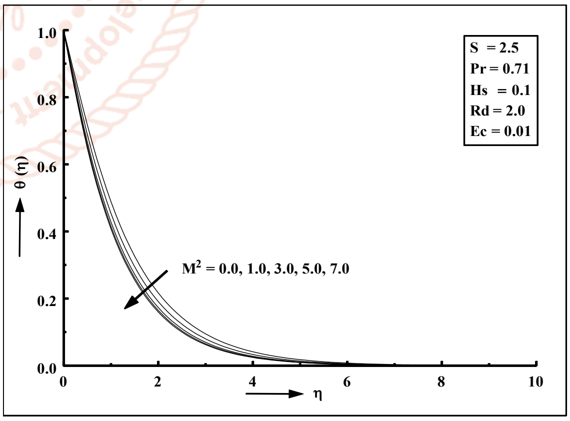

Figure 7. Dimensionless temperature profiles for various values of M^2

The dimensionless temperature distribution for various values of Prandtl number is highlighted in Figure 8. As Prandtl number increases, the thermal diffusivity decreases which reduces the energy transfer ability. Hence an increase in Prandtl number values leads to the decrease in temperature and the thickness of the thermal boundary layer becomes thin. Figure 9 represents the dimensionless temperature for various values of Heat generation parameter. In the presence of Heat generation parameter, it can be seen that heat energy is generated in thermal boundary layer which cause the temperature to increase with an enhancement in the value of Heat generation parameter.

