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RP-108: Formulation of Solutions of Standard Congruence of Higher Degree modulo a Multiple of Composite Power Integer

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ABSTRACT
In this study, the solutions of a standard congruence of higher degree modulo a multiple of composite power integer is considered for formulation and is formulated. The formula developed is proved and verified true. Oral calculation of solutions are also possible. Formulation is the merit of the paper.

KEYWORDS: *Binomial expansion, Congruence of Higher Degree, Composite modulus, Formulation*

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INRODUCTION

Congruence is a topic of Number Theory. The author has written more than 90 research papers on the formulation of congruence such as quadratic, cubic, bi-quadratic and congruence of higher degree of prime and composite modulus and published in different reputed International Journals. Even the author found a special type of congruence of higher degree of composite modulus yet not formulated. The author wished to consider such congruence and tried to find a successful formulation of the solutions of the congruence.

Books of Number theory containing congruence of higher degree are referred [1], [2]. No formulation of the said congruence is found. Only the research papers of the author published are seen [3], [4], …….[9]. Here the author wishes to formulate the congruence of the type: $x^n \equiv a^n \ (mod\ b.a^m);\ m \geq n$.

PROBLEM-STATEMENT

Here the problem is-
"To find formulation of the solutions of the congruence of higher degree of the type:
$x^n \equiv a^n \ (mod\ b.a^m);\ m \geq n,$
in two cases:
Case-I: when n is even positive integer,
Case-II: when n is odd positive integer".

ANALYSIS & RESULT

Case-I: Let n be an even positive integer and consider the congruence under consideration: $x^n \equiv a^n \ (mod\ b.a^m);\ m > n.$
Also consider that $x \equiv b.a^{m-n+1}k \pm a \ (mod\ b.a^m)$
Then, $x^n \equiv (b.a^{m-n+1}k \pm a)^n \ (\text{mod } b.a^m)$

$$\equiv (b.a^{m-n+1}k)^n + n(b.a^{m-n+1}k)^{n-1}.a + \frac{n.(n-1)}{2}(b.a^{m-n+1}k)^{n-2}.a^2 + \cdots \ldots \ldots \ldots .. + n.b.a^{m-n+1}k.a^{n-1} + a^n \ (mod\ b.a^m)$$

$$\equiv a^n + b.a^m(\ldots\ldots) \ (mod\ b.a^m)$$

$$\equiv a^n \ (mod\ b.a^m)$$

Thus, it can be said that $x \equiv b.a^{m-n+1}k \pm a \ (mod\ b.a^m)$ is a solution of the congruence.
But, if one has the value $k = a^{n-1}$, then $x \equiv b.a^{m-n+1}k \pm a \ (mod\ b.a^m)$ reduces to

$$x \equiv b.a^{m-n+1}.a^{n-1} \pm a \ (mod\ a^m)$$

$$\equiv a^m \pm a \ (mod\ b.a^m)$$

$$\equiv 0 \pm a \ (mod\ b.a^m)$$

$$\equiv \pm a \ (mod\ b.a^m),$$

which are the same solutions as for k=0. Similarly, it can also be seen that for next higher values of k, the corresponding solutions repeats as for k=1, 2, ……., $(a^{n-1} - 1)$.

Therefore, the required solutions are given by
$x \equiv b.a^{m-4}k \pm a \ (mod\ b.a^m); k = 1, 2, \dots\dots, (a^{n-1} - 1)$.

It is also seen that for a single value of k, the congruence has two solutions and here k has a^{n-1} values. Thus, total number of solutions are definitely $2a^{n-1}$ for even n.

Case-II: Let n be an odd positive integer and consider the congruence under consideration: $x^n \equiv a^n \ (mod\ b.a^m);\ m > n$.

Also consider that $x \equiv b.a^{m-n+1}k \pm a \ (mod\ b.a^m)$
Then, $x^n \equiv (b.a^{m-n+1}k \pm a)^n$ (mod $b.a^m$)
$\equiv \pm a^n + b.a^m(\dots\dots) \ (mod\ b.a^m)$
$\equiv \pm a^n \ (mod\ b.a^m)$

Thus, $x \equiv b.a^{m-n+1}k \pm a \ (mod\ b.a^m)$ cannot be the solutions.
But if $x \equiv b.a^{m-n+1}k + a \ (mod\ b.a^m)$, then : $x^n \equiv a^n \ (mod\ b.a^m)$.

Thus, it can be said that $x \equiv b.a^{m-n+1}k + a \ (mod\ b.a^m)$ is a solution of the congruence.