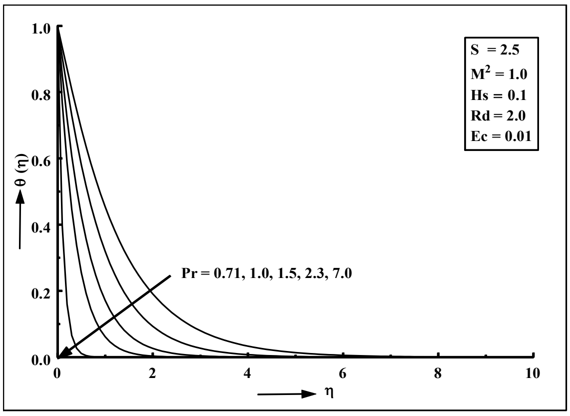

Figure 8. Dimensionless temperature profiles for various values of *Pr*

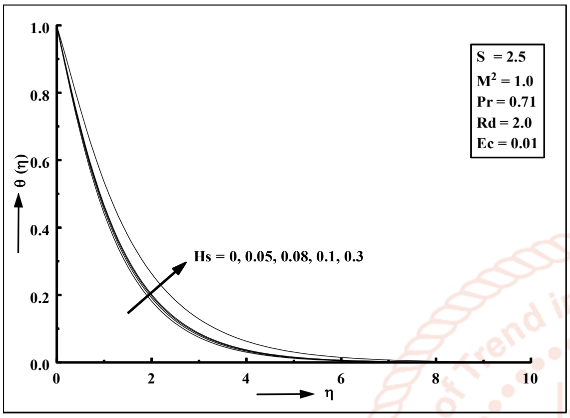

Figure 9. Dimensionless temperature profiles for various values of *Hs*

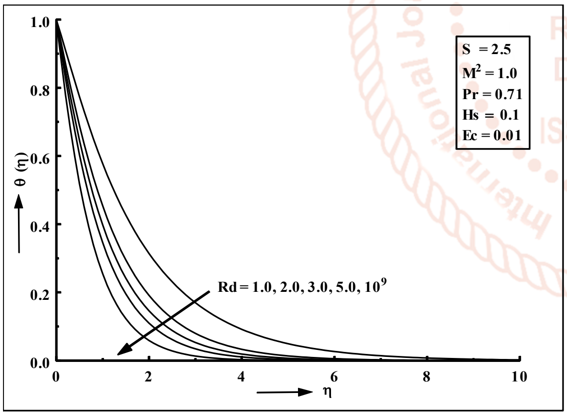

Figure 10. Dimensionless temperature profiles for various values of *Rd*

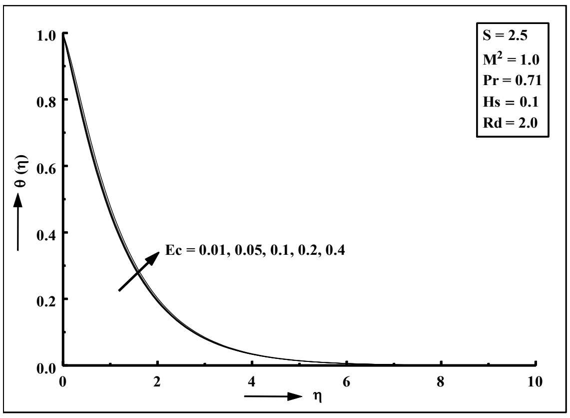

Figure 11. Dimensionless temperature profiles For various values of *Ec*

Table1. Skin friction coefficient $F''(0)$ for different values of S

S	M²	Present result	Bhattacharyya [18]
2.0	2.0	2.414214	2.414300
3.0		3.302775	3.302750
4.0		4.236068	4.236099

Table2. Comparison of numerical and analytical values of $F''(0)$

S	M²	Numerical Method	Analytical Method
2.0	1.0	2.000000	2.00000
2.5		2.500000	2.50000
3.0		3.000000	3.00000
4.0		4.000000	4.00000
4.5		4.500000	4.50000
2.5	0.0	2.001000	2.00000
	1.0	2.500000	2.50000
	3.0	3.137459	3.13750
	5.0	3.608849	3.60850

Table3. Comparison of numerical and analytical results of - $\theta'(0)$ for various physical parameters with *S* = 2.5, M² = 1.0, *Pr* = 0.71, *Hs* = 0.05, *Rd* = 2.0, *Ec* = 0.01

Physical parameters		Numerical Method	Analytical Method
S	2.0	0.281597	0.281597
	2.5	0.638486	0.638486
	3.0	0.932549	0.932550
	3.5	1.198544	1.198550
	4.0	1.449131	1.449131
M²	0.0	0.554714	0.554715
	1.0	0.638486	0.638486
	3.0	0.706830	0.706830
	5.0	0.742207	0.742208
	7.0	0.765438	0.765438
Pr	0.71	0.638486	0.638486
	1.00	0.962278	0.962279
	1.50	1.565711	1.565710
	2.30	2.608395	2.608390
	7.00	9.338060	9.338060
Hs	0.00	0.667866	0.667866
	0.05	0.638486	0.638486
	0.08	0.619929	0.619930
	0.10	0.607121	0.607121
	0.30	0.449650	0.449650
Rd	1.0	0.427164	0.427165
	2.0	0.638486	0.638486
	3.0	0.757715	0.757716
	5.0	0.886851	0.886852
	10^9	1.177781	1.177780
Ec	0.01	0.638486	0.638486
	0.20	0.514117	0.514117
	0.40	0.383203	0.383203
	0.60	0.252288	0.252289
	0.80	0.121374	0.121375

Figure 10 elucidates the effect of Radiation parameter on temperature distribution. It is evident that the effect of Radiation parameter is to lower the temperature for its ascending values with higher temperature at the wall. It is clearly seen that the Radiation parameter decreases the thermal boundary layer thickness. The radiation should be at its minimum in order to facilitate the cooling process.

Figure 11 portrays the temperature distribution for various values of Eckert number. It is evident that the thermal boundary layer is broadened as a result of step up in the values of Eckert number, which conveys the fact that the dissipative energy becomes more important with an enhancement in temperature.

To validate the solution obtained numerically, the numerical values of Skin friction coefficient for different values of S are compared with the results obtained by Bhattacharyya [18] and are shown in Table 1. Good agreement is observed between these two results.

Table4. Non-dimensional rate of heat transfer for various physical parameters

S	M^2	Pr	Hs	Rd	Ec	$\frac{Nu_x}{\sqrt{Re_x}}$
2.0 2.5 3.0 3.5 4.0	1.0	0.71	0.1	2.0	0.01	0.375393 1.011869 1.517016 1.968265 2.390873
2.5	0.0 1.0 3.0 5.0 7.0	0.71	0.1	2.0	0.01	0.864323 1.011869 1.130880 1.192080 1.232134
2.5	1.0	0.71 1.00 1.50 2.30 7.00	0.1	2.0	0.01	1.011869 1.551747 2.558023 4.297410 15.521040
2.5	1.0	0.71	0.0 0.1 0.2 0.3 0.4	2.0	0.01	1.113110 1.011869 0.894230 0.749416 0.544803
2.5	1.0	0.71	0.1	1.0 2.0 3.0 5.0 10^9	0.01	0.922676 1.011869 1.049265 1.083753 1.146642
2.5	1.0	0.71	0.1	2.0	0.01 0.05 0.10 0.20 0.40	1.011869 0.967961 0.913078 0.803313 0.583780

In order to ensure the accuracy of numerical scheme, the numerical and analytical values of $F''(0)$ and $-\theta'(0)$ for various physical parameters are compared and elucidated in Tables 2 and 3 respectively. The table values show that the numerical results are in excellent agreement with the analytical results obtained.

The numerical values of non-dimensional rate of heat transfer for various physical parameters are displayed in Table 4. From Table 4, it is inferred that the increasing values of the Suction parameter, Magnetic parameter and Prandtl number increases the non-dimensional rate of heat transfer. It is further noted that the dimensionless rate of heat transfer increases due to increasing effect of Radiation parameter whereas it gets reduced due to increasing Heat generation parameter and Eckert number.

6. Conclusion

A numerical study has been conducted on MHD flow and thermal transport characteristics over a shrinking surface in the presence of energy dissipation due to Joule heating and viscous dissipation, heat generation and thermal radiation. The Rosseland diffusion flux model has been used to simulate the radiative heat flux. A parametric study is performed to illustrate the influence of physical parameters on velocity and temperature distributions. Numerical results are obtained for skin friction coefficient as well as local Nusselt number for some values of governing parameters. The numerical scheme applied is validated by comparing the numerical results obtained with the results found using analytical method. In the absence of radiation and dissipation effects (Rd = 10^9, Ec = 0.0) , the author's results for constant temperature are identical to that of Bhattacharyya [18]. All the profiles tend to zero asymptotically which satisfies the far field boundary conditions. Some of the important findings drawn from the present analysis are listed as follows:

- The longitudinal velocity accelerates due to the increasing values of Suction parameter. When the shrinking surface is prescribed with no uniform temperature the Suction parameter diminishes the temperature and hence its boundary layer thickness also decreases. Moreover, the increasing effect of Suction parameter is to enhance the dimensionless rate of heat transfer.
- The increasing effect of Magnetic parameter is to accelerate the dimensionless longitudinal velocity and skin friction coefficient. The temperature of the fluid is suppressed when the strength of the Magnetic parameter is high whereas it enhances the non-dimensional rate of heat transfer.
- The increasing effect of Prandtl number is to reduce the temperature of the fluid and hence the thermal boundary layer thickness becomes thinner. More amount of heat is transferred from the surface due to increase in Prandtl number.
- Increase in magnitude of Heat generation parameter is to generate temperature and decrease the thermal boundary layer thickness. The dimensionless rate of heat transferred to the fluid is observed to be less when the influence of Heat generation parameter is high.
- The rise in the value of Radiation parameter diminishes the temperature of the fluid and hence reduction in the thermal boundary layer thickness is observed. The dimensionless rate of heat transfer is more due to the significant effect of Radiation parameter.
- The energy dissipation (being indicated by Eckert number) due to Joule heating and viscous dissipation has the tendency to thicken the thermal boundary

layer, so as to raise the temperature and reduce the dimensionless heat transfer rate from the surface.

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