But, if one has the value $k = a^{n-1}$, then $x \equiv b.a^{m-n+1}k + a \ (mod\ b.a^m)$ reduces to
$x \equiv b.a^{m-n+1}.a^{n-1} + a \ (mod\ b.a^m)$
$\equiv b.a^m + a \ (mod\ b.a^m)$
$\equiv 0 + a \ (mod\ b.a^m)$
$\equiv a \ (mod\ b.a^m)$,

which is the same solution as for k=0. Similarly, it can also be seen that for next higher values of k, the corresponding solutions repeats as for k=1, 2, …….,$(a^{n-1} - 1)$.

Therefore, the required solutions are given by
$x \equiv b.a^{m-n+1}k + a \ (mod\ b.a^m); k = 1, 2, \dots\dots, (a^{n-1} - 1)$.

It is also seen that for a single value of k, the congruence has one solution and here k has a^{n-1} values. Thus, total number of solutions are definitely a^{n-1} for odd n.

One thing is notable that if $m \le n$, then the congruence reduces to $x^n \equiv a^n \ (mod\ b.a^n)$
$i.e\ .x^n \equiv 0 \ (mod\ b.a^n)$.

It is seen that for $x \equiv at \ (mod\ b.a^n)$, are the solutions, t being a positive integer.

ILLUSTRATIONS
Consider the congruence $x^5 \equiv 243 \ (mod\ 3645)$.
It can be written as: $x^5 \equiv 3^5 (mod\ 5.3^6)$.
It is of the type: $x^n \equiv a^n (mod\ b.a^m); m \ge n$.
As n is an odd positive integer, the congruence has only $a^{n-1} = 3^4 = 81\ solutions$.

Solutions are given by
$x \equiv b.a^{m-n+1}k + a \ (mod\ .ba^m); k = 0, 1, 2, \dots\dots.(a^{n-1} - 1)$.
$\equiv 5.3^{6-5+1}k + 3 \ (mod\ 5.3^6); k = 0, 1, 2, 3, \dots\dots..79, 80 \ (mod\ 3645)$
$\equiv 5.3^2k + 3 \ (mod\ 3645)$
$\equiv 45k + 3 \ (mod\ 3645)$
$\equiv 0 + 3; 45 + 3; 90 + 3; 135 + 3; \dots\dots\dots\dots..;\ 3555 + 3; 3600 + 3 (mod 3645)$
$\equiv 3, 48, 93, 138, \dots\dots\dots, 3558, 3603 \ (mod\ 3645)$.

These are the required 81 solutions.

Consider one more example. Let it be $x^6 \equiv 6^6 \ (mod\ 3.6^8)$.
It is of the type: $x^n \equiv a^n (mod\ b.a^m); m > n$.

As $n = 6$, *an even positive integer*, *it has* $2a^{n-1} = 2.6^5 = 2.1296 = 15{,}515$ *incongruent* solutions. Those solutions are given by
$x \equiv b.a^{m-n+1}k \pm a \ (mod\ b.a^m); k = 0, 1, 2, \dots\dots.(a^5 - 1$.

$\equiv 3.6^{8-6+1}k \pm 6 \ (mod\ 3.6^8); k = 0, 1, 2, 3, \dots\dots\dots\dots\dots\dots, (7776 - 1)$.
$\equiv 3.6^3k \pm 6 \ (mod\ 5038848)$
$\equiv 648k \pm 6 \ (mod\ 5038848)$
$\equiv 0 \pm 6; 648 \pm 6; 1296 \pm 6; \dots\dots\dots\dots\dots\dots..$
$\equiv \pm 6; 642, 654; 1290, 1302; \dots\dots\dots\dots\dots\dots\ (mod\ 5038848)$.
These are the 7775 incongruent solutions.

COLCLUSION
Thus, it can be concluded that the standard congruence of higher degree of the type
$x^n \equiv a^n \ (mod\ b.a^m); m \ge n$, has $2a^{n-1}$ solutions, if n is an even positive integer, given by $x \equiv b.a^{m-n+1}k \pm a \ (mod\ b.a^m); k = 0, 1, 2, \dots..(a^{n-1} - 1)$.

But the congruence has only a^{n-1} solutions, if n is an odd positive integer, given by
$x \equiv b.a^{m-n+1}k + a \ (mod\ b.a^m); k = 0, 1, 2, \dots..(a^{n-1} - 1$.

MERIT OF THE PAPER
In this study, a standard congruence of higher degree of composite modulus is formulated. The formula is tested and verified by true. The solutions can be obtained in a short time. Formulation is the merit of the paper.

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